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ÖNSÖZ

Değerli YÖS adayları;

Türkiye'deki üniversitelerde eğitim görmek isteyen yabancı uyruklu öğrencilerin Yabancı Uyruklu Öğrenci Sınavı (**YÖS**)'na girmeleri gerekmektedir.

YÖS, temel öğrenme becerileri testi ve **Türkçe testi** olmak üzere iki ayrı testten oluşmaktadır. Öğrencilerin, temel öğrenme becerileri testinde yer alan matematik sorularını doğru yanıtlaması, bu sınavlardan yüksek puan almalarını sağlayacaktır.

YÖS'e hazırlanan öğrencilere **18 yıldır** kurs, yayın desteği sağlayan ve rehberlik hizmetleri sunan **Metropol Eğitim Kurumları**, öğrencilerin bu gereksinimlerini karşılamak amacıyla materyaller(kitap, deneme sınavı ve testler) hazırlamaktadır.

Bu kitapta; **Polinomlar, İkinci Dereceden Denklemler, Eşitsizlikler, Fonksiyonlar, Trigonometri, Karmaşık Sayılar, Logaritma, Tümevarım, Diziler ve Seriler, Permütasyon, Kombinasyon ve Binom Açılımı, Matris ve Determinant, Limit, Türev ve İntegral** konularını içeren **744** çözümlü örnek, **324** alıştırma, **504** test sorusu ve **2001 – 2013** yılları arasındaki **YÖS** soruları da çözümlü olarak yer almaktadır.

Sınavlara hazırlanan tüm öğrencilere başarılar diliyor, **"TÜRKİYE'YE HOŞGELDİNİZ!"** diyoruz.

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PREFACE

Dear YÖS candidates;

Foreign students, who want to study at Turkish universities, are required to take the Examination for Foreign Students (**YÖS**).

YÖS consists of two different sections, including the basic learning skills test and Turkish language test. The correct answering of the mathematical questions in the basic skills test will help students have a significantly high score.

Providing **YÖS** students with a comprehensive training support both by courses and publications for over **18 years**, **Metropol Educational Institutions** prepare materials (books, trial exams, and practice tests) that comply with the educational needs of these students.

This book includes **Polynomials, Quadratic Equations, Inequalities, and Functions, Trigonometry, Complex Numbers, Logarithm, Induction, Sequences, and Series, Permutation, Combination, and Binomial Expansion, Matrix and Determinant, Limits, Derivative, and Integral.** This book includes **744** examples with solutions, **324** examples, and **504** test questions. Moreover, there are the **YÖS** and **TCS** questions with solutions between the years **2001** and **2013**.

We wish you success in exams to all students. **Welcome to TURKEY.**

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ÜNİTE 1

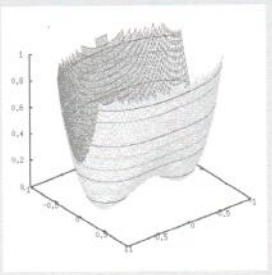
UNIT 1

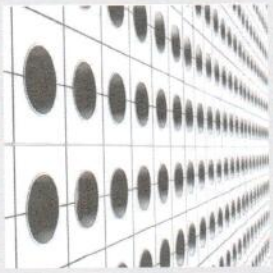
Polinomlar, ikinci dereceden denklemler, eşitsizlikler ve fonksiyonlar bu ünitede ayrıntılı anlatılmıştır. Polinom, fonksiyonun özel durumudur. Bu nedenle, fonksiyonları iyi öğrendiyseniz, polinomlarda zorlanmazsınız. Benzer şekilde, ikinci dereceden denklemler, eşitsizlikler ve fonksiyonlar, birinci dereceden denklem ve eşitsizliklerin genelleştirilmiş halidir. Ayrıca çarpanlara ayırma konusuna, ikinci dereceden denklemler konusunda kısaca değinilmiştir.

Bu ünite; 130 çözümlü örnek, 54 alıştırma, ve 88 test sorusundan oluşmaktadır.

In this unit, polynomials, quadratic equations, inequalities and functions are explained in detailed. Polynomial is special situation of function. Moreover, if you learned functions, polynomials are very easy. Similarly, quadratic equations, inequalities and functions are generalized of the first order equations, inequalities. Also factorization is explanation shortly in quadratic equation.

This unit includes; 130 examples, 54 exercises, and 88 test questions.





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BÖLÜM 1
CHAPTER

TARİHSEL NOT / HISTORICAL NOTE

Diophantus of Alexandria [MÖ/BC 250]

Yunan matematikçi. Bir veya daha fazla bilinmeyene sahip denklemlerin çözümü için geometrik yöntemleri önermiştir. Arithmetica adlı kitabında yüzden fazla problemin çözümü için örnekler vermiş, çözüm için genel bir yöntem göstermiştir. Bu denklemler, **Diophantine denklemleri** olarak bilinir.

Greek mathematician whose work displayed an algebraic approach to the solution of equations in one or more unknowns, unlike earlier Greek methods that were more geometrical. In the books of Arithmetica that survive, particular numerical examples of more than 100 problems are solved, probably to indicate the general methods of solution. These are mostly of the kind now referred to as **Diophantine equations.**

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POLYNOMIALS

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POLİNOMLAR
POLYNOMIALS

BÖLÜM 1
CHAPTER

Tanım : $a_0, a_1, a_2 \ldots\ldots a_n \in R$, $n \in N$ olmak üzere,
$P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ biçiminde tanımlanan ifadelere *polinom* denir.

NOT

Polinomlar, fonksiyonların $n \in N$ durumundaki özel halidir.

✓ *$x^{1/2} + x^2$ fonksiyondur, polinom değildir.*

✓ *$x + x^2$ fonksiyondur ve polinomdur.*

Etkinlik

a. $P(x) = 4x^6 - 3x^{-2} + 6x + 1$ ifadesi bir polinom değildir. Çünkü, $-3x^{-2}$'li terimde x'in üssü olan "–2" doğal sayı değildir.

b. $P(x) = 6x^6 + 7x^3 + x^{\frac{1}{2}} + 3$ ifadesi bir polinom değildir. Çünkü $x^{\frac{1}{2}}$'li terimde x'in üssü olan "$\frac{1}{2}$" doğal sayı değildir.

c. $P(x) = 4x^2 + 2x - 3x^3$ ifadesi bir polinomdur.

▼
- ✓ Polinomun terimleri: $a_0, a_1x, a_2x^2, \ldots, a_nx^n$
- ✓ Polinomun katsayıları: $a_0, a_1, a_2, \ldots, a_n$
- ✓ Başkatsayı: a_n
- ✓ Sabit terim: a_0
- ✓ **Derece:** Polinomda yer alan terimlerden derecesi en büyük olan terimin derecesi, polinomun derecesidir ve der (P(x)) ile gösterilir.

Etkinlik

a. $3x^2 - 4x^3 + 5$ polinomunun,
- ✓ terimleri: $3x^2, -4x^3, 5$
- ✓ katsayıları: 3, –4, 5
- ✓ başkatsayısı: –4
- ✓ sabit terimi: 5
- ✓ derecesi: 3

b. $x^2 - 3x + 4$ polinomunun derecesi, 2, 1, 0 derecelerinden en büyüğüdür. O halde, bu polinomun derecesi 2'dir.

c. $3x^2y + 2x^3y^2 + 3xy^2$ polinomunun derecesi 2 + 1, 3 + 2, 1 + 2 toplamlarından en büyüğüdür. O halde, bu polinomun derecesi 5'tir.

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NOTE

Polynomial is a special case of the functions in situation $n \in N$.

✓ *$x^{1/2} + x^2$ is function, is not polynomial.*

✓ *$x + x^2$ is function and polynomial.*

Definition: A polynomial is an expression of the form $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ where $a_0, a_1, a_2 \ldots a_n \in R$ and $n \in N$.

Activity

1. $P(x) = 4x^6 - 3x^{-2} + 6x + 1$ is not a polynomial. Because the power of the term $-3x^{-2}$ is -2, but -2 is not a natural number .

2. $P(x) = 6x^6 + 7x^3 + x^{\frac{1}{2}} + 3$ is not a polynomial. Because the power of the $\frac{1}{2}$ term x is "$\frac{1}{2}$", "$\frac{1}{2}$" is not a natural number.

3. $P(x) = 4x^2 + 2x - 3x^3$ is a polynomial.

▼
- ✓ Terms of polynomial: $a_0, a_1x, a_2x^2, \ldots, a_nx^n$
- ✓ Coefficients of polynomial: $a_0, a_1, a_2, \ldots, a_n$
- ✓ Leading coefficient: a_n
- ✓ Constant term: a_0
- ✓ **Degree:** The degree of a polynomial is the greatest of the degrees of its terms that is denoted by der (P(x)).

Activity

a. The polynomial $3x^2 - 4x^3 + 5$,
- ✓ terms: $3x^2, -4x^3, 5$
- ✓ coefficients: $3, -4, 5$
- ✓ leading coefficient: -4
- ✓ constant term: 5
- ✓ degree: 3

b. The degree of the polynomial $x^2 - 3x + 4$ is the greatest of the degrees 2, 1, 0, thus it is 2.

c. The degree of the polynomial $3x^2y + 2x^3y^2 + 3xy^2$ is the greatest of $2 + 1, 3 + 2, 1 + 2$; thus it is 5.

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Sabit Polinom

$a_0 \neq 0$ ve $a_1 = a_2 = \ldots = a_n = 0$ ise, P(x) sabit bir polinomdur.

NOT

Sabit bir polinomun derecesi sıfırdır.

Sıfır Polinomu

$a_n = a_{n-1} = \ldots = a_0 = 0$ ise, P(x) bir sıfır polinomudur.

NOT

Bir sıfır polinomunun derecesi belirsizdir.

ÖRNEK 1

P(x) bir polinomdur.

$P(x) = ax^6 + (a - b + 3)x^{-3} + (a + b - 9)x^{-2} + bx \Rightarrow$ **P(x) = ?**

A) $2x^6 + 3x$ B) $3x^6 + 6x$ C) $5x^6 + 4x$ D) $2x^6 + 5x$ E) $3x^6 + 9x$

ÇÖZÜM

Yanıt **B**

$$\begin{array}{r} a - b + 3 = 0 \\ +\ a + b - 9 = 0 \\ \hline 2a - 6 = 0 \end{array}$$

$a = 3$ ve $b = 6 \Rightarrow P(x) = 3x^6 + 6x$

PEKİŞTİRME

P(x) bir polinomdur.

$P(x) = 3x^{\frac{n^2+8}{n}} + 4x \Rightarrow \sum n = ?$

YANIT : 15

ÖRNEK 2

$P(x) = x^3 - x^{n-2} + 3x^4$,

$der(P(x)) = 5 \Rightarrow$ **n = ?**

A) 1 B) 3 C) 4 D) 5 E) 7

ÇÖZÜM

Yanıt **E**

$der(P(x)) = 5 \Rightarrow n - 2 = 5$

$n = 7$

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NOTE

The degree of a constant polynomial is zero.

Constant Polynomial

If $a_0 \neq 0$ and $a_1 = a_2 = \ldots = a_n = 0$, P(x) is a constant polynomial.

NOTE

The degree of a zero polynomial is indefinite.

Zero Polynomial

If $a_n = a_{n-1} = \ldots = a_0 = 0$; P(x) is a zero polynomial.

REINFORCE

P(x) is a polynomial.

$P(x) = 3x^{\frac{n^2+8}{n}} + 4x \Rightarrow \sum n = ?$

ANSWER : 15

EXAMPLE 1

P(x) is a polynomial.

$P(x) = ax^6 + (a - b + 3)x^{-3} + (a + b - 9)x^{-2} + bx \Rightarrow$ **P(x) = ?**

A) $2x^6 + 3x$ B) $3x^6 + 6x$ C) $5x^6 + 4x$

D) $2x^6 + 5x$ E) $3x^6 + 9x$

SOLUTION

The answer is **B**

$$\begin{array}{r} a - b + 3 = 0 \\ +\ a + b - 9 = 0 \\ \hline 2a - 6 = 0 \end{array}$$

$a = 3$ and $b = 6 \Rightarrow P(x) = 3x^6 + 6x$

EXAMPLE 2

$P(x) = x^3 - x^{n-2} + 3x^4$,

$der(P(x)) = 5 \Rightarrow$ **n = ?**

A) 1 B) 3 C) 4 D) 5 E) 7

SOLUTION

The answer is **E**

$der(P(x)) = 5 \Rightarrow n - 2 = 5$

$n = 7$

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ÖRNEK 3

P(x) bir polinomdur.

$P(x) = 3x^{n-2} + 5 \cdot x^{2-n} + 9x \Rightarrow$ **P(3) = ?**

A) 30 B) 32 C) 35 D) 37 E) 38

ÇÖZÜM

Yanıt **C**

P(x) bir polinom $\Rightarrow$ $n - 2 \geq 0$ $\quad 2 - n \geq 0$

$n \geq 2$ $\quad 2 \geq n$

$\Rightarrow n = 2$

$P(x) = 3 + 5 + 9x = 8 + 9x$

$P(3) = 8 + 9 \cdot 3 = 35$

ÖRNEK 4

$P(x) = -3x^3 + 4x^2 - x + m + 1,$

$P(2) = 4 \Rightarrow$ **m = ?**

A) 9 B) 11 C) 13 D) 15 E) 17

ÇÖZÜM

Yanıt **C**

$P(2) = -3 \cdot 2^3 + 4 \cdot 2^2 - 2 + m + 1 = 4$ [x yerine 2 yaz.]

$-3 \cdot 8 + 4 \cdot 4 - 2 + m + 1 = 4$ [İndirge.]

$-24 + 16 - 2 + m + 1 = 4$ [Sadeleştir.]

$-9 + m = 4$ [Benzer terimleri topla.]

$m = 9 + 4$ [Her iki tarafa 4 ekle.]

$m = 13$ [Sonuç]

PEKİŞTİRME

$P(x) = 3x^2 + 4x - 5,$

$\Rightarrow P(1) = ?$

YANIT : 2

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EXAMPLE 3

P(x) is a polynomial.

$P(x) = 3x^{n-2} + 5 \cdot x^{2-n} + 9x \Rightarrow$ **P(3) = ?**

A) 30 B) 32 C) 35 D) 37 E) 38

SOLUTION

The answer is **C**

P(x) is a polynomial $\Rightarrow$ $n - 2 \geq 0$ $\quad 2 - n \geq 0$

$n \geq 2 \quad 2 \geq n$

$\Rightarrow n = 2$

$P(x) = 3 + 5 + 9x = 8 + 9x$

$P(3) = 8 + 9 \cdot 3 = 35$

REINFORCE

$P(x) = 3x^2 + 4x - 5$,

$\Rightarrow P(1) = ?$

EXAMPLE 4

$P(x) = -3x^3 + 4x^2 - x + m + 1$,

$P(2) = 4 \Rightarrow$ **m = ?**

A) 9 B) 11 C) 13 D) 15 E) 17

ANSWER : 2

SOLUTION

The answer is **C**

$P(2) = -3 \cdot 2^3 + 4 \cdot 2^2 - 2 + m + 1 = 4$ [Substitute 2 for x.]

$-3 \cdot 8 + 4 \cdot 4 - 2 + m + 1 = 4$ [Reduce.]

$-24 + 16 - 2 + m + 1 = 4$ [Simplify.]

$-9 + m = 4$ [Combine like terms.]

$m = 9 + 4$ [Add 4 to both sides.]

$m = 13$ [Conclusion]

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ÖRNEK 5

$P(x) = 5x^6 - 4x^3 + 11 \Rightarrow P(\sqrt[3]{2}) = ?$

A) 20 B) 23 C) 26 D) 29 E) 31

ÇÖZÜM

Yanıt **B**

$P(\sqrt[3]{2}) = 5\cdot(\sqrt[3]{2})^6 - 4\cdot(\sqrt[3]{2})^3 + 11$

$= 5 \cdot 4 - 4 \cdot 2 + 11$

$= 20 - 8 + 11 = 23$

PEKİŞTİRME

$P(2x + 3) = 3x^2 - 4x + 1,$

$\Rightarrow P(1) = ?$

YANIT : 8

ÖRNEK 6

$n \in Z^+,$

$P(x) = 5\cdot(x - 2)^{2n} - 7\cdot(2 - x)^{2n-1} \Rightarrow P(1) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM

Yanıt **A**

$P(1) = 5 \cdot (1 - 2)^{2n} - 7 \cdot (2 - 1)^{2n-1}$

$= 5 \cdot (-1)^{2n} - 7 \cdot 1^{2n-1}$

$= 5 \cdot 1 - 7 \cdot 1$

$= 5 - 7 = -2$

PEKİŞTİRME

$P(3x + 1) = 6x + 9,$

$\Rightarrow P(x) = ?$

YANIT : 2x + 7

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REINFORCE

$P(2x + 3) = 3x^2 - 4x + 1,$

$\Rightarrow P(1) = ?$

ANSWER : 8

EXAMPLE 5

$P(x) = 5x^6 - 4x^3 + 11 \Rightarrow P(\sqrt[3]{2}) = ?$

A) 20 B) 23 C) 26 D) 29 E) 31

SOLUTION

The answer is **B**

$$P(\sqrt[3]{2}) = 5\cdot(\sqrt[3]{2})^6 - 4\cdot(\sqrt[3]{2})^3 + 11$$
$$= 5 \cdot 4 - 4 \cdot 2 + 11$$
$$= 20 - 8 + 11 = 23$$

REINFORCE

$P(3x + 1) = 6x + 9,$

$\Rightarrow P(x) = ?$

ANSWER : 2x + 7

EXAMPLE 6

$n \in Z^+,$

$P(x) = 5\cdot(x - 2)^{2n} - 7\cdot(2 - x)^{2n-1} \Rightarrow P(1) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

SOLUTION

The answer is **A**

$$P(1) = 5 \cdot (1 - 2)^{2n} - 7 \cdot (2 - 1)^{2n-1}$$
$$= 5 \cdot (-1)^{2n} - 7 \cdot 1^{2n-1}$$
$$= 5 \cdot 1 - 7 \cdot 1$$
$$= 5 - 7 = -2$$

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POLİNOMLARIN EŞİTLİĞİ

Dereceleri aynı olan polinomların eşit olması için, aynı dereceli terimlerin katsayıları eşit olmalıdır.

$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$

$Q(x) = b_n x^n + b_{n-1}x^{n-1} + \ldots + b_1 x + b_0$

$P(x) = Q(x) \Rightarrow a_n = b_n\ ,\ a_{n-1} = b_{n-1} \ldots,\ a_1 = b_1\ ,\ a_0 = b_0$

ÖRNEK 7

$P(x) = (x^2 + 1)\cdot(x + 3)$,

$Q(x) = x^3 + ax^2 + bx + c$,

$P(x) = Q(x) \Rightarrow$ **a + b + c = ?**

A) 30 B) 25 C) 20 D) 15 E) 7

ÇÖZÜM

Yanıt **E**

$(x^2 + 1)\cdot(x + 3) = x^3 + ax^2 + bx + c$

$x^3 + 3x^2 + 1x + 3 = x^3 + ax^2 + bx + c$

$a = 3,\ b = 1,\ c = 3 \Rightarrow a + b + c = 3 + 1 + 3 = 7$

ÖRNEK 8

$P(x) = (x-2)(x^2 + px + 3) + x - 5$

$Q(x) = x^3 + 3x^2 + bx + c$

$P(x) = Q(x) \Rightarrow$ **b + p = ?**

A) –5 B) –4 C) –3 D) –2 E) –1

ÇÖZÜM

Yanıt **E**

$P(x) = x^3 + x^2(p-2) + (4 - 2p)x - 11$

$Q(x) = x^3 + 3x^2 + bx + c$

$$P(x) = Q(x) \Rightarrow \begin{cases} p - 2 = 3 \Rightarrow p = 5 \\ 4 - 2p = b \Rightarrow b = 4 - 10 = -6 \\ c = -11 \end{cases}$$

$b + p = 5 - 6 = -1$

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EQUALITY of POLYNOMIALS

To the polynomials with same degree are equal, the coefficients of the terms with same degree must be equal.

$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$

$Q(x) = b_n x^n + b_{n-1}x^{n-1} + \ldots + b_1 x + b_0$

$P(x) = Q(x) \Rightarrow a_n = b_n\ ,\ a_{n-1} = b_{n-1} \ldots,\ a_1 = b_1\ ,\ a_0 = b_0$

EXAMPLE 7

$P(x) = (x^2 + 1)\cdot(x + 3)$,

$Q(x) = x^3 + ax^2 + bx + c$,

$P(x) = Q(x) \Rightarrow$ **a + b + c = ?**

A) 30 B) 25 C) 20 D) 15 E) 7

SOLUTION

The answer is **E**

$(x^2 + 1)\cdot(x + 3) = x^3 + ax^2 + bx + c$

$x^3 + 3x^2 + 1x + 3 = x^3 + ax^2 + bx + c$

$a = 3,\ b = 1,\ c = 3 \Rightarrow a + b + c = 3 + 1 + 3 = 7$

EXAMPLE 8

$P(x) = (x-2)(x^2 + px + 3) + x - 5$

$Q(x) = x^3 + 3x^2 + bx + c$

$P(x) = Q(x) \Rightarrow$ **b + p = ?**

A) –5 B) –4 C) –3 D) –2 E) –1

SOLUTION

The answer is **E**

$P(x) = x^3 + x^2(p-2) + (4 - 2p)x - 11$

$Q(x) = x^3 + 3x^2 + bx + c$

$$P(x) = Q(x) \Rightarrow \begin{cases} p - 2 = 3 \Rightarrow p = 5 \\ 4 - 2p = b \Rightarrow b = 4 - 10 = -6 \\ c = -11 \end{cases}$$

$b + p = 5 - 6 = -1$

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POLİNOMLARDA DÖRT İŞLEM

Toplama ve Çıkarma İşlemi

İki polinom toplanırken (veya çıkarılırken) dereceleri aynı olan terimlerin katsayıları toplanır (veya çıkarılır) ve toplam (veya fark) polinomu elde edilir.

NOT

$p > q$
$der(P(x)) = p$
$der(Q(x)) = q$
$\Rightarrow der(P(x) \mp Q(x)) = p$

Etkinlik

a. $P(x) = 2x^3 - 4x^2 + 5x - 1$
$Q(x) = 6x^2 - 4x + 3$

$$P(x)+Q(x) = \underbrace{2x^3-4x^2+5x-1}_{P(x)} + \underbrace{6x^2-4x+3}_{Q(x)}$$
$$= 2x^3+2x^2+x+2$$
$$P(x)-Q(x) = \underbrace{2x^3-4x^2+5x-1}_{P(x)} - \underbrace{(6x^2-4x+3)}_{Q(x)}$$
$$= 2x^3-4x^2+5x-1-6x^2+4x-3$$
$$= 2x^3-10x^2+9x-4$$

b. $P(x) = x^4 - x^3 + 2x^2 + 3x + 2$
$Q(x) = -x^4 + 3x^2 - x + 5$

$$P(x)+Q(x) = \underbrace{x^4-x^3+2x^2+3x+2}_{P(x)} + \underbrace{(-x^4+3x^2-x+5)}_{Q(x)}$$
$$= -x^3+5x^2+2x+7$$
$$P(x)-Q(x) = \underbrace{x^4-x^3+2x^2+3x+2}_{P(x)} - \underbrace{(-x^4+3x^2-x+5)}_{Q(x)}$$
$$= x^4-x^3+2x^2+3x+2+x^4-3x^2+x-5$$
$$= 2x^4-x^3-x^2+4x-3$$

c. $P(x) = 3x^2 + 5x^3 - 4x$
$Q(x) = 4x^2 + x^3 - 3x$
$P(x) + Q(x) = 6x^3 + 7x^2 - 7x \Rightarrow der\,(P(x) + Q(x)) = 3$
$P(x) - Q(x) = -x^2 + 4x^3 - x \Rightarrow der\,(P(x) - Q(x)) = 3$

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FOUR OPERATIONS on POLYNOMIALS

Addition and Subtraction

To find the sum (or difference) of two polynomials, we add (or subtract) the coeffients of the combine like terms of the same degree.

NOTE

$p > q$

$der(P(x)) = p$

$der(Q(x)) = q$

$\Rightarrow der(P(x) \mp Q(x)) = p$

Activity

a. $P(x) = 2x^3 - 4x^2 + 5x - 1$

$Q(x) = 6x^2 - 4x + 3$

$$P(x) + Q(x) = \underbrace{2x^3 - 4x^2 + 5x - 1}_{P(x)} + \underbrace{6x^2 - 4x + 3}_{Q(x)}$$

$$= 2x^3 + 2x^2 + x + 2$$

$$P(x) - Q(x) = \underbrace{2x^3 - 4x^2 + 5x - 1}_{P(x)} - \underbrace{(6x^2 - 4x + 3)}_{Q(x)}$$

$$= 2x^3 - 4x^2 + 5x - 1 - 6x^2 + 4x - 3$$

$$= 2x^3 - 10x^2 + 9x - 4$$

b. $P(x) = x^4 - x^3 + 2x^2 + 3x + 2$

$Q(x) = -x^4 + 3x^2 - x + 5$

$$P(x) + Q(x) = \underbrace{x^4 - x^3 + 2x^2 + 3x + 2}_{P(x)} + \underbrace{(-x^4 + 3x^2 - x + 5)}_{Q(x)}$$

$$= -x^3 + 5x^2 + 2x + 7$$

$$P(x) - Q(x) = \underbrace{x^4 - x^3 + 2x^2 + 3x + 2}_{P(x)} - \underbrace{(-x^4 + 3x^2 - x + 5)}_{Q(x)}$$

$$= x^4 - x^3 + 2x^2 + 3x + 2 + x^4 - 3x^2 + x - 5$$

$$= 2x^4 - x^3 - x^2 + 4x - 3$$

c. $P(x) = 3x^2 + 5x^3 - 4x$

$Q(x) = 4x^2 + x^3 - 3x$

$P(x) + Q(x) = 6x^3 + 7x^2 - 7x \Rightarrow$ der $(P(x) + Q(x)) = 3$

$P(x) - Q(x) = -x^2 + 4x^3 - x \Rightarrow$ der $(P(x) - Q(x)) = 3$

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POLİNOMLARDA ÇARPMA

İki polinomun çarpımı; polinomlardan birinin her teriminin diğer polinomun her bir terimi ile ayrı ayrı çarpımlarından elde edilen terimlerin toplamına eşittir.

Etkinlik

$P(x) = x^2 - 3x + 4$, $Q(x) = x^3 - 2x^2 - 1$

$$P(x) \cdot Q(x) = (x^2 - 3x + 4) \cdot (x^3 - 2x^2 - 1)$$

$$= x^5 - 2x^4 - x^2 - 3x^4 + 6x^3 + 3x + 4x^3 - 8x^2 - 4$$

$$= x^5 - 5x^4 + 10x^3 - 9x^2 + 3x - 4$$

NOT

$der(P(x)) = p$, $der(Q(x)) = q \Rightarrow$

- ✓ $der(P(x) \cdot Q(x)) = p + q$
- ✓ $der(P^m(x)) = m \cdot p$
- ✓ $der(P(Q(x))) = p \cdot q$
- ✓ $der(P(x^m)) = m \cdot p$
- ✓ $der(x^m\, P(x)) = m + p$

ÖRNEK 9

$P(x) = (3x^2 - 2)^2$

$Q(x) = (2x^2 + 1)(x - 1)$

$P(x) \cdot Q(x) = \ldots\ldots + ax^2 + \ldots$

$\Rightarrow$ **a = ?**

A) 1 B) 2 C) 4 D) 12 E) 18

ÇÖZÜM

Yanıt **C**

$P(x) \cdot Q(x) = (3x^2 - 2)^2 \cdot (2x^2 + 1)(x - 1)$

$= 18x^7 + \ldots\ldots + 12x^2 - 8x^2 + \ldots\ldots$

$\Rightarrow$ $a = 12 - 8 = 4$

POLİNOMLARDA BÖLME

Bölme Özdeşliği

$$\begin{array}{r|l} P(x) & Q(x) \\ \hline K(x) & T(x) \end{array} \Rightarrow \begin{array}{l} P(x) = \text{Bölünen} \\ Q(x) = \text{Bölen} \\ T(x) = \text{Bölüm} \\ K(x) = \text{Kalan} \end{array}$$

1. $der\,[P(x)] \geq der\,[Q(x)]$
2. $der\,[K(x)] < der\,[Q(x)]$
3. $P(x) = Q(x) \cdot T(x) + K(x)$

NOT

$der(P(x)) = p$,

$der(Q(x)) = q \Rightarrow$

$$der\left(\frac{P(x)}{Q(x)}\right) = p - q$$

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NOTE

$der(P(x)) = p$, $der(Q(x)) = q \Rightarrow$

- ✓ $der(P(x) \cdot Q(x)) = p + q$
- ✓ $der(P^m(x)) = m \cdot p$
- ✓ $der(P(Q(x))) = p \cdot q$
- ✓ $der(P(x^m)) = m \cdot p$
- ✓ $der(x^m P(x)) = m + p$

MULTIPLICATION in POLYNOMIALS

The product of two polynomials equals the sum of the terms that multiplying each terms of one polynomial with each terms of the other polynomial.

Activity

$P(x) = x^2 - 3x + 4$, $Q(x) = x^3 - 2x^2 - 1$

$$P(x) \cdot Q(x) = (x^2 - 3x + 4) \cdot (x^3 - 2x^2 - 1)$$

$$= x^5 - 2x^4 - x^2 - 3x^4 + 6x^3 + 3x + 4x^3 - 8x^2 - 4$$

$$= x^5 - 5x^4 + 10x^3 - 9x^2 + 3x - 4$$

EXAMPLE 9

$P(x) = (3x^2 - 2)^2$

$Q(x) = (2x^2 + 1)(x - 1)$

$P(x) \cdot Q(x) = \ldots\ldots + ax^2 + \ldots$

$\Rightarrow$ **a = ?**

A) 1 B) 2 C) 4 D) 12 E) 18

SOLUTION

The answer is **C**

$P(x) \cdot Q(x) = (3x^2 - 2)^2 \cdot (2x^2 + 1)(x - 1)$

$= 18x^7 + \ldots\ldots + 12x^2 - 8x^2 + \ldots\ldots$

$\Rightarrow$ $a = 12 - 8 = 4$

NOTE

$der(P(x)) = p$,

$der(Q(x)) = q \Rightarrow$

$$der\left(\frac{P(x)}{Q(x)}\right) = p - q$$

DIVISION in POLYNOMIALS

Identity of Division

$$\begin{array}{r|l} P(x) & Q(x) \\ \hline - & T(x) \\ \hline K(x) & \end{array} \Rightarrow \begin{array}{l} P(x) = \text{Dividend} \\ Q(x) = \text{Divisor} \\ T(x) = \text{Quotient} \\ K(x) = \text{Remainder} \end{array}$$

1. $der\,[P(x)] \geq der\,[Q(x)]$
2. $der\,[K(x)] < der\,[Q(x)]$
3. $P(x) = Q(x) \cdot T(x) + K(x)$

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Bölme Yöntemleri

1. **Polinom Bölmesi:** Sayılardaki bölme işlemine benzer olarak yapılır. En büyük dereceli terim yok edilerek sonuca ulaşılır.

Etkinlik

$$\begin{array}{r|l} 2x^2 - x + 1 & x - 3 \\ -(2x^2 - 6x) & 2x + 5 \rightarrow T(x) \\ \hline 5x + 1 & \\ -(5x - 15) & \\ \hline 16 \rightarrow K(x) & \end{array}$$

2. **Yapay Bölme (Horner Yöntemi):** Bir polinomun katsayıları ve polinomunun böleninin derecesi dikkate alınarak bölme işlemi yapılır.

Etkinlik

	$2x^2$	$-x$	$+1$
$x - 3 = 0$	2	-1	1
$x = 3$		$2 \cdot 3 = 6$	$5 \cdot 3 = 15$
	2	$-1+6 = 5$	$15 + 1 = 16$
		$T(x) = 2x + 5$	$K(x) = 16$

NOT

Horner yöntemi; böleni yalnızca $x - a$ şeklinde olan bir polinom için kullanılabilir. Bu yöntem, böleni $x^2 - 7$ şeklinde olan ikinci dereceden bir polinom için kullanılamaz.

ÖRNEK 10

$$\begin{array}{r|l} 3x^2 + x - 1 & x + 1 \\ - & \hline K(x) & Q(x) \end{array} \Rightarrow Q(2) + K(x) = ?$$

A) 2 B) 3 C) 4 D) 5 E) 6

ÇÖZÜM

Yanıt **D**

Çözüm 1 [Polinom Bölmesi] :

$$\begin{array}{r|l} 3x^2 + x - 1 & x + 1 \\ -(-3x^2 \mp 3x) & 3x - 2 \rightarrow Q(x) \\ \hline -2x - 1 & \\ -(+2x + 2) & \\ \hline -3 \rightarrow K(x) & \end{array}$$

$Q(x) = 3x - 2 \Rightarrow Q(2) = 3 \cdot 2 - 2 = 4$

$Q(2) + K(x) = 4 + 1 = 5$

Çözüm 2 [Horner Yöntemi] :

	3	1	-1
$x + 1 = 0$		-3	2
$x = -1$	3	-2	1
	$Q(x) = 3x - 2$		$K(x) = 1$

$Q(2) + K(x) = 4 + 1 = 5$

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Division Methods

1. **Polynomial Division:** This method is similar to the division of the numbers. The term with the maximum degree is simplify, and then the result is found.

Activity

$$\begin{array}{r|l} 2x^2 - x + 1 & x - 3 \\ \underline{-\ 2x^2 - 6x} & \boxed{2x + 5} \rightarrow T(x) \\ 5x + 1 & \\ \underline{-\ 5x - 15} & \\ \boxed{16} \rightarrow K(x) & \end{array}$$

NOTE

Horner's Method works only for divisors of the form x – a. You cannot use this method to divide a polynomial by quadratics such as $x^2 - 7$.

2. **Synthetic Division (Horner's Method):** The coefficients of polynomial and the degree of divisor is taken in this method.

Activity

	$2x^2$	$-x$	$+1$
$x - 3 = 0$			
$x = 3$	2	-1	1
		$2 \cdot 3 = 6$	$5 \cdot 3 = 15$
	2	$-1+6 = 5$	$15 + 1 = 16$
		$T(x) = 2x + 5$	$K(x) = 16$

EXAMPLE 10

$$\begin{array}{r|l} 3x^2 + x - 1 & x + 1 \\ \underline{-\qquad\qquad} & \overline{Q(x)} \\ K(x) & \end{array} \quad \Rightarrow \quad Q(2) + K(x) = ?$$

A) 2 B) 3 C) 4 D) 5 E) 6

SOLUTION

The answer is **D**

Solution 1 [Polynomial Division] :

$$\begin{array}{r|l} 3x^2 + x - 1 & x + 1 \\ \underline{-3x^2 \mp 3x} & \boxed{3x - 2} \rightarrow Q(x) \\ -2x - 1 & \\ \underline{-\ +2x + 2} & \\ \boxed{1} \rightarrow K(x) & \end{array}$$

$Q(x) = 3x - 2 \Rightarrow Q(2) = 3 \cdot 2 - 2 = 4$

$Q(2) + K(x) = 4 + 1 = 5$

Solution 2 [Horner's Method] :

$x + 1 = 0$	3	1	-1
$x = -1$		-3	2
	3	-2	1
	$Q(x) = 3x - 2$		$K(x) = 1$

$Q(2) + K(x) = 4 + 1 = 5$

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ÖRNEK 11

$\text{der}\ (P^2(x)\cdot Q(x)) = 8$,

$\text{der}\left(\frac{P(x)}{Q(x)}\right) = 1 \Rightarrow$ **der(P(x) + Q(x)) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt **C**

$\text{der}(P(x)) = p,\quad \text{der}(Q(x)) = q$

$\text{der}(P^2(x)\cdot Q(x)) = 2p + q = 8$

$\text{der}(P(x) / Q(x)) = p - q = 1$

$3p = 9 \Rightarrow p = 3,\ q = 2$

$\text{der}(P(x) + Q(x)) = \max(p, q) = 3$

P(x) Polinomunun ax + b ile Bölümünden Elde Edilen Kalan

Bir P(x) polinomunun ax + b ile bölümünden kalanı bulmak için $ax + b = 0 \Rightarrow x = -\frac{b}{a}$ belirlenir. P(x) polinomunda, x'in yerine $-\frac{b}{a}$ yazılırsa işlemin sonucu kalanı verir.

ax + b birinci dereceden bir polinom olduğu için kalan sıfırıncı dereceden bir polinom, yani; reel bir sayı olmak zorundadır.

$$\begin{array}{r|l} P(x) & ax+b \\ \hline - & \\ \hline K(x) & \end{array} \quad \Rightarrow \quad K(x) = P\left(\frac{-b}{a}\right)$$

ÖRNEK 12

$P(x) = x^3 - 2x^2 + ax + 8$

$\begin{array}{r|l} P(x) & x-2 \\ \hline - & \\ \hline 18 & \end{array}$

$\Rightarrow$ **a = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt **E**

$x - 2 = 0 \Rightarrow x = 2$

$P(2) = 2^3 - 2\cdot 2^2 + a\cdot 2 + 8$

$18 = 8 - 8 + 2a + 8$

$2a = 10 \Rightarrow a = 5$

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EXAMPLE 11

$\text{der}\ (P^2(x)\cdot Q(x)) = 8$,

$\text{der}\left(\frac{P(x)}{Q(x)}\right) = 1 \Rightarrow$ **der(P(x) + Q(x)) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION

The answer is C

$\text{der}(P(x)) = p, \quad \text{der}(Q(x)) = q$

$\text{der}(P^2(x)\cdot Q(x)) = 2p + q = 8$

$\text{der}(P(x) / Q(x)) = p - q = 1$

$3p = 9 \Rightarrow p = 3,\ q = 2$

$\text{der}(P(x) + Q(x)) = \max(p, q) = 3$

The Remainder of the Polynomial P(x) Divided by ax + b

To find the remainder of P(x) divided by (ax + b) the following is determined:

$ax + b = 0 \Rightarrow x = -\frac{b}{a}$

If we write $-\frac{b}{a}$ instead of x in P(x), then the result gives the remainder. Since ax + b is a first order polynomial, the remainder must always be equal to a real number, which is a zero polynomial.

$$\begin{array}{r|l} P(x) & ax+b \\ \hline - & \\ \hline K(x) & \end{array} \quad \Rightarrow \quad K(x) = P\left(\frac{-b}{a}\right)$$

EXAMPLE 12

$P(x) = x^3 - 2x^2 + ax + 8$

$$\begin{array}{r|l} P(x) & x-2 \\ \hline - & \\ \hline 18 & \end{array}$$

$\Rightarrow$ **a = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION

The answer is E

$x - 2 = 0 \Rightarrow x = 2$

$P(2) = 2^3 - 2\cdot 2^2 + a\cdot 2 + 8$

$18 = 8 - 8 + 2a + 8$

$2a = 10 \Rightarrow a = 5$

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ÖRNEK 13

$P(3x+4) = x^3 + x^2 - x + 9 \Rightarrow$

$P(x) \mid x+2$

$-$

$?$

A) 13 B) 9 C) 11 D) 10 E) 7

ÇÖZÜM

Yanıt **E**

$x + 2 = 0 \Rightarrow x = -2$

$3x + 4 = -2$

$3x = -6 \Rightarrow x = -2$

$P(3\cdot(-2) + 4) = (-2)^3 + (-2)^2 - (-2) + 9$

$P(-2) = -8 + 4 + 2 + 9 = 7$

ÖRNEK 14

$P(x) \mid x-4$

$-$

3

$Q(x) \mid x-4$

$-$

4

$\Rightarrow$

$x\cdot P(x) + (x+1)\cdot Q(x) + x^2 + 2x \mid x-4$

$-$

$?$

A) 56 B) 48 C) 46 D) 44 E) 40

ÇÖZÜM

Yanıt **A**

$x - 4 = 0 \Rightarrow x = 4$

$P(4) = 3, Q(4) = 4$

Kalan : $4P(4) + (4+1)\cdot Q(4) + 4^2 + 2\cdot 4 = 4\cdot 3 + 5\cdot 4 + 16 + 8 = 12 + 20 + 24 = 56$

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EXAMPLE 13

$P(3x+4) = x^3 + x^2 - x + 9 \Rightarrow$

$P(x) \mid x+2$

$-$

$?$

A) 13 B) 9 C) 11 D) 10 E) 7

SOLUTION

The answer is E

$x + 2 = 0 \Rightarrow x = -2$

$3x + 4 = -2$

$3x = -6 \Rightarrow x = -2$

$P(3\cdot(-2) + 4) = (-2)^3 + (-2)^2 - (-2) + 9$

$P(-2) = -8 + 4 + 2 + 9 = 7$

EXAMPLE 14

$P(x) \mid x-4$

$-$

3

$Q(x) \mid x-4$

$-$

4

$\Rightarrow$

$x\cdot P(x) + (x+1)\cdot Q(x) + x^2 + 2x \mid x-4$

$-$

$?$

A) 56 B) 48 C) 46 D) 44 E) 40

SOLUTION

The answer is A

$x - 4 = 0 \Rightarrow x = 4$

$P(4) = 3,\ Q(4) = 4$

Remainder : $4P(4) + (4+1)\cdot Q(4) + 4^2 + 2\cdot 4 = 4\cdot 3 + 5\cdot 4 + 16 + 8 = 12 + 20 + 24 = 56$

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▼ $(x - a) \cdot P(x) = Q(x) \Rightarrow Q(a) = 0$

ÖRNEK 15

$(x - 1)P(x) = x^4 + ax^3 + 3x - 7$

$\Rightarrow a = ?$

A) 2 B) 3 C) 5 D) 7 E) 8

ÇÖZÜM

Yanıt **B**

$x - 1 = 0 \Rightarrow x = 1$

$1^4 + a \cdot 1^3 + 3 \cdot 1 - 7 = (1 - 1)P(1)$

$1 + a + 3 - 7 = 0$

$a = 3$

ÖRNEK 16

$(x - 2)\,P(x) = x^4 - ax^2 + 2x + 8 \Rightarrow$

P(x)	x – 1
–	
?	

A) –4 B) –3 C) –1 D) 3 E) 5

ÇÖZÜM

Yanıt **A**

$x - 2 = 0 \Rightarrow x = 2$

$(2 - 2)P(2) = 16 - 4a + 4 + 8$

$0 = 28 - 4a$

$a = 7$

$x - 1 = 0 \Rightarrow x = 1$

$(1 - 2)P(1) = 1 - 7 + 2 + 8$

$-1 \cdot P(1) = 4$

$P(1) = -4$

Kalan = $P(1) = -4$

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▼ $(x-a)\cdot P(x) = Q(x) \Rightarrow Q(a) = 0$

EXAMPLE 15

$(x-1)P(x) = x^4 + ax^3 + 3x - 7$

$\Rightarrow$ **a = ?**

A) 2 B) 3 C) 5 D) 7 E) 8

SOLUTION

The answer is **B**

$x - 1 = 0 \Rightarrow x = 1$

$1^4 + a\cdot 1^3 + 3\cdot 1 - 7 = (1-1)P(1)$

$1 + a + 3 - 7 = 0$

$a = 3$

EXAMPLE 16

$(x-2)\,P(x) = x^4 - ax^2 + 2x + 8 \Rightarrow$

P(x)	x − 1
−	
?	

A) −4 B) −3 C) −1 D) 3 E) 5

SOLUTION

The answer is **A**

$x - 2 = 0 \Rightarrow x = 2$

$(2-2)P(2) = 16 - 4a + 4 + 8$

$0 = 28 - 4a$

$a = 7$

$x - 1 = 0 \Rightarrow x = 1$

$(1-2)P(1) = 1 - 7 + 2 + 8$

$-1\cdot P(1) = 4$

$P(1) = -4$

Remainder = $P(1) = -4$

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P(x) Polinomunun $x^n \mp a$ ile Bölümünden Elde Edilen Kalan

Bir P(x) polinomunun $x^n \pm a$ ile bölümünden kalanını bulmak için $x^n \pm a = 0$'dan $x^n = \pm a$ bulunur. Polinomda x^n yerine $\pm a$ değeri yerine yazılarak kalan bulunur.

ÖRNEK 17

$P(x) = x^{16} - 2x^{11} + 6x^6 + 3 \Rightarrow$

$P(x) \mid x^5 + 2$

$-$

$?$

A) $-28x + 3$ B) $4x + 9$ C) $-17x + 21$

D) $-14x + 7$ E) $21x + 18$

ÇÖZÜM

Yanıt **A**

$x^5 + 2 = 0 \Rightarrow x^5 = -2$

$P(x) = x \cdot x^{15} - 2x \cdot x^{10} + 6x \cdot x^5 + 3$

$= x(x^5)^3 - 2x(x^5)^2 + 6x(x^5) + 3$

Kalan $= x(-2)^3 - 2x(-2)^2 + 6x(-2) + 3 = -8x - 8x - 12x + 3 = -28x + 3$

P(x) Polinomunun $(x - a) \cdot (x - b) \cdot (x - c)\ldots$ Çarpımı ile Bölünebilmesi

P(x) polinomu $(x - a)\cdot(x - b)\cdot(x - c)\ldots$ çarpımı ile tam bölünebiliyorsa $(x - a)$, $(x - b)$, $(x - c)$ ... çarpanlarıyla da ayrı ayrı tam bölünebilir.

ÖRNEK 18

$P(x) \mid x - 2$, $P(x) \mid x - 3$ $\Rightarrow$ $P(x) \mid (x - 2) \cdot (x - 3)$

$-$ $-$ $-$

5 9 $?$

A) $2x - 7$ B) $4x + 8$ C) $4x - 3$

D) $8x + 9$ E) $12x - 1$

ÇÖZÜM

Yanıt **C**

$K(x) = mx + n$

$x - 2 = 0 \Rightarrow x = 2 \Rightarrow P(2) = 5 \Rightarrow 2m + n = 5$

$x - 3 = 0 \Rightarrow x = 3 \Rightarrow P(3) = 9 \Rightarrow 3m + n = 9$

$-m = -4$

$m = 4$ ve $n = -3 \Rightarrow K(x) = 4x - 3$

NOT

1. *P(x) ve Q(x) polinomları x – a'ya bölündüğünde kalanlar sırayla A ve B olsun.*
   - *a. P(x) ± Q(x) polinomu (x – a) ile bölündüğünde kalan A ± B dir.*
   - *b. P(x) · Q(x) polinomu (x – a) ile bölündüğünde kalan A · B'dir.*
2. *P(x) polinomu (x – a) ile bölündüğünde kalan A, (x – b) ile bölündüğünde kalan B dir. P(x) polinomu (x – a)(x – b)'ye bölündüğünde kalan K(x) = mx + n biçimindedir.*

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The Remainder of P(x) Divided by $x^n \mp a$

To find the remainder of P(x) divided by $(x^n \pm a)$, insert $(x^n = \pm a)$ in the polynomial P(x), which writing $\pm a$ instead of x^n in the polynomial.

EXAMPLE 17

$P(x) = x^{16} - 2x^{11} + 6x^6 + 3 \Rightarrow$

$$\begin{array}{r|l} P(x) & x^5 + 2 \\ \hline - & \\ \hline ? & \end{array}$$

A) $-28x + 3$ B) $4x + 9$ C) $-17x + 21$

D) $-14x + 7$ E) $21x + 18$

SOLUTION

The answer is **A**

$x^5 + 2 = 0 \Rightarrow x^5 = -2$

$P(x) = x \cdot x^{15} - 2x \cdot x^{10} + 6x \cdot x^5 + 3$

$= x(x^5)^3 - 2x(x^5)^2 + 6x(x^5) + 3$

Remainder $= x(-2)^3 - 2x(-2)^2 + 6x(-2) + 3 = -8x - 8x - 12x + 3 = -28x + 3$

NOTE

1. *The remainders of P(x) and Q(x) divided by (x–a) are equal to A and B, respectively .*
   - **a.** *The remainder of $P(x) \pm Q(x)$ divided by (x–a) is equal to $A \pm B$.*
   - **b.** *The remainder of $P(x) \cdot Q(x)$ divided by (x–a) equal to $A \cdot B$.*
2. *The remainders of P(x) divided by (x – a) and (x – b) are equal to A and B, respectively. Then the remainder of P(x) divided by $(x - a) \cdot (x - b)$ is in the form of $K(x) = mx + n$.*

Divided by $(x - a) \cdot (x - b) \cdot (x - c)\ldots$ of the Polynomial P(x)

If the polynomial P(x) is divided by $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$, it is divided by the factor $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$

EXAMPLE 18

$$\left.\begin{array}{r|l} P(x) & x - 2 \\ \hline - & \\ \hline 5 & \end{array}\right. , \quad \left.\begin{array}{r|l} P(x) & x - 3 \\ \hline - & \\ \hline 9 & \end{array}\right\} \Rightarrow \begin{array}{r|l} P(x) & (x-2)\cdot(x-3) \\ \hline - & \\ \hline ? & \end{array}$$

A) $2x - 7$ B) $4x + 8$ C) $4x - 3$

D) $8x + 9$ E) $12x - 1$

SOLUTION

The answer is **C**

$K(x) = mx + n$

$x - 2 = 0 \Rightarrow x = 2 \Rightarrow P(2) = 5 \Rightarrow 2m + n = 5$

$x - 3 = 0 \Rightarrow x = 3 \Rightarrow P(3) = 9 \Rightarrow 3m + n = 9$

$-m = -4$

$m = 4$ ve $n = -3 \Rightarrow K(x) = 4x - 3$

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ÖRNEK 19

$$\begin{array}{r|l} P(x) & x-3 \\ \hline - & Q(x) \\ \hline 4 & \end{array} \quad , \quad \begin{array}{r|l} Q(x) & x+3 \\ \hline - & \\ \hline 2 & \end{array} \quad \Rightarrow \quad \begin{array}{r|l} \mathbf{P(x)} & \mathbf{x^2-9} \\ \hline - & \\ \hline ? & \end{array}$$

A) $3x + 4$ B) $2x - 2$ C) $2x + 7$

D) $4x + 1$ E) $5x + 4$

ÇÖZÜM

Yanıt **B**

$$\begin{array}{r|l} P(x) & x-3 \\ \hline - & Q(x) \\ \hline 4 & \end{array} \Rightarrow P(x) = (x-3)\cdot Q(x) + 4$$

$$\begin{array}{r|l} Q(x) & x+3 \\ \hline - & T(x) \\ \hline 2 & \end{array} \Rightarrow Q(x) = (x+3)\cdot T(x) + 2$$

$$P(x) = (x-3)\,[(x+3)\cdot T(x) + 2] + 4 = (x^2-9)\cdot T(x) + 2x - 6 + 4$$
$$= (x^2-9)\cdot T(x) + 2x - 2$$

Kalan: $2x - 2$

P(x) Polinomunun $(ax + b)^n$ ile Bölünebilmesi

P(x) polinomu $(ax + b)^n$ polinomu ile tam bölünebiliyorsa, $P'(x)$, $P''(x)$, $P'''(x)$ ... $P^{(n-1)}(x)$ polinomları da $ax + b$ ile tam bölünebilir.

Burada $P'(x)$, $P''(x)$, ... , $P^{(n-1)}(x)$ polinomları P(x) polinomunun 1, 2, 3. ve $(n - 1)$. mertebeden türevleridir.

$$\begin{array}{r|l} P(x) & (ax+b)^n \\ \hline - & \\ \hline 0 & \end{array} \Rightarrow P\left(-\frac{b}{a}\right) = 0$$

$$P'\left(-\frac{b}{a}\right) = 0$$

$$P''\left(-\frac{b}{a}\right) = 0$$

$$\vdots$$

$$P^{(n-1)}\left(-\frac{b}{a}\right) = 0$$

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EXAMPLE 19

$$\begin{array}{r|l} P(x) & x-3 \\ \hline - & Q(x) \\ \hline 4 & \end{array} \quad , \quad \begin{array}{r|l} Q(x) & x+3 \\ \hline - & \\ \hline 2 & \end{array} \quad \Rightarrow \quad \begin{array}{r|l} \mathbf{P(x)} & \mathbf{x^2-9} \\ \hline - & \\ \hline ? & \end{array}$$

A) $3x + 4$ B) $2x - 2$ C) $2x + 7$

D) $4x + 1$ E) $5x + 4$

SOLUTION

The answer is **B**

$$\begin{array}{r|l} P(x) & x-3 \\ \hline = & Q(x) \\ \hline 4 & \end{array} \Rightarrow P(x) = (x-3) \cdot Q(x) + 4$$

$$\begin{array}{r|l} Q(x) & x+3 \\ \hline = & T(x) \\ \hline 2 & \end{array} \Rightarrow Q(x) = (x+3) \cdot T(x) + 2$$

$$P(x) = (x-3)\,[(x+3) \cdot T(x) + 2] + 4 = (x^2 - 9) \cdot T(x) + 2x - 6 + 4$$

$$= (x^2 - 9) \cdot T(x) + 2x - 2$$

Remainder: $2x - 2$

Divided by $(ax + b)^n$ of a Polynomial P(x)

If the polynomial P(x) is divided by $(ax + b)^n$, the polynomial $P'(x)$, $P''(x)$, $P'''(x)$ ... $P^{(n-1)}(x)$ is divided by $ax + b$ where $P'(x)$, $P''(x)$, $P'''(x)$ ... $P^{(n-1)}(x)$ are 1, 2, ... $(n - 1)$. order derivatives of the polynomial P(x).

$$\begin{array}{r|l} P(x) & (ax+b)^n \\ \hline = & \\ \hline 0 & \end{array} \Rightarrow P\left(-\frac{b}{a}\right) = 0$$

$$P'\left(-\frac{b}{a}\right) = 0$$

$$P''\left(-\frac{b}{a}\right) = 0$$

$$\vdots$$

$$P^{(n-1)}\left(-\frac{b}{a}\right) = 0$$

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ÖRNEK 20

$P(x) = x^3 - 2x^2 + 3x - 1$

$P(x)$ bölü $(x+1)^2$ kalan ?

A) $6x + 5$ B) $8x + 3$ C) $10x + 3$ D) $10x + 6$ E) $12x + 3$

ÇÖZÜM

Yanıt **C**

Çözüm 1

$$\begin{array}{r|l} x^3 - 2x^2 + 3x - 1 & x^2 + 2x + 1 \\ -x^3 \mp 2x^2 \mp x & \hline x - 4 \\ \hline -4x^2 + 2x - 1 & \\ +4x^2 + 8x + 4 & \\ \hline 10x + 3 & \end{array}$$

Çözüm 2

$x^2 + 2x + 1 = 0 \Rightarrow x^2 = -2x - 1$

$x^3 - 2x^2 + 3x - 1 = (-2x-1)x - 2(-2x-1) + 3x - 1$

$= -2x^2 - x + 4x + 2 + 3x - 1$

$= -2(-2x-1) + 6x + 1$

$= 4x + 2 + 6x + 1$

$= 10x + 3$

ÖRNEK 21

$P(x) = x^2 - 3ax + 4$,

$P(x)$ bölü $(x-1)^2$ kalan 0 $\Rightarrow$ **a = ?**

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{4}{3}$ E) $\frac{5}{3}$

ÇÖZÜM

Yanıt **E**

$x - 1 = 0 \Rightarrow x = 1$

$P(1) = 0 \Rightarrow 1^2 - 3a \cdot 1 + 4 = 0$

$a = \frac{5}{3}$

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EXAMPLE 20

$P(x) = x^3 - 2x^2 + 3x - 1$

$P(x) \mid (x+1)^2$

$= \;?$

A) $6x + 5$ B) $8x + 3$ C) $10x + 3$ D) $10x + 6$ E) $12x + 3$

SOLUTION

The answer is C

Solution 1

$x^3 - 2x^2 + 3x - 1 \mid x^2 + 2x + 1$

$-x^3 \mp 2x^2 \mp x \quad\quad x - 4$

$-4x^2 + 2x - 1$

$+4x^2 + 8x + 4$

$10x + 3$

Solution 2

$x^2 + 2x + 1 = 0 \Rightarrow x^2 = -2x - 1$

$x^3 - 2x^2 + 3x - 1 = (-2x-1)x - 2(-2x-1) + 3x - 1$

$= -2x^2 - x + 4x + 2 + 3x - 1$

$= -2(-2x-1) + 6x + 1$

$= 4x + 2 + 6x + 1$

$= 10x + 3$

EXAMPLE 21

$P(x) = x^2 - 3ax + 4,$

$P(x) \mid (x-1)^2 \quad \Rightarrow \mathbf{a = ?}$

$= \;0$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{4}{3}$ E) $\frac{5}{3}$

SOLUTION

The answer is E

$x - 1 = 0 \Rightarrow x = 1$

$P(1) = 0 \Rightarrow 1^2 - 3a \cdot 1 + 4 = 0$

$a = \frac{5}{3}$

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ÖRNEK 22

$P(x) = x^3 + ax^2 + b$

$P(x) \mid (x-1)^2 \Rightarrow a \cdot b = ?$

$-$

0

A) $-\frac{1}{2}$ B) $-\frac{3}{4}$ C) 0 D) $\frac{1}{2}$ E) $\frac{3}{4}$

ÇÖZÜM

Yanıt **B**

Çözüm 1

$$\begin{array}{r|l} x^3 + ax^2 + b & x^2 - 2x + 1 \\ -x^3 - 2x^2 + x & x + (a+2) \\ \hline (a+2)x^2 - x + b & \\ -\ (a+2)x^2 - 2(a+2)x + a + 2 & \\ \hline (2a+3)x - a + b - 2 = 0 & \end{array}$$

$2a + 3 = 0 \Rightarrow a = -\frac{3}{2}$

$-a + b - 2 = 0 \Rightarrow b - a = 2 \Rightarrow b = 2 - \frac{3}{2} = \frac{1}{2}$

$a \cdot b = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$

Çözüm 2

$x^2 - 2x + 1 = 0 \Rightarrow x^2 = 2x - 1$

$P(x) = x^3 + ax^2 + b = x^2 \cdot x + ax^2 + b = (2x-1)x + ax^2 + b = 0$

$2x^2 - x + ax^2 + b = 0$

$2(2x-1) - x + a(2x-1) + b = 0$

$3x - 2 + 2ax - a + b = 0$

$(3 + 2a)x - 2 - a + b = 0$

$3 + 2a = 0 \qquad b - a = 2$

$a = -\frac{3}{2} \qquad b = 2 - \frac{3}{2} = \frac{1}{2} \Rightarrow a \cdot b = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$

Çözüm 3

$K(x) = 0 \Rightarrow P(1) = 0 \Rightarrow 1 + a + b = 0 \Rightarrow a + b = -1$

$P'(1) = 0 \Rightarrow 3 \cdot 1^2 + 2a \cdot 1 = 0 \Rightarrow a = -\frac{3}{2}, \; b = \frac{1}{2}$

$\Rightarrow a \cdot b = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$

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EXAMPLE 22

$P(x) = x^3 + ax^2 + b$

$P(x) \mid (x-1)^2 \Rightarrow a \cdot b = ?$

0

A) $-\frac{1}{2}$ B) $-\frac{3}{4}$ C) 0 D) $\frac{1}{2}$ E) $\frac{3}{4}$

SOLUTION

The answer is B

Solution 1

$$\begin{array}{r|l} x^3 + ax^2 + b & x^2 - 2x + 1 \\ -x^3 - 2x^2 + x & x + (a+2) \\ \hline (a+2)x^2 - x + b & \\ -\ (a+2)x^2 - 2(a+2)x + a + 2 & \\ \hline (2a+3)x - a + b - 2 = 0 & \end{array}$$

$2a + 3 = 0 \Rightarrow a = -\frac{3}{2}$

$-a + b - 2 = 0 \Rightarrow b - a = 2 \Rightarrow b = 2 - \frac{3}{2} = \frac{1}{2}$

$a \cdot b = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$

Solution 2

$x^2 - 2x + 1 = 0 \Rightarrow x^2 = 2x - 1$

$P(x) = x^3 + ax^2 + b = x^2 \cdot x + ax^2 + b = (2x-1)x + ax^2 + b = 0$

$2x^2 - x + ax^2 + b = 0$

$2(2x-1) - x + a(2x-1) + b = 0$

$3x - 2 + 2ax - a + b = 0$

$(3 + 2a)x - 2 - a + b = 0$

$3 + 2a = 0 \qquad b - a = 2$

$a = -\frac{3}{2} \qquad b = 2 - \frac{3}{2} = \frac{1}{2} \Rightarrow a \cdot b = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$

Solution 3

$K(x) = 0 \Rightarrow P(1) = 0 \Rightarrow 1 + a + b = 0 \Rightarrow a + b = -1$

$P'(1) = 0 \Rightarrow 3 \cdot 1^2 + 2a \cdot 1 = 0 \Rightarrow a = -\frac{3}{2}, \; b = \frac{1}{2}$

$\Rightarrow a \cdot b = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$

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ÖRNEK 23

$P(-1) = 10,$

$P(1) = 4 \Rightarrow$

$$\begin{array}{r|l} P(x) & x^2+1 \\ \hline - & x+2 \\ \hline ? & \end{array}$$

A) $2x + 1$ B) $-5x + 3$ C) $-3x + 7$

D) $8x + 4$ E) $-2x - 4$

ÇÖZÜM

Yanıt **B**

$P(x) = (x^2 + 1)(x + 2) + ax + b$

$x = 1 \Rightarrow P(1) = 6 + a + b = 4 \Rightarrow a + b = -2$

$x = -1 \Rightarrow P(-1) = 2 - a + b = 10 \Rightarrow \underset{+}{} \begin{array}{r} b - a = 8 \\ \hline 2b = 6 \\ b = 3 \end{array}$

$a = -5 \Rightarrow ax + b = -5x + 3$

ÖRNEK 24

$P(x) = x^3 - 3x^2 + 4x - 9$

$$\begin{array}{r|l} P(x) & x^2-x+1 \\ \hline - & \\ \hline ? & \end{array}$$

A) $2x + 5$ B) $x + 9$ C) $x - 7$

D) $3x - 5$ E) $5x + 8$

ÇÖZÜM

Yanıt **C**

$x^2 - x + 1 = 0 \Rightarrow x^2 = x - 1$

P(x) teki x^2 yerine $x - 1$ yazılırsa aşağıdaki eşitlik bulunur.

$P(x) = x \cdot x^2 - 3x^2 + 4x - 9$

Kalan : $x(x-1) - 3(x-1) + 4x - 9 = x^2 - x - 3x + 3 + 4x - 9$

$= x - 1 - 6 = x - 7$

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EXAMPLE 23

$P(-1) = 10$,

$P(1) = 4 \Rightarrow$

$$\begin{array}{r|l} P(x) & x^2+1 \\ \hline - & x+2 \\ \hline ? & \end{array}$$

A) $2x + 1$ B) $-5x + 3$ C) $-3x + 7$

D) $8x + 4$ E) $-2x - 4$

SOLUTION

The answer is **B**

$P(x) = (x^2 + 1)(x + 2) + ax + b$

$x = 1 \Rightarrow P(1) = 6 + a + b = 4 \Rightarrow a + b = -2$

$x = -1 \Rightarrow P(-1) = 2 - a + b = 10 \Rightarrow + \; b - a = 8$

$2b = 6$

$b = 3$

$a = -5 \Rightarrow ax + b = -5x + 3$

EXAMPLE 24

$P(x) = x^3 - 3x^2 + 4x - 9$

$$\begin{array}{r|l} P(x) & x^2-x+1 \\ \hline - & \\ \hline ? & \end{array}$$

A) $2x + 5$ B) $x + 9$ C) $x - 7$

D) $3x - 5$ E) $5x + 8$

SOLUTION

The answer is **C**

$x^2 - x + 1 = 0 \Rightarrow x^2 = x - 1$

If we write $x-1$ instead of x^2 in $P(x)$, we obtain

$P(x) = x \cdot x^2 - 3x^2 + 4x - 9$

Remainder : $x(x-1) - 3(x-1) + 4x - 9 = x^2 - x - 3x + 3 + 4x - 9$

$= x - 1 - 6 = x - 7$

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Polinomların Basit Kesirlere Ayrılması

Payın derecesinin paydanın derecesinden küçük ve paydanın çarpanlara ayrıldığı durumlarda basit kesirlere ayırma yöntemi kullanılır.

NOT

Bu yöntem integralde çok sık kullanılır.

ÖRNEK 25

$$\frac{3x-1}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1} \Rightarrow A \cdot B = ?$$

A) 1 B) 2 C) 0 D) –1 E) –2

ÇÖZÜM

Yanıt **B**

$$\frac{3x-1}{x^2-2x-3} = \frac{A}{\underset{(x+1)}{x-3}} + \frac{B}{\underset{(x-3)}{x+1}}$$

$$3x-1 = A(x+1)+B(x-3)$$

$$= (A+B)x + A - 3B$$

$$\Rightarrow \quad A + B = 3$$

$$-/\ A - 3B = -1$$

$$4B = 4$$

$$B = 1\ ,\ A = 2 \Rightarrow A \cdot B = 1 \cdot 2 = 2$$

ÖRNEK 26

$$\frac{5x^2+3x+2}{x^3+x} = \frac{Ax+B}{x^2+1} + \frac{C}{x} \Rightarrow A+B \cdot C = ?$$

A) 3 B) 4 C) 5 D) 6 E) 9

ÇÖZÜM

Yanıt **E**

$$\frac{5x^2+3x+2}{x^3+x} = \frac{Ax+B}{\underset{(x)}{x^2+1}} + \frac{C}{\underset{(x^2+1)}{x}}$$

$$5x^2 + 3x + 2 = Ax^2 + Bx + Cx^2 + C$$

$$A + C = 5\ ,\ B = 3$$

$$C = 2 \Rightarrow A + 2 = 5$$

$$A = 3$$

$$A + B \cdot C = 3 + 3 \cdot 2 = 9$$

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NOTE

This method is very frequently in integral.

Discrimination to Partial Fractions of Polynomials

This method is apply that the degree of numarator is less than the degree of denominator and denominator is factorization.

EXAMPLE 25

$$\frac{3x-1}{x^2-2x-3}=\frac{A}{x-3}+\frac{B}{x+1} \Rightarrow A\cdot B=?$$

A) 1 B) 2 C) 0 D) –1 E) –2

SOLUTION

The answer is B

$$\frac{3x-1}{x^2-2x-3}=\frac{A}{\underset{(x+1)}{x-3}}+\frac{B}{\underset{(x-3)}{x+1}}$$

$$3x-1=A(x+1)+B(x-3)$$

$$=(A+B)x+A-3B$$

$$\Rightarrow \quad A+B=3$$

$$-/\ A-3B=-1$$

$$4B=4$$

$$B=1\ ,\ A=2 \Rightarrow A\cdot B=1\cdot 2=2$$

EXAMPLE 26

$$\frac{5x^2+3x+2}{x^3+x}=\frac{Ax+B}{x^2+1}+\frac{C}{x} \Rightarrow A+B\cdot C=?$$

A) 3 B) 4 C) 5 D) 6 E) 9

SOLUTION

The answer is E

$$\frac{5x^2+3x+2}{x^3+x}=\frac{Ax+B}{\underset{(x)}{x^2+1}}+\frac{C}{\underset{(x^2+1)}{x}}$$

$$5x^2+3x+2=Ax^2+Bx+Cx^2+C$$

$$A+C=5\ ,\ B=3$$

$$C=2 \Rightarrow A+2=5$$

$$A=3$$

$$A+B\cdot C=3+3\cdot 2=9$$

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1. $P(x) = x + 4$,

$Q(x) = x^2 - 5x \Rightarrow$ **P(x) + Q(x) = ?**

A) $x^2 - 4x$

B) $x^2 - 4$

C) $x^2 + 4x + 4$

D) $(x - 2)^2$

E) $(x + 4)^2$

2. $P(x - 4) = 2x^2 + 3x + 4 \Rightarrow$ **P(1) = ?**

A) 69

B) 70

C) 71

D) 72

E) 73

3. $P(x - 3) = x^3 + 2x^2 - x + a$,

$P(-1) = 5 \Rightarrow$ **a = ?**

A) – 10

B) – 9

C) – 8

D) –7

E) – 6

4. $P(x^2) = x^4 - 1$,

$Q(\sqrt{x}) = x + 1 \Rightarrow$ **P(x) · Q(x) = ?**

A) $x^4 + 1$

B) $x^4 - 1$

C) $x^2 - 1$

D) $x^2 + 1$

E) $x^5 - 1$

5. $m \in Z^+$,

$P(x) = x^{2m+2} + x^{2m+1} + 2 \Rightarrow$ **P (–1) = ?**

A) 2

B) 3

C) 4

D) 5

E) 6

6. $(x^3 - 4x^2 + 3x) \cdot (x^2 - 5x + 1) = \ldots\ldots + a \cdot x^4 + \ldots\ldots$

$\Rightarrow$ **a = ?**

A) – 11

B) – 10

C) – 9

D) – 8

E) – 7

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7. $P(4x-1) = x^3 - x^2 + 2x - 5 \Rightarrow$

$P(x) \mid x-3$

$-$

$?$

A) 2 B) 1 C) 0

D) – 1 E) –3

8. $9x^2 + 3x + 7 = (3x+1) \cdot Q(x) + a \Rightarrow$ **a = ?**

A) 1

B) 2

C) 4

D) 7

E) 9

9. $P(2x+5) = (2x^2 + 3x - 1) \cdot Q(x+1)$

$Q(-1) = 3 \Rightarrow$ **P(1) = ?**

A) 1

B) 2

C) 3

D) 4

E) 5

10.

$P(x) \mid (3x-4)^2$ $\Rightarrow$ $P(x) \mid 3x-4$

$-$ $-$

$12x + 7$ $?$

A) 15 B) 17 C) 19

D) 21 E) 23

11. $P(x) = x^3 - x^2 + ax + b$

$P(x) \mid x^2 - 3x + 2$

$-$

0

$\Rightarrow$ **a + b = ?**

A) 5 B) 4 C) 2

D) 0 E) –2

12. $P(x) = -x^2 + 3x$,

$Q(x) = 5x^2 - x + 2$

$\Rightarrow$ **P [Q (1)] + Q [P (–1)] = ?**

A) 36

B) 48

C) 50

D) 56

E) 68

YANITLAR / ANSWERS							
1. D	2. A	3. B	4. B	5. A	6. C	7. E	8. D
9. C	10. E	11. D	12. E				

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1. $P(x) + Q(x) = \underbrace{x+4}_{P(x)} + \underbrace{x^2-5x}_{Q(x)}$

$= x^2 - 4x + 4$

$= (x-2)^2$

Yanıt / The Answer is D

2. $P(x-4) = 2x^2 + 3x + 4$

$x - 4 = 1 \Rightarrow x = 5$

$P(5-4) = 2 \cdot 5^2 + 3 \cdot 5 + 4$

$= 2 \cdot 25 + 15 + 4$

$= 50 + 15 + 4$

$= 69$

Yanıt / The Answer is A

3. $P(x-3) = x^3 + 2x^2 - x + a$

$x = 2 \Rightarrow P(2-3) = 2^3 + 2 \cdot 2^2 - 2 + a$

$P(-1) = 8 + 2 \cdot 4 - 2 + a$

$P(-1) = 8 + 8 - 2 + a$

$P(-1) = 14 + a$

$5 = 14 + a \Rightarrow a = -9$

Yanıt / The Answer is B

4. $P(x^2) = (x^2)^2 - 1,$

$P(x) = x^2 - 1,$

$Q\left(\sqrt{x^2}\right) = x^2 + 1$

$Q(x) = x^2 + 1$

$P(x) \cdot Q(x) = (x^2-1) \cdot (x^2+1)$

$= x^4 + x^2 - x^2 - 1$

$= x^4 - 1$

Yanıt / The Answer is B

5. $P(x) = x^{2m+2} + x^{2m+1} + 2$

$P(-1) = (-1)^{2m+2} + (-1)^{2m+1} + 2$

$= 1 + (-1) + 2$

$= 2$

Yanıt / The Answer is A

6. $(x^3 - 4x^2 + 3x) \cdot (x^2 - 5x + 1)$

$-5x^4 - 4x^4 = ax^4$

$-9x^4 = ax^4$

$a = -9$

Yanıt / The Answer is C

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7. $x-3=0 \Rightarrow x=3$

$4x-1=3$

$4x=4 \Rightarrow x=1$

$P(4\cdot 1-1)=1^3-1^2+2\cdot 1-5$

$P(3)=1-1+2-5$

$=-3$

Yanıt / The Answer is E

8. $3x+1=0 \Rightarrow x=-\frac{1}{3}$

$9\left(-\frac{1}{3}\right)^2+3\left(-\frac{1}{3}\right)+7=\left(3\left(-\frac{1}{3}\right)+1\right)\cdot Q\left(-\frac{1}{3}\right)+a$

$9\cdot\frac{1}{9}-3\cdot\frac{1}{3}+7=a$

$a=1-1+7$

$a=7$

Yanıt / The Answer is D

9. $P(2x+5)=(2x^2+3x-1)\cdot Q(x+1)$

$x=-2 \Rightarrow$

$P(2\cdot(-2)+5)=(2\cdot(-2)^2+3\cdot(-2)-1)\cdot Q(-2+1)$

$P(1)=(2\cdot 4-6-1)\cdot Q(-1)$

$P(1)=Q(-1)$

$P(1)=3$

Yanıt / The Answer is C

10. $3x-4=0 \Rightarrow x=\frac{4}{3}$

$P(x)=(3x-4)^2\cdot Q(x)+12x+7$

$x=\frac{4}{3} \Rightarrow P\left(\frac{4}{3}\right)=\underbrace{\left(\frac{4}{3}\cdot 3-4\right)^2}_{0}\cdot Q\left(\frac{4}{3}\right)+12\cdot\frac{4}{3}+7$

Kalan (Remainder) $=0+4\cdot 4+7=23$

Yanıt / The Answer is E

11. $x^2-3x+2=(x-1)(x-2)$

$x-1=0 \Rightarrow x=1$

$P(1)=1-1+a+b=0$

$\Rightarrow a+b=0$

Yanıt / The Answer is D

12. $Q(1)=5\cdot 1-1+2$, $P(-1)=-(-1)^2+3\cdot(-1)$

$Q(1)=6$, $P(-1)=-1-3$

$P(-1)=-4$

$P[Q(1)]+Q[P(-1)]=P(6)+Q(-4)$

$=-6^2+3\cdot 6+5\cdot(-4)^2-(-4)+2$

$=-36+18+5\cdot 16+4+2$

$=-36+18+80+4+2$

$=68$

Yanıt / The Answer is E

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YÖS Soruları / YÖS Questions

1. $P(x) = (x^2 - 1)\cdot Q(x) + 4x + 5,$

$P(x) = (x - 1)\cdot K(x) + R$

$\Rightarrow$ **R = ?**

A) 7 B) 8 C) 9 D) 10 E) 11

[ANKARA ÜNİV. YÖS 2013]

2. $P(x) = 0$

$P(x) = (2a - b)x^3 + (b - 4)x^2 - a + b - 2c$

$\Rightarrow$ **c = ?**

A) –2 B) 0 C) 1 D) 3 E) 5

[MARMARA ÜNİV. YÖS 2013]

3.

$P(x) \,|\, \frac{x+1}{A(x)}$ = -6 $\qquad$ $P(x) \,|\, \frac{x-2}{B(x)}$ = 6

$P(x) \,|\, \frac{(x+1)(x-2)}{C(x)}$ = $ax+b$

ax + b = ?

A) 4x + 2 B) 4x – 2 C) 2x – 2 D) x – 4 E) 2x + 4

[İSTANBUL ÜNİV. YÖS 2013]

4. $\begin{array}{r|l} x^5 - 3x^3 - x & x^2 + 2 \\ \hline K(x) & Q(x) \end{array}$ $\Rightarrow$ **Q(x) = ?**

A) $2x^3 - 1$ B) $x^3 - x$ C) $x^3 - 2x^2$ D) $x^3 + x$ E) $x^3 - 5x$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

5. $x^3 + mx - 2 = (x - 1)\cdot P(x)$

$\Rightarrow$ **P(2) = ?**

A) 8 B) 5 C) 3 D) 10 E) 13

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

6. $(x + 1)\cdot P(x) + k = x^3 + x^2 - 4x$ $\Rightarrow$ **k = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

[MARMARA ÜNİV. YÖS 2012]

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7. $\frac{P(x-2)}{Q(x+2)} = x^2 - x + 4$

$Q(5) = 1$

P(1) = ?

A) 4 B) 10 C) 16 D) 24 E) 34

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

8. $P(x) \cdot P(x+7) = x^2 - 5x - 6$

P(10) = ?

A) 0 B) 1 C) 2 D) 3 E) 4

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

9. $\frac{x+1}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2}$

A + B = ?

A) –2 B) –1 C) 0 D) 1 E) 2

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

10. $x^6 - x^4 + x^2 + m \;\big|\; x^2 - 1$

$\underline{-}$

$0 \qquad\qquad B(x)$

$\Rightarrow$ **m = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

[YÖS 2010]

11. $(x^2 + 1)P(x) = ax^3 + (b - 2)x + a - 1$

a·b = ?

A) –6 B) –4 C) 3 D) 6 E) 8

[YÖS 2009]

YANITLAR / ANSWERS							
1. C	2. C	3. B	4. E	5. A	6. D	7. B	8. E
9. D	10. B	11. C					

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1. $x = 1 \Rightarrow P(1) = 0 \cdot Q(1) + 4 + 5 = 9$

$x = 1 \Rightarrow P(1) = (1 - 1) \cdot K(1) + R$

$9 = 0 + R$

$9 = R$

Yanıt / The Answer is C

2. $P(x) = (2a - b)x^3 + (b - 4)x^2 - a + b - 2c$

$P(x) = 0 \Rightarrow$

$0 = (2a - b)x^3 + (b - 4)x^2 - a + b - 2c$

$0x^3 + 0x^2 + 0x + 0 = (2a - b)x^3 + (b - 4)x^2 - a + b - 2c$

$b - 4 = 0$, $0 = 2a - b$, $-a + b - 2c = 0$

$b = 4$, $b = 2a$, $-2 + 4 - 2c = 0$

$a = 2$ $2c = 2$

$c = 1$

Yanıt / The Answer is C

3.

$$P(x) \;\big|\; \frac{x+1}{A(x)} \quad \Rightarrow \quad P(-1) = -6$$
$$= \;\; -6$$

$$P(x) \;\big|\; \frac{x-2}{B(x)} \quad \Rightarrow \quad P(2) = 6$$
$$= \;\; 6$$

$$P(x) \;\big|\; \frac{(x+1)(x-2)}{C(x)} \quad \Rightarrow \quad P(x) = (x+1)(x-2) \cdot C(x) + ax + b$$
$$= \;\; ax+b$$

$x = -1 \Rightarrow -a + b = -6$

$x = 2 \Rightarrow 2a + b = 6$

$a = 4, \quad b = -2$

$\Rightarrow ax + b = 4x - 2$

Yanıt / The Answer is B

4.

$$\begin{array}{r|l} x^5 - 3x^3 - x & x^2 + 2 \\ \hline x^5 + 2x^3 & x^3 - 5x \\ \hline -5x^3 - x & \\ -5x^3 - 10x & \\ \hline 9x & \end{array}$$

$\Rightarrow Q(x) = x^3 - 5x$

Yanıt / The Answer is E

5. $x = 1 \Rightarrow 1^3 + m \cdot 1 - 2 = (1 - 1) \cdot P(1)$

$1 + m - 2 = 0$

$m = 1$

$x = 2 \Rightarrow 2^3 + 1 \cdot 2 - 2 = (2 - 1) \cdot P(2)$

$8 + 2 - 2 = P(2)$

$8 = P(2)$

Yanıt / The Answer is A

6. $(x + 1) \cdot P(x) + k = x^3 + x^2 - 4x$

$x = -1 \Rightarrow (-1 + 1) \cdot P(-1) + k = (-1)^3 + (-1)^2 - 4 \cdot (-1)$

$0 \cdot P(-1) + k = 4$

$k = 4$

Yanıt / The Answer is D

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7. $x-2=1 \Rightarrow x=3$

$$\frac{P(1)}{Q(5)}=3^2-3+4 \Rightarrow P(1)=10\cdot Q(5)$$

$$Q(5)=1 \Rightarrow P(1)=1 \Rightarrow P(1)=10\cdot 1=10$$

Yanıt / The Answer is B

8. $P(x)\cdot P(x+7)=(x-6)\cdot(x+1)$

$P(x)=x-6$

$P(10)=10-6=4$

Yanıt / The Answer is E

9. $$\frac{x+1}{x^2-3x+2}=\frac{A}{\underset{(x-2)}{x-1}}+\frac{B}{\underset{(x-1)}{x-2}}$$

$A(x-2)+B(x-1)=x+1$

$A+B=1$

Yanıt / The Answer is D

10.
$$\begin{array}{r|l} x^6-x^4+x^2+m & x^2-1 \\ \hline - & B(x) \\ \hline 0 & \end{array}$$

$x^2-1=0 \Rightarrow x^2=1$

$x^6-x^4+x^2+m=(x^2)^3-(x^2)^2+x^2+m=0$

$1^3-1^2+1+m=0$

$1+m=0$

$m=-1$

Yanıt / The Answer is B

11. $x^2+1=0 \Rightarrow x^2=-1$

$a\cdot(-1)\cdot x+(b-2)x+a-1=0$

$(b-a-2)x+a-1=0$

$\Rightarrow b-a=2\ ,\ a=1,\ b=3 \Rightarrow a\cdot b=1\cdot 3=3$

Yanıt / The Answer is C

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1. P(x) bir polinomdur. / P(x) is a polynomial.

$P(x) = x^{\frac{n+12}{n}} + 5x^{n-4} + 3x + 1$

$\Rightarrow \sum n = ?$

Yanıt / The Answer is **22**

2. $\left.\begin{array}{l} \text{der}(P^3(x)\cdot Q^2(x)) = 18 \\ \text{der}\left(\dfrac{P^3(x)}{Q(x)}\right) = 9 \end{array}\right\} \Rightarrow \textbf{der (P(x))} = ?$

Yanıt / The Answer is **4**

3. $P(x) = 3x^5 - x - 1 \Rightarrow \textbf{P(1)} = ?$

Yanıt / The Answer is **1**

4. $P(x) = x^7 + 4x + a,\ P(2) = 0 \Rightarrow \textbf{a} = ?$

Yanıt / The Answer is **–136**

5. $P(x + 3) = x^3 - 2x - 3 \Rightarrow \textbf{P (–2)} = ?$

Yanıt / The Answer is **–118**

6. $P(x, y) = x^3 \cdot y - x^2 \cdot y^2 + 2 \cdot y^4$

$\Rightarrow \textbf{P}(\sqrt{5}, -\sqrt{5}) = ?$

Yanıt / The Answer is **0**

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7. $P(2x+1) = 2x+5 \Rightarrow$ **P (x) = ?**

Yanıt / The Answer is **x + 4**

8. $\left.\begin{array}{l} P(x) = x^2 + 2x \\ Q(x) = x-3 \end{array}\right\} \Rightarrow$ **P[Q(4)] + 2 · Q [P(1)] = ?**

Yanıt / The Answer is **3**

9. $\left.\begin{array}{l} P(x+2) = 3x^3 + ax^2 + x + 1 \\ P(3) = 8 \end{array}\right\} \Rightarrow$ **a = ?**

Yanıt / The Answer is **3**

10. $\left.\begin{array}{l} P(x) = x^2 + 4x - 5 \\ Q(x) = 4 - 4x \end{array}\right\} \Rightarrow \dfrac{P(x)+Q(x)}{(x-1)(x+1)} = ?$

Yanıt / The Answer is **1**

11. $P(x+2) = x^3 - ax^2 + 4x + b - 4$

$P(x) \mid x-2$

$-$

0

$\Rightarrow$ **b = ?**

Yanıt / The Answer is **4**

12. $P(x+3) = x^3 + 2x^2 + x - 5$

$\Rightarrow$ $P(x) \mid x+1$

$-$

$?$

Yanıt / The Answer is **–41**

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13. $\left.\begin{array}{l}\dfrac{P(2x-1)}{Q(x+1)} = 2x^2 - 3x + 1 \\ Q(0) = 2\end{array}\right\} \Rightarrow$ $\begin{array}{r|l} P(x) & x+3 \\ \hline - & \\ \hline ? & \end{array}$

Yanıt / The Answer is **12**

14. $P(2x-1) = x^4 - 3x^3 + 2x + a$

$\begin{array}{r|l} P(2x-1) & x-1 \\ \hline - & \\ \hline 4 & \end{array}$ $\Rightarrow$ **a = ?**

Yanıt / The Answer is **4**

15. $P(x-2) = x^5 - 2x^2 + 4x + 6$

$\Rightarrow$ $\begin{array}{r|l} P(x) & x+3 \\ \hline - & \\ \hline ? & \end{array}$

Yanıt / The Answer is **–1**

16. $P(7x) = -21x + 11$

$\Rightarrow$ $\begin{array}{r|l} P(x) & x+5 \\ \hline - & \\ \hline ? & \end{array}$

Yanıt / The Answer is **26**

17. $\left.\begin{array}{r|l} P(x) & x-2 \\ \hline - & \\ \hline -1 & \end{array} \quad \begin{array}{r|l} P(x) & x+3 \\ \hline - & \\ \hline -16 & \end{array}\right\} \Rightarrow$ $\begin{array}{r|l} P(x) & x^2+x-6 \\ \hline - & \\ \hline ? & \end{array}$

Yanıt / The Answer is **3x – 7**

18. $\begin{array}{r|l} P(x) & x-2 \\ \hline - & \\ \hline -5 & \end{array} \quad \begin{array}{r|l} P(x) & x+3 \\ \hline - & \\ \hline -20 & \end{array} \Rightarrow \begin{array}{r|l} P(x) & x^2+x-6 \\ \hline - & \\ \hline ? & \end{array}$

Yanıt / The Answer is **3x – 11**

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1. $P(2-3x) = -2x^7 + 5x^3 + 2x^2 + 8 \Rightarrow$ **P(5) = ?**

A) 3
B) 4
C) 5
D) 6
E) 7

2. $P(x^3 - 1) = 3x^2 - 4x + 1 \Rightarrow$ **P(7) = ?**

A) 2
B) 3
C) 4
D) 5
E) 6

3. $P(x) = (3x - 4) \cdot Q(x) + 3$,
$P(4) = 19 \Rightarrow$ **Q (4) = ?**

A) 1
B) 2
C) 3
D) 4
E) 5

4. $P(x) = ax + b \Rightarrow$ **P(1) – P(2) = ?**

A) –a
B) – b
C) 2b
D) a
E) b

5. $\left.\begin{array}{l} P(x+2) = 2x^4 - 3x - 2 \\ Q(x) = 3x^2 - 2x + 2 \end{array}\right\} \Rightarrow$ **P(0) · Q (0) = ?**

A) 56 B) 64 C) 72
D) 76 E) 80

6. $P(1-x) = 2x^2 + 2 \Rightarrow$ **P(x) = ?**

A) $2x^2 - 4$
B) $2x^2 + 4x$
C) $2x^2 - 4x$
D) $2x^2 + 4x + 4$
E) $2x^2 - 4x + 4$

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7. $P(x) = 2\sqrt{2}x + 12 \Rightarrow P(\sqrt{2}) = ?$

A) 5

B) 8

C) 16

D) $4\sqrt{2}$

E) $\sqrt{2} + 12$

8. $P(x, y) = 4x^2y^3 - 2x^2 + y^2x + 16 \Rightarrow P(-1, 1) = ?$

A) -2

B) 7

C) 17

D) 19

E) 20

9. $P(x-2) = 2x^2 - 3x - 6 \Rightarrow P(\sqrt{2}) = ?$

A) $-\sqrt{2} + 1$

B) $-3\sqrt{2}$

C) $2\sqrt{2}$

D) $4\sqrt{2} + 1$

E) $5\sqrt{2}$

10. $\left.\begin{array}{l} P(x+2) = 4x^3 + mx + 5 \\ P(4) = 51, \end{array}\right\} \Rightarrow m = ?$

A) 2

B) 4

C) 5

D) 6

E) 7

11. $(x + 3) \cdot P(x + 3) + 2 = x^3 + ax + 5 \Rightarrow P(3) = ?$

A) -2

B) -1

C) 0

D) 1

E) 2

12. $P(x + 1) = (x^2 + 2x - 1) \cdot Q(x) + x - 2$

$P(2) = 11 \Rightarrow Q(1) = ?$

A) 2

B) 3

C) 4

D) 5

E) 6

YANITLAR / ANSWERS							
1. E	2. D	3. B	4. A	5. C	6. E	7. C	8. C
9. E	10. E	11. D	12. E				

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1. $P(x^2) = x^4 + 5x^2 + 8 \Rightarrow$ **P(–3) = ?**

A) 1
B) 2
C) 3
D) –2
E) –3

2. $P(x+1) = x^2 - 3x \Rightarrow$ **P(x) = ?**

A) $x^2 - 2x + 1$
B) $x^2 - 5x + 4$
C) $2x^2 - 6x + 3$
D) $x^2 - 4x + 2$
E) $x^2 - 5x - 2$

3. $\left.\begin{array}{l} P(x) = 3x^2 + 8x \\ Q(x) = 4x + 3 \end{array}\right\} \Rightarrow$ **P[Q(1)] + Q[P(3)] = ?**

A) 210
B) 310
C) 410
D) 510
E) 610

4. $\left.\begin{array}{l} P(x) = 4x^2 + 7x - 8 \\ Q(x) = 3x^3 + 5x^2 - 4x - 7 \\ P(x) + Q(x) = T(x) \end{array}\right\} \Rightarrow$ **T(1) = ?**

A) –2 B) –1 C) 0
D) 15 E) 24

5. $Q(x) = 3ax + 3a - 12$

$$\begin{array}{r|l} Q(x) & x-3 \\ \hline - & \\ \hline 0 & \end{array} \Rightarrow \begin{array}{r|l} \mathbf{Q(x)} & \mathbf{x+3} \\ \hline - & \\ \hline ? & \end{array}$$

A) –18 B) –14 C) –10
D) –6 E) –2

6. $P(x-2) = Q(x+2)\cdot(x^2 - x - 2)$

$$\begin{array}{r|l} Q(x) & x-3 \\ \hline - & \\ \hline -4 & \end{array} \Rightarrow \begin{array}{r|l} \mathbf{P(x)} & \mathbf{x+1} \\ \hline - & \\ \hline ? & \end{array}$$

A) 18 B) 14 C) 10
D) 8 E) 4

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7. $\frac{25x-9}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \Rightarrow A+B=?$

A) 5
B) 8
C) 12
D) 17
E) 25

8. $\left.\begin{matrix} P(3)=6 \\ P(4)=10 \end{matrix}\right\} \Rightarrow$ $P(x)$ | $x^2-7x+12$; $-$; $?$

A) $4x+2$
B) $4x+4$
C) $4x-4$
D) $4x+6$
E) $4x-6$

9. $P(x)$ | x^3-4x^2+3x ; $-$; $3x^2+2x$ $\Rightarrow$ $P(x)$ | $x-3$; $-$; $?$

A) 30
B) 31
C) 32
D) 33
E) 34

10. $P(3x-2) = (x^2-3x-12)\cdot Q(x+5) - 4x + 12$

$P(x)$ | $x-1$; $-$; 36 $\Rightarrow$ $Q(x)$ | $x-6$; $-$; $?$

A) 6 B) 4 C) 0
D) -2 E) -9

11. $P(x+2) = Q(x-2)\cdot(x^2+x+2)$

$Q(x)$ | $x-3$; $-$; 4 $\Rightarrow$ $P(x)$ | $x-7$; $-$; $?$

A) 128 B) 64 C) 48
D) 43 E) 16

12. $\left.\begin{matrix} P(x) \mid x-3 \\ -\ 4 \\ Q(x) \mid x-3 \\ -\ 7 \end{matrix}\right\} \Rightarrow P(x)\cdot Q(x) \mid x-3$; $-$; $?$

A) 12 B) 18 C) 28
D) 36 E) 56

YANITLAR / ANSWERS							
1. B	2. B	3. C	4. C	5. A	6. D	7. E	8. E
9. D	10. D	11. A	12. C				

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BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Bombelli, Rafael [1526 – 1573]

Italyan matematikçi. I' Algebra adlı çalışmasında, kübik denklemlerin çözümünde negatif sayıların karekökle-rinin elde edildiğini ve karmaşık sayılarla çözüm yolunu ilk gösterenlerdendir.

Italian mathematician who in his book l'Algebra, seems to have been the first to go some way in working with complex numbers, when the square roots of negative numbers occurred in the solution of cubic equations.

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İKİNCİ DERECEDEN DENKLEMLER
QUADRATIC EQUATIONS

BÖLÜM 2
CHAPTER

Tanım : a, b, c ∈ R, a ≠ 0 olmak üzere, $ax^2 + bx + c = 0$ biçimindeki eşitliklere *ikinci dereceden denklem* denir.
a, b, c sayılarına *denklemin katsayıları* denir. Denklemi sağlayan sayılara *denklemin kökleri*, köklerin kümesine de *çözüm kümesi* denir.

Düşünün

$b = \frac{1}{a-2} \Rightarrow a + ab - \frac{1}{b} - 2b = ?$

İKİNCİ DERECEDEN DENKLEMLERİN ÇÖZÜM KÜMESİNİN BULUNMASI

1. ÇARPANLARA AYIRMA YÖNTEMİ

a, b, c ∈ R, a ≠ 0 olmak üzere, $ax^2 + bx + c = 0$ denklemi çarpanlarına ayrılıyorsa denklemin kökleri çarpanlara ayrılarak bulunur.

Etkinlik

1. $x^2 - x - 20 = 0$

x −5
x 4

$4x - 5x = -x$
$(x - 5)(x + 4) = 0$
$x - 5 = 0 \vee x + 4 = 0$
$x = 5 \vee x = -4$
ÇK = {−4, 5}

2. $x^2 + 2x - 35 = 0$

x 7
x −5

$7x - 5x = 2x$
$(x + 7)(x - 5) = 0$
$x + 7 = 0 \vee x - 5 = 0$
$x = -7 \quad x = 5$
ÇK = {−7, 5}

Yanıt : 3

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Think about it

$b = \frac{1}{a-2} \Rightarrow a + ab - \frac{1}{b} - 2b = ?$

Definition: An equation in the form $ax^2 + bx + c = 0$ is called an *quadratic equation* where $a, b, c \in R, a \neq 0$.

The numbers of a,b and c are *coefficients in the equation.* Numbers satisfying the equation are called *roots of the equation,* the set containing the roots are called *solution set.*

FINDING the SOLUTION SET of the QUADRATIC EQUATIONS

1. FACTORIZATION METHOD

If the equation $ax^2 + bx + c = 0$ is factorization, the roots of equation is found by the factorization method.

Activity

1. $x^2 - x - 20 = 0$

$4x - 5x = -x$

$(x - 5)(x + 4) = 0$

$x - 5 = 0 \vee x + 4 = 0$

$x = 5 \vee x = -4$

$SS = \{-4, 5\}$

2. $x^2 + 2x - 35 = 0$

$7x - 5x = 2x$

$(x + 7)(x - 5) = 0$

$x + 7 = 0 \vee x - 5 = 0$

$x = -7 \quad x = 5$

$SS = \{-7, 5\}$

Answer : 3

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ÖRNEK 1

$2x^2 + 3x + 1 = 0 \Rightarrow$ ÇK = ?

A) $\{-1, -2\}$ B) $\left\{\frac{1}{2}, -1\right\}$ C) $\left\{-1, -\frac{1}{2}\right\}$

D) $\left\{\frac{1}{2}, 1\right\}$ E) $\left\{1, -\frac{1}{2}\right\}$

PEKİŞTİRME

$3x^2 + x - t = 0$

$x_1 = 1 \Rightarrow t = ?$

ÇÖZÜM

Yanıt C

$2x^2 + 3x + 1 = 0$

$\downarrow \quad \downarrow$

$2x \quad 1$

$x \quad 1$

$2x^2 + 3x + 1 = (2x + 1)\cdot(x + 1) = 0 \Rightarrow \quad 2x+1=0 \quad x+1=0$

$x = -\frac{1}{2} \quad x = -1$

ÇK = $\left\{-1, -\frac{1}{2}\right\}$

YANIT : 4

ÖRNEK 2

$3x^2 + 2x - 1 = 0 \Rightarrow$ ÇK = ?

A) $\{-1\}$ B) $\left\{-1, \frac{1}{3}\right\}$ C) $\left\{\frac{1}{3}\right\}$ D) $\varnothing$ E) $\left\{1, -\frac{1}{3}\right\}$

ÇÖZÜM

Yanıt B

$3x^2 + 2x - 1 = (3x - 1)\cdot(x + 1) = 0$

$\downarrow \quad \downarrow$

$3x \quad -1 \qquad 3x-1=0 \;,\; x+1=0$

$x \quad +1 \qquad x = \frac{1}{3} \quad x = -1$

ÇK = $\left\{-1, \frac{1}{3}\right\}$

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REINFORCE

$3x^2 + x - t = 0$

$x_1 = 1 \Rightarrow t = ?$

ANSWER : 4

EXAMPLE 1

$2x^2 + 3x + 1 = 0 \Rightarrow SS = ?$

A) $\{-1, -2\}$ B) $\left\{\frac{1}{2}, -1\right\}$ C) $\left\{-1, -\frac{1}{2}\right\}$

D) $\left\{\frac{1}{2}, 1\right\}$ E) $\left\{1, -\frac{1}{2}\right\}$

SOLUTION

The answer is **C**

$2x^2 + 3x + 1 = 0$

$\downarrow \quad \downarrow$

$2x \quad 1$

$x \quad 1$

$2x^2 + 3x + 1 = (2x + 1)\cdot(x + 1) = 0 \Rightarrow 2x+1=0 \quad x+1=0$

$x = -\frac{1}{2} \quad x = -1$

$SS = \left\{-1, -\frac{1}{2}\right\}$

EXAMPLE 2

$3x^2 + 2x - 1 = 0 \Rightarrow SS = ?$

A) $\{-1\}$ B) $\left\{-1, \frac{1}{3}\right\}$ C) $\left\{\frac{1}{3}\right\}$ D) $\varnothing$ E) $\left\{1, -\frac{1}{3}\right\}$

SOLUTION

The answer is **B**

$3x^2 + 2x - 1 = (3x - 1)\cdot(x + 1) = 0$

$\downarrow \quad \downarrow$

$3x \quad -1 \quad 3x-1=0 \ , \ x+1=0$

$x \quad +1 \quad x = \frac{1}{3} \quad x = -1$

$SS = \left\{-1, \frac{1}{3}\right\}$

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2. İKİNCİ DERECEDEN DENKLEME DÖNÜŞTÜRÜLEBİLEN DENKLEMLER

A. $f(x) \cdot g(x) = 0 \Rightarrow f(x) = 0$ veya $g(x) = 0$

ÖRNEK 3

$x^3 - 9x = 0 \Rightarrow$ ÇK = ?

A) {–3, 0, 3} B) {–2, 0, 2} C) {–1, 0, 1}

D) {–4, 0, 4} E) ∅

ÇÖZÜM

Yanıt A

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x = 0$ veya $x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x - 3 = 0$, $x + 3 = 0$

$x = 3$ $x = -3$

ÇK = {–3, 0, 3}

PEKİŞTİRME

$x^2 + x^3 + x + 1 = 0 \Rightarrow$ ÇK = ?

YANIT : {–1}

ÖRNEK 4

$\frac{2x^2 - 1}{x - 3} = x + 3 + \frac{17}{x - 3} \Rightarrow$ ÇK = ?

A) {–3, 3} B) {–3} C) {3} D) ∅ E) R

ÇÖZÜM

Yanıt B

$\frac{2x^2 - 1}{x - 3} = \frac{x + 3}{\underset{(x-3)}{1}} + \frac{17}{x - 3}$

$\frac{2x^2 - 1}{x - 3} = \frac{x^2 - 9 + 17}{x - 3}$

$2x^2 - 1 = x^2 + 8$

$x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x = 3$, $x = -3$

$x \neq 3 \Rightarrow$ ÇK = {–3}

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2. THE EQUATIONS THAT CAN BE CONVERTED INTO QUADRATIC EQUATIONS

A. $f(x) \cdot g(x) = 0 \Rightarrow f(x) = 0$ or $g(x) = 0$

REINFORCE

$x^2 + x^3 + x + 1 = 0 \Rightarrow SS = ?$

ANSWER : {–1}

EXAMPLE 3

$x^3 - 9x = 0 \Rightarrow$ **SS = ?**

A) {–3, 0, 3} B) {–2, 0, 2} C) {–1, 0, 1} D) {–4, 0, 4} E) ∅

SOLUTION The answer is **A**

$$x^3 - 9x = 0$$
$$x(x^2 - 9) = 0$$

$x = 0$ or $x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x - 3 = 0$, $x + 3 = 0$

$x = 3$ $x = -3$

$SS = \{-3, 0, 3\}$

EXAMPLE 4

$\frac{2x^2 - 1}{x - 3} = x + 3 + \frac{17}{x - 3} \Rightarrow$ **SS = ?**

A) {–3, 3} B) {–3} C) {3} D) ∅ E) R

SOLUTION The answer is **B**

$$\frac{2x^2 - 1}{x - 3} = \frac{x + 3}{\underset{(x-3)}{1}} + \frac{17}{x - 3}$$

$$\frac{2x^2 - 1}{x - 3} = \frac{x^2 - 9 + 17}{x - 3}$$

$$2x^2 - 1 = x^2 + 8$$

$$x^2 - 9 = 0$$

$$(x - 3)(x + 3) = 0$$

$$x = 3 \text{ , } x = -3$$

$x \neq 3 \Rightarrow SS = \{-3\}$

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B. Değişken Değiştirerek Çözülebilen Denklemler

ÖRNEK 5

$8x^6 - 35x^3 + 27 = 0 \Rightarrow$ ÇK = ?

A) $\left\{\frac{3}{2}\right\}$ B) $\left\{\frac{2}{3}, 1\right\}$ C) $\left\{1, \frac{3}{2}\right\}$ D) $\left\{-1, \frac{3}{2}\right\}$ E) $\{2\}$

ÇÖZÜM

Yanıt C

$x^3 = t \Rightarrow x^6 = t^2$

$8x^6 - 35x^3 + 27 = 8t^2 - 35t + 27 = 0$

$8t \downarrow \quad -27 \downarrow$

$t \quad -1$

$(8t-27)(t-1) = 0 \Rightarrow t = \frac{27}{8}, \quad t = 1$

$x^3 = \frac{27}{8}, \quad x^3 = 1$

$x = \frac{3}{2} \qquad x = 1 \Rightarrow ÇK = \left\{\frac{3}{2}, 1\right\}$

ÖRNEK 6

$2\left(\frac{1}{x} - 1\right)^2 + 5\left(\frac{1}{x} - 1\right) = 3 \Rightarrow$ ÇK = ?

A) $\left\{\frac{1}{2}, \frac{2}{3}\right\}$ B) $\left\{\frac{1}{2}, -\frac{2}{3}\right\}$ C) $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$

D) $\left\{-1, -\frac{3}{2}\right\}$ E) $\{1, -1\}$

ÇÖZÜM

Yanıt C

$\frac{1}{x} - 1 = t \Rightarrow 2t^2 + 5t = 3$

$2t^2 + 5t - 3 = 0$

$(2t-1)(t+3) = 0$

$2t - 1 = 0 \qquad t + 3 = 0$

$t = \frac{1}{2} \qquad t = -3$

$\frac{1}{x} - 1 = t \Rightarrow \frac{1}{x} - 1 = \frac{1}{2} \qquad \frac{1}{x} - 1 = -3$

$\frac{1}{x} = \frac{3}{2} \qquad \frac{1}{x} = -2$

$x = \frac{2}{3} \qquad x = -\frac{1}{2}$

$ÇK = \left\{-\frac{1}{2}, \frac{2}{3}\right\}$

PEKİŞTİRME

$\frac{x-1}{x+1} = \frac{x+1}{x-1} + \frac{8}{3} \Rightarrow ÇK = ?$

YANIT : {–2, 1/2}

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B. The Equations That Can Be Solved By Changing the Variables

EXAMPLE 5

$8x^6 - 35x^3 + 27 = 0 \Rightarrow$ **SS = ?**

A) $\left\{\frac{3}{2}\right\}$ B) $\left\{\frac{2}{3}, 1\right\}$ C) $\left\{1, \frac{3}{2}\right\}$ D) $\left\{-1, \frac{3}{2}\right\}$ E) $\{2\}$

SOLUTION

The answer is **C**

$x^3 = t \Rightarrow x^6 = t^2$

$8x^6 - 35x^3 + 27 = 8t^2 - 35t + 27 = 0$

$8t \quad -27$

$t \quad -1$

$(8t-27)(t-1) = 0 \Rightarrow t = \frac{27}{8}, \quad t = 1$

$x^3 = \frac{27}{8}, \quad x^3 = 1$

$x = \frac{3}{2} \qquad x = 1 \Rightarrow SS = \left\{\frac{3}{2}, 1\right\}$

REINFORCE

$\frac{x-1}{x+1} = \frac{x+1}{x-1} + \frac{8}{3} \Rightarrow SS = ?$

EXAMPLE 6

$2\left(\frac{1}{x} - 1\right)^2 + 5\left(\frac{1}{x} - 1\right) = 3 \Rightarrow$ **SS = ?**

A) $\left\{\frac{1}{2}, \frac{2}{3}\right\}$ B) $\left\{\frac{1}{2}, -\frac{2}{3}\right\}$ C) $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$

D) $\left\{-1, -\frac{3}{2}\right\}$ E) $\{1, -1\}$

ANSWER : {–2, 1/2}

SOLUTION

The answer is **C**

$\frac{1}{x} - 1 = t \Rightarrow 2t^2 + 5t = 3$

$2t^2 + 5t - 3 = 0$

$(2t-1)(t+3) = 0$

$2t - 1 = 0 \qquad t + 3 = 0$

$t = \frac{1}{2} \qquad t = -3$

$\frac{1}{x} - 1 = t \Rightarrow \frac{1}{x} - 1 = \frac{1}{2} \qquad \frac{1}{x} - 1 = -3$

$\frac{1}{x} = \frac{3}{2} \qquad \frac{1}{x} = -2$

$x = \frac{2}{3} \qquad x = -\frac{1}{2}$

$SS = \left\{-\frac{1}{2}, \frac{2}{3}\right\}$

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C. Köklü Denklemler

Bu tür denklemler çözülürken, verilen denklem $\sqrt{f(x)} = g(x)$ biçiminde yazılır ve her iki tarafın kök kuvveti alınarak kökten kurtarılır. Eğer kök kuvveti çift ise bulunan x'ler denklemde yerine yazılarak denklemi sağlayıp sağlamadığına bakılır. Denklemi sağlayan x'ler çözüm kümesini oluşturur.

ÖRNEK 7

$\sqrt{3x+1} - x = -1 \Rightarrow$ ÇK = ?

A) {1} B) {2} C) {3} D) {4} E) {5}

ÇÖZÜM

Yanıt E

$\sqrt{3x+1} - x = -1$ [Verilen denklem]

$\sqrt{3x+1} = x - 1$ [Her tarafa x'e ekle.]

$\left(\sqrt{3x+1}\right)^2 = (x-1)^2$ [Her taraftan kare al.]

$3x + 1 = x^2 - 2x + 1$

$x^2 - 5x = 0$ [Benzer terimleri topla.]

$x(x-5) = 0$

$x = 0 \ , \ x - 5 = 0$

$x = 5$

$x = 0 \Rightarrow \sqrt{3\cdot 0+1} - 0 = -1$

$1 - 0 = -1$

$1 \neq -1$

$x = 5 \Rightarrow \sqrt{3\cdot 5+1} - 5 = -1$

$4 - 5 = -1$

$-1 = -1$

ÇK = {5}

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NOT

$x_1 = x_2 \Rightarrow$ çift katlı kök (çakışık kök.)

$x_1 = -x_2 \Rightarrow$ simetrik kök ($b = 0$).

C. The Equations Involving Radicals

To the solving this equation, we write the form $\sqrt{f(x)} = g(x)$ to the given equation and the radical is reduce which take the both sides indices. If index is odd, we write in the equation of the finding values x. If the value x is satisfying equation, this value in solution set.

EXAMPLE 7

$\sqrt{3x+1} - x = -1 \Rightarrow$ **SS = ?**

A) {1} B) {2} C) {3} D) {4} E) {5}

SOLUTION

The answer is **E**

$\sqrt{3x+1} - x = -1$ [Given equation]

$\sqrt{3x+1} = x - 1$ [Add x both sides.]

$\left(\sqrt{3x+1}\right)^2 = (x-1)^2$ [Square the both sides.]

$3x+1 = x^2 - 2x + 1$

$x^2 - 5x = 0$ [Combine like terms.]

$x(x-5) = 0$

$x = 0 \ , \ x - 5 = 0$

$x = 5$

$x = 0 \Rightarrow \sqrt{3\cdot 0+1} - 0 = -1$

$1 - 0 = -1$

$1 \neq -1$

$x = 5 \Rightarrow \sqrt{3\cdot 5+1} - 5 = -1$

$4 - 5 = -1$

$-1 = -1$

$SS = \{5\}$

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3. DİSKRİMİNANT YÖNTEMİ

Çarpanlara ayırma yöntemi, çarpanlara ayrılabilen ikinci dereceden denklemlerde etkilidir. Fakat birçok ikinci dereceden denklem, çarpanlara ayrılmayabilir. Bu tür denklemleri çözmek için, genel bir formül diskriminant yöntemidir.

Bu önemli formülü, aşağıdaki gibi tam kareye tamamlayarak elde ederiz.

$ax^2 + bx + c = 0$ [ikinci dereceden denklem]

$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$ [Ortak parantez]

$a \neq 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ [Sadeleştir.]

$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$ [Tamkareye tamamla.]

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ [Her iki tarafa $\frac{b^2}{4a^2} - \frac{c}{a}$ ekle.]

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$ [Her iki taraftan karekök al.]

$x = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$ [Sadeleştir.]

$\Delta = b^2 - 4ac \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$ [Sonuç]

$ax^2 + bx + c = 0$ denklemi için;

$\Delta = b^2 - 4ac$ sayısına denklemin *diskriminantı* denir.

$ax^2 + bx + c = 0$ denkleminde,

a. $\Delta > 0$ ise, denklemin birbirinden farklı iki reel çözümü vardır.

Bu kökler aşağıdaki gibidir.

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a},$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

b. $\Delta = 0$ ise, denklemin birbirine eşit iki çözümü vardır.

Bu kökler aşağıdaki gibidir.

$$x_1 = x_2 = -\frac{b}{2a}$$

c. $\Delta < 0$ ise denklemin reel kökü yoktur, karmaşık kök vardır. Çözüm kümesi boş kümedir.

NOTE

$x_1 = x_2 \Rightarrow$ double root (eclipsed root)

$x_1 = -x_2 \Rightarrow$ symmetric root ($b = 0$).

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3. THE DISCRIMINANT METHOD

The factorization method are efficient for special quadratic equations that are factorable. There are, however, many quadratic equations that cannot be solved efficiently by factorization. The general formula is the discriminant method that can be used to solve any quadratic equation.

We can develop this important formula by a process called completing the square, as follows.

$ax^2 + bx + c = 0$ [Quadratic equation]

$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$ [Common paranthesis]

$a \neq 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ [Simplify.]

$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$ [Complete the square.]

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ [Add $\frac{b^2}{4a^2} - \frac{c}{a}$ to both sides.]

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$ [Take the square root to both sides.]

$x = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$ [Simplify.]

$\Delta = b^2 - 4ac \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$ [Conclusion]

For the equation $ax^2 + bx + c = 0$,
$\Delta = b^2 - 4ac$ is called the *discriminant* of the equation.

In equation $ax^2 + bx + c = 0$,

a. If $\Delta > 0$, the equation has two distinct real solutions.
These roots are as follows:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a},$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

b. If $\Delta = 0$, there are two equal repeated real solutions.
These roots are as follows:

$$x_1 = x_2 = -\frac{b}{2a}$$

c. If $\Delta < 0$, the equation has imaginary solutions. The solution set is an empty set.

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Etkinlik

a. $x^2 + 3x + 1 = 0 \Rightarrow$ **ÇK = ?**

$\Delta = 3^2 - 4 \cdot 1 \cdot 1 = 5 > 0$

$\Rightarrow x_{1,2} = \dfrac{-3 \pm \sqrt{5}}{2}$

b. $4x^2 + 4x^2 + 1 = 0 \Rightarrow$ **ÇK = ?**

$\Delta = 4^2 - 4 \cdot 4 = 0$

$\Rightarrow x_1 = x_2 = \dfrac{-4}{8} = -\dfrac{1}{2}$

c. $2x^2 + x + 1 = 0 \Rightarrow$ **ÇK = ?**

$\Delta = 1^2 - 4 \cdot 2 \cdot 1$

$= 1 - 8$

$= -7 < 0$

$\Rightarrow$ ÇK = $\varnothing$

ÖRNEK 8

$2x^2 - 5x + 3 = 0 \Rightarrow$ **ÇK = ?**

A) $\left\{1, -\dfrac{3}{2}\right\}$ B) $\left\{1, \dfrac{3}{2}\right\}$ C) $\left\{\dfrac{4}{3}, 1\right\}$ D) $\left\{-1, \dfrac{3}{2}\right\}$ E) $\left\{\dfrac{4}{3}, -1\right\}$

ÇÖZÜM

Yanıt **B**

$2x^2 - 5x + 3 = 0$

$a = 2,\ b = -5,\ c = 3$

$\Delta = b^2 - 4ac$

$\Delta = (-5)^2 - 4 \cdot 2 \cdot 3$

$\Delta = 1 > 0$

$x_1 = \dfrac{-b - \sqrt{\Delta}}{2a} = \dfrac{-(-5) - \sqrt{1}}{2 \cdot 2} = \dfrac{5-1}{4} = \dfrac{4}{4} = 1$

$x_2 = \dfrac{-b + \sqrt{\Delta}}{2a} = \dfrac{-(-5) + \sqrt{1}}{2 \cdot 2} = \dfrac{5+1}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

ÇK = $\left\{1, \dfrac{3}{2}\right\}$

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Activity

a. $x^2 + 3x + 1 = 0 \Rightarrow$ **SS = ?**

$\Delta = 3^2 - 4 \cdot 1 \cdot 1 = 5 > 0$

$\Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{5}}{2}$

b. $4x^2 + 4x^2 + 1 = 0 \Rightarrow$ **SS = ?**

$\Delta = 4^2 - 4 \cdot 4 = 0$

$\Rightarrow x_1 = x_2 = \frac{-4}{2} = -\frac{1}{2}$

c. $2x^2 + x + 1 = 0 \Rightarrow$ **SS = ?**

$\Delta = 1^2 - 4 \cdot 2 \cdot 1$

$= 1 - 8$

$= -7 < 0$

$\Rightarrow SS = \varnothing$

EXAMPLE 8

$2x^2 - 5x + 3 = 0 \Rightarrow$ **SS = ?**

A) $\left\{1, -\frac{3}{2}\right\}$ B) $\left\{1, \frac{3}{2}\right\}$ C) $\left\{\frac{4}{3}, 1\right\}$ D) $\left\{-1, \frac{3}{2}\right\}$ E) $\left\{\frac{4}{3}, -1\right\}$

SOLUTION

The answer is **B**

$2x^2 - 5x + 3 = 0$

$a = 2,\ b = -5,\ c = 3$

$\Delta = b^2 - 4ac$

$\Delta = (-5)^2 - 4 \cdot 2 \cdot 3$

$\Delta = 1 > 0$

$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-5) - \sqrt{1}}{2 \cdot 2} = \frac{5-1}{4} = \frac{4}{4} = 1$

$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-5) + \sqrt{1}}{2 \cdot 2} = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2}$

$SS = \left\{1, \frac{3}{2}\right\}$

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ÖRNEK 9

$9x^2 - 12x + 4 = 0 \Rightarrow$ **ÇK = ?**

A) $\left\{\frac{5}{2}\right\}$ B) $\left\{\frac{3}{2}\right\}$ C) $\{2\}$ D) $\left\{\frac{2}{3}\right\}$ E) $\{1\}$

ÇÖZÜM

Yanıt **D**

$9x^2 - 12x + 4 = 0$

$a = 9,\ b = -12,\ c = 4$

$\Delta = (-12)^2 - 4 \cdot 9 \cdot 4$

$\Delta = 144 - 144 = 0$

$$x_1 = x_2 = \frac{-b}{2a} = \frac{-(-12)}{2 \cdot 9} = \frac{12}{18} = \frac{2}{3}$$

PEKİŞTİRME

$x^2 - 2x + t + 1 = 0$

$ÇK = \{x_1, x_2\}, x_1 = x_2 \Rightarrow t = ?$

YANIT : 0

ÖRNEK 10

$x^2 + 2x + 4 = 0 \Rightarrow$ **ÇK = ?**

A) $\{1, -1\}$ B) $\{1\}$ C) $\{-1\}$ D) $\varnothing$ E) R

ÇÖZÜM

Yanıt **D**

$x^2 + 2x + 4 = 0$

$\Delta = b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot 4 = -12 < 0 \Rightarrow ÇK = \varnothing$

ÖRNEK 11

$a \in Z,$

$2x^2 - 4x + a = 0$

$\Delta > 0 \Rightarrow$ **max(a) = ?**

A) –1 B) 0 C) 1 D) 2 E) 3

ÇÖZÜM

Yanıt **C**

$\Delta = 4^2 - 4 \cdot 2 \cdot a > 0$

$16 > 8a$

$2 > a$

$\Rightarrow max(a) = 1$

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REINFORCE

$x^2 - 2x + t + 1 = 0$

$SS = \{x_1, x_2\}, x_1 = x_2 \Rightarrow t = ?$

ANSWER : 0

EXAMPLE 9

$9x^2 - 12x + 4 = 0 \Rightarrow$ **SS = ?**

A) $\left\{\frac{5}{2}\right\}$ B) $\left\{\frac{3}{2}\right\}$ C) $\{2\}$ D) $\left\{\frac{2}{3}\right\}$ E) $\{1\}$

SOLUTION The answer is **D**

$9x^2 - 12x + 4 = 0$

$a = 9, \ b = -12, \ c = 4$

$\Delta = (-12)^2 - 4 \cdot 9 \cdot 4$

$\Delta = 144 - 144 = 0$

$$x_1 = x_2 = \frac{-b}{2a} = \frac{-(-12)}{2 \cdot 9} = \frac{12}{18} = \frac{2}{3}$$

EXAMPLE 10

$x^2 + 2x + 4 = 0 \Rightarrow$ **SS = ?**

A) $\{1, -1\}$ B) $\{1\}$ C) $\{-1\}$ D) $\varnothing$ E) R

SOLUTION The answer is **D**

$x^2 + 2x + 4 = 0$

$\Delta = b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot 4 = -12 < 0 \Rightarrow SS = \varnothing$

EXAMPLE 11

$a \in Z,$

$2x^2 - 4x + a = 0$

$\Delta > 0 \Rightarrow$ **max(a) = ?**

A) -1 B) 0 C) 1 D) 2 E) 3

SOLUTION The answer is **C**

$\Delta = 4^2 - 4 \cdot 2 \cdot a > 0$

$16 > 8a$

$2 > a$

$\Rightarrow \max(a) = 1$

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KÖKLER İLE KATSAYILAR ARASINDAKİ BAĞINTILAR

$ax^2 + bx + c = 0$ denkleminde $\Delta > 0$ ise,

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a}, \quad x_2 = \frac{-b-\sqrt{\Delta}}{2a}$$

1. $x_1 + x_2 = \frac{-b}{a}$
2. $x_1 \cdot x_2 = \frac{c}{a}$
3. $|x_1 - x_2| = \frac{\sqrt{\Delta}}{|a|}$

SONUÇLAR

1. $\frac{1}{x_1} + \frac{1}{x_2} = \frac{-b}{c}$
2. $x_1^2 x_2 + x_2^2 x_1 = x_1 \cdot x_2 (x_1 + x_2)$
$= \frac{c}{a} \cdot \frac{-b}{a} = \frac{-bc}{a^2}$
3. $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2$
$= \frac{b^2}{a^2} - \frac{2c}{a}$

ÖRNEK 12

$2x^2 - 3x - 1 = 0 \Rightarrow x_1 + x_2 + x_1 \cdot x_2 = ?$

A) 1 B) $\frac{1}{2}$ C) 0 D) $\frac{3}{2}$ E) $\frac{5}{2}$

ÇÖZÜM

Yanıt **A**

$2x^2 - 3x - 1 = 0 \Rightarrow a = 2,\ b = -3,\ c = -1$

$$x_1 + x_2 = -\frac{b}{a} = -\frac{-3}{2} = \frac{3}{2}$$

$$x_1 \cdot x_2 = \frac{c}{a} = -\frac{1}{2}$$

$$x_1 + x_2 + x_1 \cdot x_2 = \frac{3}{2} - \frac{1}{2} = 1$$

ÖRNEK 13

$x^2 + 7x + m = 7,\ x_1 \cdot x_2 = 10 \Rightarrow m = ?$

A) 11 B) 13 C) 15 D) 17 E) 19

ÇÖZÜM

Yanıt **D**

$x^2 + 7x + m - 7 = 0 \Rightarrow a = 1,\ b = 7,\ c = m - 7$

$$x_1 \cdot x_2 = 10$$

$$\frac{c}{a} = 10$$

$$\frac{m-7}{1} = 10$$

$$m = 17$$

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CONCLUSIONS

1. $\frac{1}{x_1}+\frac{1}{x_2}=\frac{-b}{c}$

2. $x_1^2x_2+x_2^2x_1=x_1\cdot x_2(x_1+x_2)$

$$=\frac{c}{a}\cdot\frac{-b}{a}=\frac{-bc}{a^2}$$

3. $x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2$

$$=\frac{b^2}{a^2}-\frac{2c}{a}$$

RELATIONSHIPS BETWEEN ROOTS and COEFFICIENTS

If $\Delta > 0$ in the equation $ax^2 + bx + c = 0$,

$$x_1=\frac{-b+\sqrt{\Delta}}{2a},\ x_2=\frac{-b-\sqrt{\Delta}}{2a}$$

1. $x_1+x_2=\frac{-b}{a}$

2. $x_1\cdot x_2=\frac{c}{a}$

3. $|x_1-x_2|=\frac{\sqrt{\Delta}}{|a|}$

EXAMPLE 12

$2x^2-3x-1=0 \Rightarrow x_1+x_2+x_1\cdot x_2=?$

A) 1 B) $\frac{1}{2}$ C) 0 D) $\frac{3}{2}$ E) $\frac{5}{2}$

SOLUTION

The answer is A

$2x^2-3x-1=0 \Rightarrow a=2,\ b=-3,\ c=-1$

$$x_1+x_2=-\frac{b}{a}=-\frac{-3}{2}=\frac{3}{2}$$

$$x_1\cdot x_2=\frac{c}{a}=-\frac{1}{2}$$

$$x_1+x_2+x_1\cdot x_2=\frac{3}{2}-\frac{1}{2}=1$$

EXAMPLE 13

$x^2+7x+m=7$, $x_1\cdot x_2=10 \Rightarrow m=?$

A) 11 B) 13 C) 15 D) 17 E) 19

SOLUTION

The answer is D

$x^2+7x+m-7=0 \Rightarrow a=1,\ b=7,\ c=m-7$

$$x_1\cdot x_2=10$$

$$\frac{c}{a}=10$$

$$\frac{m-7}{1}=10$$

$$m=17$$

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ÖRNEK 14

$2x^2 - 7x + 3 = 0$, ÇK = $\{x_1, x_2\} \Rightarrow x_1^2 \cdot x_2 + x_1 \cdot x_2^2 = ?$

A) $\frac{9}{4}$ B) $\frac{13}{4}$ C) $\frac{15}{4}$ D) $\frac{17}{4}$ E) $\frac{21}{4}$

ÇÖZÜM

Yanıt E

$a = 2, b = -7, c = 3 \Rightarrow x_1 \cdot x_2 = \frac{c}{a} = \frac{3}{2}$

$x_1 + x_2 = \frac{-b}{a} = \frac{7}{2}$

$x_1^2 \cdot x_2 + x_1 \cdot x_2^2 = x_1 \cdot x_2 (x_1 + x_2)$

$= \frac{7}{2} \cdot \frac{3}{2}$

$= \frac{21}{4}$

PEKİŞTİRME

$x^2 + kx + 25 = 0$, SS = $\{x_1, x_2\}$,

$\frac{1}{\sqrt{x_1}} + \sqrt{x_2} = 3 \Rightarrow k = ?$

YANIT : –41/4

ÖRNEK 15

$2x^2 - 4x - 5 = 0 \Rightarrow \frac{x_1}{1-x_1} + \frac{x_2}{1-x_2} = ?$

A) –6 B) –5 C) –4 D) –3 E) –2

ÇÖZÜM

Yanıt E

$\frac{x_1}{1-x_1} + \frac{x_2}{1-x_2} = \frac{x_1 - x_1x_2 + x_2 - x_1x_2}{1 - x_2 - x_1 + x_1x_2}$

$\left(\frac{}{(1-x_2)} \quad \frac{}{(1-x_1)}\right)$

$= \frac{x_1 + x_2 - 2x_1x_2}{1 + x_1x_2 - (x_1 + x_2)}$

$= \frac{2 - 2 \cdot \frac{-5}{2}}{1 - \frac{5}{2} - 2} = \frac{7}{\frac{-7}{2}} = -2$

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EXAMPLE 14

$2x^2 - 7x + 3 = 0$, $SS = \{x_1, x_2\} \Rightarrow x_1^2 \cdot x_2 + x_1 \cdot x_2^2 = ?$

A) $\frac{9}{4}$ B) $\frac{13}{4}$ C) $\frac{15}{4}$ D) $\frac{17}{4}$ E) $\frac{21}{4}$

SOLUTION

The answer is E

$a = 2, b = -7, c = 3 \Rightarrow x_1 \cdot x_2 = \frac{c}{a} = \frac{3}{2}$

$x_1 + x_2 = \frac{-b}{a} = \frac{7}{2}$

$x_1^2 \cdot x_2 + x_1 \cdot x_2^2 = x_1 \cdot x_2 (x_1 + x_2)$

$= \frac{7}{2} \cdot \frac{3}{2}$

$= \frac{21}{4}$

REINFORCE

$x^2 + kx + 25 = 0$, $ÇK = \{x_1, x_2\}$,

$\frac{1}{\sqrt{x_1}} + \sqrt{x_2} = 3 \Rightarrow k = ?$

ANSWER : –41/4

EXAMPLE 15

$2x^2 - 4x - 5 = 0 \Rightarrow \frac{x_1}{1-x_1} + \frac{x_2}{1-x_2} = ?$

A) –6 B) –5 C) –4 D) –3 E) –2

SOLUTION

The answer is E

$\frac{x_1}{\underset{(1-x_2)}{1-x_1}} + \frac{x_2}{\underset{(1-x_1)}{1-x_2}} = \frac{x_1 - x_1x_2 + x_2 - x_1x_2}{1 - x_2 - x_1 + x_1x_2}$

$= \frac{x_1 + x_2 - 2x_1x_2}{1 + x_1x_2 - (x_1 + x_2)}$

$= \frac{2 - 2 \cdot \frac{-5}{2}}{1 - \frac{5}{2} - 2} = \frac{7}{\frac{-7}{2}} = -2$

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KÖKLERİ VERİLEN İKİNCİ DERECEDEN DENKLEMİN YAZILMASI

$ax^2+bx+c=0$

$a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=0 \Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0 \quad [a\neq 0]$

$x_1+x_2=-\frac{b}{a}\ ,\ x_1\cdot x_2=\frac{c}{a}$

$\Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$

$x^2-(x_1+x_2)x+x_1\cdot x_2=0.$

NOT

Kökleri x_1 ve x_2 olan ikinci dereceden bir denklem genel olarak aşağıdaki gibi yazılır.

$(x-x_1)\cdot(x-x_2)=0$

NOT

İkinci dereceden bir denklemin kökleri rasyonel değil ise, kökler eşlenik ifadedir.

$ax^2+bx+c=0$

$ÇK=\{x_1, x_2\}$

$x_1=\sqrt{a}-\sqrt{b} \Rightarrow x_2=\sqrt{a}+\sqrt{b}$

ÖRNEK 16

$f(x)=ax^2+bx+c,$

$f(2)=f(-1)=0,$

$f(3)=8 \Rightarrow$ **f(x) = ?**

A) $2x^2-2x-4$

B) $2x^2-2x+4$

C) $2x^2+2x-4$

D) $2x^2+2x+4$

E) x^2-2x-4

ÇÖZÜM

Yanıt **A**

$f(x)=a\cdot(x-x_1)\cdot(x-x_2)$

$f(x)=a\cdot(x-2)\cdot(x+1)$

$f(x)=a\cdot(x^2-x-2)$

$f(3)=8 \Rightarrow a(3^2-3-2)=8$

$4\cdot a=8$

$a=2$

$\Rightarrow f(x)=2\cdot(x^2-x-2)=2x^2-2x-4$

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NOTE

In generally, we can write a quadratic equation as follows that the roots x_1 and x_2.

$$(x - x_1)\cdot(x - x_2) = 0$$

WRITING QUADRATIC EQUATION THAT GIVEN ROOTS

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad [a \neq 0]$$

$$x_1 + x_2 = -\frac{b}{a}\ ,\ x_1 \cdot x_2 = \frac{c}{a}$$

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 - (x_1 + x_2) + x_1 \cdot x_2 = 0.$$

NOTE

If the roots of a second order equation are not rational then they are conjugate.

$ax^2 + bx + c = 0$

$SS = \{x_1 , x_2\}$

$x_1 = \sqrt{a} - \sqrt{b} \Rightarrow x_2 = \sqrt{a} + \sqrt{b}$

EXAMPLE 16

$f(x) = ax^2 + bx + c,$

$f(2) = f(-1) = 0,$

$f(3) = 8 \Rightarrow$ **f(x) = ?**

A) $2x^2 - 2x - 4$

B) $2x^2 - 2x + 4$

C) $2x^2 + 2x - 4$

D) $2x^2 + 2x + 4$

E) $x^2 - 2x - 4$

SOLUTION

The answer is **A**

$f(x) = a\cdot(x - x_1) \cdot (x - x_2)$

$f(x) = a\cdot(x - 2)\cdot(x + 1)$

$f(x) = a\cdot(x^2 - x - 2)$

$f(3) = 8 \Rightarrow a(3^2 - 3 - 2) = 8$

$4\cdot a = 8$

$a = 2$

$\Rightarrow f(x) = 2\cdot(x^2 - x - 2) = 2x^2 - 2x - 4$

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1. $x^2 + 4x - 21 = 0 \Rightarrow$ **ÇK (SS) = ?**

A) {–7, –3}
B) {–3, 7}
C) {–4, – 3}
D) {–7, 3}
E) {3, 7}

2. $x^2 - x - 6 = 0 \Rightarrow$ **ÇK (SS) = ?**

A) {–3, –2}
B) {–2, –4}
C) {–2, 3}
D) {–3, 2}
E) {2 , 3}

3. $x^2 + 5x = 0 \Rightarrow$ **ÇK (SS) = ?**

A) {–5, 0}
B) {–5}
C) {0, 5}
D) {–3, 2}
E) {2, 3}

4. $x^2 + 4 = 0 \Rightarrow$ **ÇK (SS) = ?**

A) R
B) Ø
C) {–2}
D) {– 4}
E) {4}

5. $x \cdot (2 - x) + m + 1 = 0$,
$x_1 = x_2 \Rightarrow$ **m = ?**

A) – 4
B) – 2
C) 0
D) 2
E) 4

6. $2 \cdot (x - 3)^2 - (x - 3) = x - 3 \Rightarrow$ **$x_1 + x_2$ = ?**

A) 5
B) 6
C) 7
D) 8
E) 9

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7. $x^{\frac{4}{3}} - 5 \cdot x^{\frac{2}{3}} - 36 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {24}
B) {13}
C) {27}
D) {37}
E) {43}

8. $x^2 + (m-2)\,x + m = 0$
$(x_1 - 1) \cdot (x_2 - 1) = 1 \Rightarrow$ **m = ?**

A) – 2
B) – 1
C) 0
D) 1
E) 2

9. $x^2 - (m+2)\,x + 2m = 0$,
$\frac{1}{x_1} + \frac{1}{x_2} = \frac{5}{6} \Rightarrow$ **m = ?**

A) – 3
B) – 1
C) 0
D) 1
E) 3

10. $x^2 + (2m-4)\,x + m - 9 = 0$,
$x_1 = -5 \Rightarrow$ **x_2 = ?**

A) – 2
B) – 1
C) 0
D) 1
E) 2

11. $\frac{x-10}{x^2+x} - \frac{x}{x+1} + \frac{4}{x} = 0 \Rightarrow$ **ÇK (SS) = ?**

A) {–1, 2}
B) {1 , 2}
C) {1, 3}
D) {2 , 3}
E) {–4, –3}

12. $k > 0$,
$3x^2 - 2kx + 12 = 0$,
$x_1 = x_2 \Rightarrow$ **k = ?**

A) 4
B) 5
C) 6
D) 7
E) 8

YANITLAR / ANSWERS

1. D	2. C	3. A	4. B	5. B	6. C	7. C	8. D
9. E	10. D	11. D	12. C				

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1. $x^2 + 4x - 21 = 0$

$(x + 7)(x - 3) = 0$

$x_1 = -7\ ,\ x_2 = 3$

$\Rightarrow$ ÇK (SS) $= \{-7, 3\}$

Yanıt / The Answer is D

2. $x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x_1 = 3\ ,\ x_2 = -2$

$\Rightarrow$ ÇK (SS) $= \{-2, 3\}$

Yanıt / The Answer is C

3. $x^2 + 5x = 0$

$x \cdot (x + 5) = 0$

$x = 0$ veya (or) $x + 5 = 0$

$x = -5$

ÇK (SS) $= \{-5\ ,\ 0\}$

Yanıt / The Answer is A

4. $x^2 + 4 = 0$

$x^2 = -4 \Rightarrow$ ÇK (SS) $= \emptyset$

Yanıt / The Answer is B

5. $x \cdot (2 - x) + m + 1 = 0$

$2x - x^2 + m + 1 = 0$

$-x^2 + 2x + m + 1 = 0$

$x_1 = x_2 \Rightarrow \Delta = 0$

$\Delta = 2^2 - 4 \cdot (-1) \cdot (m + 1)$

$= 4 + 4 \cdot (m + 1)$

$= 4 + 4m + 4$

$= 8 + 4m$

$8 + 4m = 0$

$4m = -8$

$m = -2$

Yanıt / The Answer is B

6. $2 \cdot (x - 3)^2 - (x - 3) = x - 3$

$x - 3 = t \Rightarrow 2t^2 - t = t$

$2t^2 - 2t = 0$

$2t \cdot (t - 1) = 0$

$\Rightarrow t = 0 \wedge t = 1$

$t = 0 \Rightarrow x = 3$

$t = 1 \Rightarrow x = 4$

$x_1 + x_2 = 3 + 4 = 7$

Yanıt / The Answer is C

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7. $x^{\frac{4}{3}} - 5 \cdot x^{\frac{2}{3}} - 36 = 0$

$x^{\frac{2}{3}} = t \Rightarrow$

$t^2 - 5t - 36 = 0$

$(t-9)(t+4) = 0$

$t-9=0 \quad , \quad t+4=0$

$t = 9 \qquad t = -4$

$x^{\frac{2}{3}} = 9 \qquad \sqrt[3]{x^2} = -4 \Rightarrow ÇK(SS) = \varnothing$

$x^2 = 9^3 \Rightarrow x^2 = (3^3)^2$

$x = 27$

$\Rightarrow ÇK(SS) = \{27\}$

Yanıt / The Answer is C

8. $x^2 + (m-2)\,x + m = 0$

$(x_1 - 1) \cdot (x_2 - 1) = 1$

$x_1 \cdot x_2 - x_1 - x_2 + 1 = 1$

$x_1 \cdot x_2 - (x_1 + x_2) = 0$

$\frac{m}{1} - \left(-\frac{(m-2)}{1}\right) = 0$

$m + m - 2 = 0$

$2m = 2$

$m = 1$

Yanıt / The Answer is D

9. $x^2 - (m+2)\,x + 2m = 0$

$\frac{1}{x_1} + \frac{1}{x_2} = \frac{5}{6}$

$\frac{x_1 + x_2}{x_1 \cdot x_2} = \frac{5}{6}$

$\frac{m+2}{2m} = \frac{5}{6} \Rightarrow 6m + 12 = 10m$

$4m = 12 \Rightarrow m = 3$

Yanıt / The Answer is E

10. $x^2 + (2m-4) \cdot x + m - 9 = 0$

$x_1 = -5 \Rightarrow (-5)^2 + (2m-4) \cdot (-5) + m - 9 = 0$

$25 - 10m + 20 + m - 9 = 0$

$-9m + 36 = 0$

$-9m = -36$

$m = 4$

$x^2 + 4x - 5 = 0$

$(x+5)\,(x-1) = 0$

$x = -5$ veya (or) $x = 1$

Yanıt / The Answer is D

11. $\frac{x-10}{x^2+x} - \underbrace{\frac{x}{x+1}}_{(x)} + \underbrace{\frac{4}{x}}_{(x+1)} = 0$

$\frac{x - 10 - x^2 + 4x + 4}{x^2 + x} = 0$

$x - 10 - x^2 + 4x + 4 = 0$

$-x^2 + 5x - 6 = 0$

$(x-2)(x-3) = 0$

$x_1 = 2 \quad , \quad x_2 = 3$

$\Rightarrow ÇK(SS) = \{2\,,\,3\}$

Yanıt / The Answer is D

12. $x_1 = x_2 \Rightarrow \Delta = 0$

$\Delta = (-2k)^2 - 4 \cdot 3 \cdot 12$

$\Delta = 4k^2 - 144$

$4k^2 - 144 = 0$

$4k^2 = 144$

$k^2 = 36$

$k = 6$

Yanıt / The Answer is C

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YÖS Soruları / YÖS Questions

1. $m > 0$

$m - \frac{6}{m} = 1$

$\Rightarrow$ **m = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

2. $x^2 + (m^2 - 16)x - 2 + m = 0$

$x_1 + x_2 = 0 \Rightarrow$ **m = ?**

A) –4 B) –2 C) 0 D) 2 E) 4

[İSTANBUL ÜNİV. YÖS 2013]

3. $f(x) = x^2 - ax + 4$

$f(x_1) = f(x_2) = 0$

$\frac{2}{x_1} + x_2 = 3$

a = ?

A) –2 B) –1 C) 2 D) 3 E) 4

[YÖS 2009]

4. $f(x) = 2x^2 + 3x - 1$

$f(x_1) = f(x_2) = 0$

$\frac{1}{x_1} + \frac{1}{x_2} = ?$

A) –3 B) $\frac{-3}{2}$ C) $\frac{-1}{2}$ D) 2 E) 3

[YÖS 2008]

5. $2^{4x+1} - 2^{2x+2} = 96 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3 D) $\frac{3}{2}$ E) $\frac{5}{2}$

[YÖS 2007]

6. $f(x) = x^2 - (a + 1)x + 3a + 4$

$f(x_1) = f(x_2) = 0$

$x_1 + x_2 = k$

$x_1 \cdot x_2 = m \Rightarrow$ **m = ?**

A) $k + 1$ B) $3k + 1$ C) $1 - 3k$ D) $\frac{5k-2}{3}$ E) $\frac{6k+1}{2}$

[YÖS 2007]

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7. $f(x) = x + \frac{2}{x} - 3$

$f(x_1) = f(x_2) = 0$

$x_1 + x_2 = ?$

A) –4 B) –3 C) 1 D) 2 E) 3

[YÖS 2006]

8. $x^2 - x - 2 = 0$

$x^4 - 2x^3 + x^2 = ?$

A) 2 B) 3 C) 4 D) 6 E) 8

[YÖS 2005]

9. $\frac{1}{x+1} + \frac{1}{x^2+2x+1} = 0$

$x = ?$

A) –8 B) –4 C) –2 D) 2 E) 4

[YÖS 2005]

10. $x^2 - 2x + 8a = 0$

$x_1 \cdot x_2 \cdot (x_1 + x_2) = -48 \Rightarrow a = ?$

A) –6 B) –3 C) –2 D) 4 E) 6

[YÖS 2004]

YANITLAR / ANSWERS							
1. C	2. A	3. E	4. E	5. D	6. B	7. E	8. C
9. C	10. B						

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1. $m > 0$

$$m - \frac{6}{m} = 1 \Rightarrow m^2 - m - 6 = 0$$

$$(m-3)(m+2) = 0$$

$$m_1 = 3, \quad m_2 = -2$$

$m > 0 \Rightarrow m = 3$

Yanıt / The Answer is C

---

2. $x^2 + (m^2 - 16)x - 2 + m = 0$

$x_1 + x_2 = 0$

$$-\frac{b}{a} = 0$$

$$16 - m^2 = 0$$

$$(4-m)(4+m) = 0$$

$m_1 = 4 \quad m_2 = -4$

$m = 4 \Rightarrow x \in R$

Yanıt / The Answer is A

3.

$$\frac{2}{x_1} + x_2 = \frac{2 + x_1 \cdot x_2}{x_1} = 3$$

$$= \frac{2+4}{x_1} = 3 \Rightarrow x_1 = 2$$

$f(2) = 2^2 - 2a + 4 = 0 \Rightarrow a = 4$

Yanıt / The Answer is E

4.

$$\frac{1}{x_1} + \frac{1}{x_2} = -\frac{b}{c}$$

$$= \frac{-3}{-1} = 3$$

Yanıt / The Answer is E

---

5. $2^{4x+1} - 2^{2x+2} = 96$

$2 \cdot 2^{4x} - 4 \cdot 2^{2x} = 96$

$2^{2x} = u$

$2u^2 - 4u = 96$

$u^2 - 2u - 48 = 0$

$u_1 = 8 \quad , \quad u_2 = -6$

$2^{2x} = 2^3 \quad , \quad 2^{2x} \neq -6$

$$x = \frac{3}{2}$$

Yanıt / The Answer is D

---

6. $x^2 - (a+1)x + 3a + 4 = 0$

$\Rightarrow x_1 + x_2 = a + 1 = k \Rightarrow a = k - 1$

$x_1 \cdot x_2 = 3a + 4 = m$

$= 3(k-1) + 4 = m$

$3k + 1 = m$

Yanıt / The Answer is B

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7. $x+\frac{2}{x}-3=0$

$x^2-3x+2=0$

$\Rightarrow x_1+x_2=-\frac{(-3)}{1}=3$

Yanıt / The Answer is E

8. $x^2-x-2=0$

$(x-2)\cdot(x+1)=0$

$x_1=2 \quad x_2=-1$

$x=2 \Rightarrow 2^4-2\cdot2^3+2^2=4$

$x=-1 \Rightarrow (-1)^4-2(-1)^3+(-1)^2=4$

Yanıt / The Answer is C

9. $\frac{1}{x+1}+\frac{1}{(x+1)^2}=0 \Rightarrow \frac{x+1+1}{(x+1)^2}=0$

$\frac{x+2}{(x+1)^2}=0 \Rightarrow x+2=0$

$x=-2$

Yanıt / The Answer is C

10. $x_1\cdot x_2\cdot(x_1+x_2)=-48$

$x_1\cdot x_2=\frac{c}{a}, \; x_1+x_2=\frac{-b}{a}$

$\frac{8a}{1}\cdot\frac{2}{1}=-48$

$16a=-48$

$a=-3$

Yanıt / The Answer is B

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1. $x^2 - 6x - 3 = 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is $3 \mp 2\sqrt{3}$

2. $x^3 - 3x^2 - 4x = 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is **{–1, 0, 4}**

3. $2x^2 + x + 3 = 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is ∅

4. $x^2 - 15x + K = 0$, $\frac{x_1}{x_2} = \frac{2}{3} \Rightarrow x_1 \cdot x_2 = ?$

Yanıt / The Answer is **54**

5. $x^2 + 4x - 2m = 0$ ÇK(SS) = {2,n} $\Rightarrow$ **n = ?**

Yanıt / The Answer is **–6**

6. $x^2 - 2x + 6 + m = 0$

$x_1 = x_2 \Rightarrow$ **m = ?**

Yanıt / The Answer is **–5**

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7. $x^6 - 26x^3 - 27 = 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is **{–1, 3}**

8. $m \in Z^+$

$f(x) = 2x^2 - (1 + 2m)x + m^2 - 4$

$f(x_1) = f(x_2) = 0$

$x_1 = 1 \Rightarrow$ **x_2 = ?**

Yanıt / The Answer is $\frac{5}{2}$

9. $f(x) = x^2 - (2a - 1)x + a^2 - 1$

$f(x_1) = f(x_2) = 0$

$x_1 = x_2 \Rightarrow$ **a = ?**

Yanıt / The Answer is $\frac{5}{4}$

10. $f(x) = 5x^2 - 5x - 2,$

$f(x_1) = f(x_2) = 0$

$\Rightarrow$ **$(3x_1 - 2x_2)\cdot(3x_2 - 2x_1)$ = ?**

Yanıt / The Answer is **–16**

11. $3^{2x} - 2\cdot3^{x+1} - 27 = 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is **{2}**

12. $(2x - 4)^2 = (3x + 2)^2 \Rightarrow$ **ÇK (SS) = ?**

Yanıt / The Answer is $\left\{-6, \frac{2}{5}\right\}$

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13. $(x+3)^4 + 2 \cdot (x+3)^2 - 3 = 0 \Rightarrow$ **ÇK (SS) = ?**

Yanıt / The Answer is **{–4, –2}**

14. $9^x - 4 \cdot 3^{x+1} + 27 = 0 \Rightarrow$ **ÇK (SS) = ?**

Yanıt / The Answer is **{1, 2}**

15. $x^4 - 5x^2 - 36 = 0 \Rightarrow$ **ÇK (SS) = ?**

Yanıt / The Answer is **{–3, 3}**

16. $x^2 - 4(m+1)x + 3m - 6 = 0,\ x_1 = 2 \Rightarrow$ **m = ?**

Yanıt / The Answer is **–2**

17. $3x^2 - 10x + 2 = 0 \Rightarrow$ **$x_1^{-1} + x_2^{-1}$ = ?**

Yanıt / The Answer is **5**

18. $x^2 + (m+2)x + (m-4) = 0$

$2x_1 + 2x_2 + 3x_1 x_2 = 5 \Rightarrow$ **m = ?**

Yanıt / The Answer is **21**

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1. $\frac{x^2-7x+10}{x-5}=0 \Rightarrow$ **ÇK(SS) = ?**

A) {2 ,5}

B) {5}

C) {2}

D) Ø

E) R

2. $x^2+mx+n=0$,

ÇK (SS) = {m, n} $\Rightarrow$ **m + n = ?**

A) -2

B) -1

C) $-\frac{1}{2}$

D) $\frac{3}{2}$

E) 3

3. $x^2-3x-2=0$,

ÇK(SS) = $\{x_1, x_2\}$

$\Rightarrow \frac{2}{x_1+2}+\frac{2}{x_2+2}=?$

A) $\frac{4}{7}$

B) $\frac{5}{3}$

C) $\frac{7}{4}$

D) 2

E) 1

4. $x^2+2x-m=0$,

ÇK(SS) = $\{x_1, x_2\}$ ve(and)

$x_1 \cdot x_2 - 2x_1 - 2x_2 = 3 \Rightarrow$ **m = ?**

A) 1

B) 3

C) 5

D) 7

E) 9

5. $x-\sqrt{2x+7}=4 \Rightarrow$ **ÇK(SS) = ?**

A) {9}

B) {7}

C) {5}

D) Ø

E) R

6. $\left(\frac{2x+1}{x}\right)^2+2\cdot\left(\frac{2x+1}{x}\right)-3=0 \Rightarrow$ **ÇK (SS) = ?**

A) $\left\{-1, -\frac{1}{5}\right\}$

B) {–3, 4}

C) $\left\{-\frac{1}{2}, 1\right\}$

D) $\left\{\frac{1}{5}, 3\right\}$

E) {–1, 4}

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7. $\sqrt{2-\sqrt{3x}} = 1 \Rightarrow$ **ÇK(SS) = ?**

A) {1}

B) $\left\{\frac{1}{2}\right\}$

C) $\left\{\frac{1}{3}\right\}$

D) $\left\{\frac{2}{3}\right\}$

E) {0}

8. $x^2 - (2 - m)\,x - m + 3 = 0$,

ÇK(SS) = $\{x_1, x_2\}$

$x_1 \cdot (2x_2 - 1) + x_2\,(2x_1 - 1) = 1 \Rightarrow$ **m = ?**

A) 9

B) 8

C) 6

D) 5

E) 3

9. $(m - 1)\,x^2 + mx + 3 - 4m = 0$,

$f(x_1) = f(x_2) = 0$, $x_1 = -3 \Rightarrow$ **x_2 = ?**

A) 8

B) 3

C) $\frac{4}{5}$

D) $\frac{3}{2}$

E) $\frac{4}{3}$

10. $2x^2 - (m - 3)\,x - 2m + 10 = 0$,

$f(x_1) = f(x_2) = 0$, $x_1 + x_2 = 3 \Rightarrow$ **m = ?**

A) 4

B) 5

C) 7

D) 8

E) 9

11. $(x + 3)^4 + 2\,(x + 3)^2 - 3 = 0 \Rightarrow$ **ÇK (SS) = ?**

A) {0, 1, 2}

B) {–1 , 0, 1, 2}

C) {– 2, 1, 3}

D) {–2, –4}

E) {– 3, – 4}

12. $x^2 + \frac{1}{x^2} + x + \frac{1}{x} - 4 = 0 \Rightarrow$ **Σx = ?**

A) – 4

B) – 2

C) – 1

D) 0

E) 1

YANITLAR / ANSWERS							
1. C	2. B	3. C	4. A	5. A	6. A	7. C	8. E
9. D	10. E	11. D	12. C				

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1. $x^2 - 8x + 16 = 0 \Rightarrow$ **ÇK (SS) = ?**

A) {2, 4}

B) {–2}

C) {2}

D) {–4}

E) {4}

2. $ax^2 + bx + c = 0$, ÇK(SS) = $\{x_1, x_2\}$

$x_1 = 2 - \sqrt{3} \Rightarrow \dfrac{x_1 + x_2}{x_1 \cdot x_2} = ?$

A) 1

B) $2\sqrt{3}$

C) 2

D) 4

E) $4\sqrt{3}$

3. $x^2 - 5x + m = 0$, ÇK(SS) = $\{x_1, x_2\}$

$x_2 = x_1 + 1 \Rightarrow$ **m = ?**

A) – 6

B) – 4

C) 0

D) 3

E) 6

4. $x^2 - (a + 4)x - a^2 + 3 = 0$, ÇK(SS) = $\{x_1, x_2\}$

$x_1 + x_2 = 2 \Rightarrow$ **$x_1 \cdot x_2$ = ?**

A) – 3

B) – 2

C) –1

D) 2

E) 3

5. $x^2 + (a + b - 10)x + (2a - b + 5) = 0$, ÇK(SS) = $\{x_1, x_2\}$

$x_1 + x_2 = 8$, $x_1 \cdot x_2 = 21 \Rightarrow$ **a = ?**

A) 6

B) 7

C) 8

D) 10

E) 21

6. $\dfrac{(3-x)(x-2)}{(3-x)-(x-2)} = x \Rightarrow$ **ÇK (SS) = ?**

A) $\{\sqrt{6}\}$

B) $\{-\sqrt{6}, \sqrt{6}\}$

C) $\{-2\sqrt{3}, 2\sqrt{3}\}$

D) {2,3}

E) {–6, 6}

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7. $x^2 - 3x + m - 2 = 0$, ÇK(SS) = $\{x_1, x_2\}$

$x_1^2 - x_2^2 = 6 \Rightarrow$ **m = ?**

A) -4

B) $\frac{3}{4}$

C) $\frac{6}{5}$

D) $\frac{13}{4}$

E) $\frac{15}{4}$

8. $2x^2 - 7x + m - 4 = 0$, $f(x_1) = f(x_2) = 0$,

$3x_1 + 4x_2 = 1 \Rightarrow$ **m = ?**

A) –243

B) –124

C) 0

D) 124

E) 243

9. $3^x + \frac{1}{3^x} - \frac{10}{3} = 0$

ÇK(SS) = $\{x_1, x_2\} \Rightarrow$ $x_1 + x_2$ **= ?**

A) -2

B) -1

C) 0

D) 1

E) 2

10. $x^2 - 3mx - 3m + 1 = 0$, $f(x_1) = f(x_2) = 0$,

$2x_1 + x_2 = 1 \Rightarrow$ **m = ?**

A) $-\frac{1}{2}$

B) $-\frac{1}{3}$

C) 0

D) $\frac{1}{2}$

E) $\frac{1}{3}$

11. $\left(x + \frac{3}{x}\right)^2 - 8 - \frac{6}{x} - 2x = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {–1, –3}

B) {3}

C) {1}

D) {1, 2}

E) {1, 3}

12. $x^2 - (m + 3)x + m + 2 = 0 \Rightarrow$ **ÇK (SS) = ?**

A) $\{2, m+2\}$

B) $\{1, m+2\}$

C) $\left\{\frac{m}{2}, \frac{m}{3}\right\}$

D) $\{1, m-2\}$

E) $\left\{\frac{2m}{3}, \frac{m}{2}\right\}$

YANITLAR / ANSWERS

1. E	2. D	3. E	4. C	5. A	6. B	7. D	8. A
9. C	10. E	11. E	12. B				

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BÖLÜM 3
CHAPTER 3

TARİHSEL NOT / HISTORICAL NOTE

Chebyshev, Pafnuty Lvovich [1821 – 1894]

Rus matematikçi. Petersburg'daki matematik okulunun kurucularındandır. Cebir,analiz ve olasılık teorisine olan katkılarıyla hatırlanır. Sayı teorisinde, 3 den büyük tüm n'ler için n ile 2n – 2 arasında en az bir asal sayının var olduğunu göstermiştir.

Russian mathematician and founder of a notable school of mathematicians in St Petersburg. His name is remembered in results in algebra, analysis and probability theory. In number theory, he proved that, for all n > 3, there is at least one prime between n and 2n-2.

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QUADRATIC INEQUALITIES

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İKİNCİ DERECEDEN EŞİTSİZLİKLER
QUADRATIC INEQUALITIES

BÖLÜM 3
CHAPTER

Tanım: $f(x) \geq 0$, $f(x) \leq 0$, $f(x) > 0$, $f(x) < 0$ ifadelerine *eşitsizlik* denir.

1. $f(x) = ax + b = 0 \Rightarrow x = -\frac{b}{a}$

x	$-\infty$		$-\frac{b}{a}$		$+\infty$
ax + b	a' nın işaretinin tersi		0	a' nın işareti ile aynı	

Tablo oluşturma:

- ✓ f(x)'in pay ve paydasını sıfır yapan değerler sayı doğrusuna yerleştiriliyormuş gibi küçükten büyüğe doğru sıralanır.
- ✓ Pay ve paydayı sıfır yapan değerlerden tek sayıda olan kökler tek katlı kök, çift sayıda olan kökler, çift katlı kök olarak adlandırılır.
- ✓ Her bileşenin en büyük dereceli terimlerinin işareti bulunarak f(x)'in işareti belirlenir.
- ✓ En sağ kısma f(x)'in işareti yazılır.
- ✓ Tek katlı köklerde sağdaki işaretle zıt; çift katlı köklerde ise sağdaki işaretle aynı işaret yazılır.

AÇIKLAMA

$\{x \mid m \leq x \leq n ;\ m, n \in R\} = [m, n]$
$\{x \mid m \leq x < n ;\ m, n \in R\} = [m, n)$
$\{x \mid m < x \leq n ;\ m, n \in R\} = (m, n]$
$\{x \mid m < x < n ;\ m, n \in R\} = (m, n)$

NOT

Eşitsizliklerde, kuvveti çift olan ifadeler dikkate alınmayabilir. Ancak bunu yaparken dikkate alınmayan ifadeyi sıfır yapan değer, eşitsizliğin çözüm kümesine eklenir veya çıkarılır. Kuvveti tek olan ifadelerde ise, kuvvet dikkate alınmaz.

ÖRNEK 1

$\frac{x-1}{2-x} > 0 \Rightarrow ? < x < ?$

A) $-2 < x < 1$ B) $1 < x < 2$ C) $-1 < x < 2$
D) $0 < x < 2$ E) $1 < x < 3$

ÇÖZÜM

Yanıt **B**

$\frac{x-1}{2-x} > 0$

$x - 1 = 0 \Rightarrow x = 1$

$2 - x = 0 \Rightarrow x = 2$

x	$-\infty$		1		2	$+\infty$
x − 1	−		0 +		+	
2 − x	+		+		0 −	
$\frac{x-1}{2-x}$	−		+		−	

$1 < x < 2$

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EXPLANATION

$\{x \mid m \le x \le n;\ m, n \in R\} = [m, n]$

$\{x \mid m \le x < n;\ m, n \in R\} = [m, n)$

$\{x \mid m < x \le n;\ m, n \in R\} = (m, n]$

$\{x \mid m < x < n;\ m, n \in R\} = (m, n)$

Definition: The expressions $f(x) \ge 0$, $f(x) \le 0$, $f(x) > 0$, $f(x) < 0$ are called *inequalities*.

1. $f(x) = ax + b = 0 \Rightarrow x = -\frac{b}{a}$

x	$-\infty$		$-\frac{b}{a}$		$+\infty$
ax + b	Opposite sign with a		0	Same sign with a	

NOTE

Expressions that even power are canceled in inequalities. But if canceling value is zero, we add or subtract to solution set of inequalities. If in expressions that odd powers, we canceled power.

Consisting Table:

- ✓ The values of f(x) that numerator and denominator is zero is ordered.
- ✓ If these roots are odd number, is named by odd root and these roots are even, is named by even root.
- ✓ The sign of every composition that maximum degree is finding and the sign of f(x) is finding.
- ✓ The sign of f(x) is writingin right part.
- ✓ Write the opposite sign in right in the odd roots and the same sign in right in the even roots.

EXAMPLE 1

$\frac{x-1}{2-x} > 0 \Rightarrow ? < x < ?$

A) $-2 < x < 1$ B) $1 < x < 2$ C) $-1 < x < 2$

D) $0 < x < 2$ E) $1 < x < 3$

SOLUTION

The answer is **B**

$\frac{x-1}{2-x} > 0$

$x - 1 = 0 \Rightarrow x = 1$

$2 - x = 0 \Rightarrow x = 2$

x	$-\infty$		1		2	$+\infty$
x − 1	−		0	+		+
2 − x	+			+	0	−
$\frac{x-1}{2-x}$	−			+		−

$1 < x < 2$

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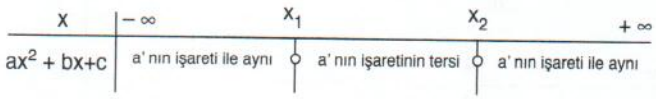
2. $f(x) = ax^2 + bx + c$

i. $\Delta > 0 \Rightarrow$



NOT

$f(x) \geq g(x)$, $f(x) > g(x)$, $f(x) < g(x)$, $f(x) \leq g(x)$ biçimindeki eşitsizlikler $f(x) - g(x) \geq 0$, $f(x) - g(x) > 0$, $f(x) - g(x) < 0$, $f(x) - g(x) \leq 0$ biçimine dönüştürüldükten sonra bilinen yöntem uygulanır.

ÖRNEK 2

$x^2 < 12 - 4x \Rightarrow$ **ÇK = ?**

A) (–6, –2) B) (–2, 6) C) (–6, 2) D) (2, 6) E) (6, 2)

ÇÖZÜM

Yanıt **C**

$x^2 < 12 - 4x$

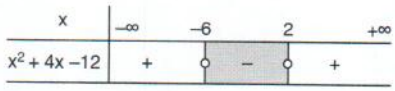
$x^2 + 4x - 12 < 0$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x + 6 = 0 \quad x - 2 = 0$

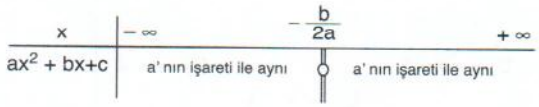
$x = -6 \quad x = 2$



ÇK = {x | $-6 < x < 2$, $x \in R$}

ÇK = (– 6, 2)

ii. $\Delta = 0 \Rightarrow$



ÖRNEK 3

$x \in Z$,

$|x + 2| < |x - 5| \Rightarrow$ **max(x) = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

ÇÖZÜM

Yanıt **B**

$(|x + 2|)^2 < (|x - 5|)^2$

$x^2 + 4x + 4 < x^2 - 10x + 25$

$14x < 21$

$x < \frac{3}{2} \Rightarrow \max(x) = 1$

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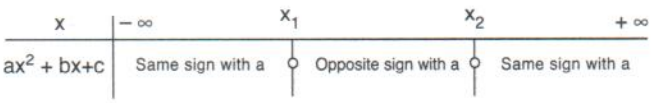
NOTE

Inequalities that in the form, $f(x) \geq g(x)$, $f(x) > g(x)$, $f(x) < g(x)$, converted into $f(x) \leq g(x)$, $f(x) - g(x) \geq 0$, $f(x) - g(x) > 0$, $f(x) - g(x) < 0$, $f(x) - g(x) \leq 0$.

2. $f(x) = ax^2 + bx + c$

i. $\Delta > 0 \Rightarrow$



EXAMPLE 2

$x^2 < 12 - 4x \Rightarrow$ **SS = ?**

A) (–6, –2) B) (–2, 6) C) (–6, 2) D) (2, 6) E) (6, 2)

SOLUTION

The answer is **C**

$x^2 < 12 - 4x$ $\qquad$ $x^2 + 4x - 12 = 0$

$x^2 + 4x - 12 < 0$ $\qquad$ $(x + 6)(x - 2) = 0$

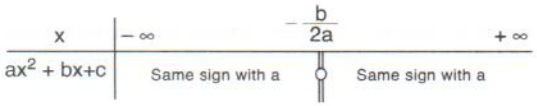
$x + 6 = 0$ $\quad$ $x - 2 = 0$

$x = -6$ $\quad$ $x = 2$

$SS = \{x \mid -6 < x < 2, x \in R\}$

$SS = (-6, 2)$

ii. $\Delta = 0 \Rightarrow$



EXAMPLE 3

$x \in Z$,

$|x + 2| < |x - 5| \Rightarrow$ **max(x) = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

SOLUTION

The answer is **B**

$(|x + 2|)^2 < (|x - 5|)^2$

$x^2 + 4x + 4 < x^2 - 10x + 25$

$14x < 21$

$x < \frac{3}{2} \Rightarrow \max(x) = 1$

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iii. $\Delta < 0 \Rightarrow$

x	$-\infty$		$+\infty$
$ax^2 + bx + c$		a' nın işareti ile aynı	

NOT

$\forall x \in R$,

✓ $a > 0, \Delta < 0 \Rightarrow ax^2 + bx + c > 0$

✓ $a < 0, \Delta < 0 \Rightarrow ax^2 + bx + c < 0$

ÖRNEK 4

$\forall x \in R$,

$(a-1)x^2 - 2(a+1)x + a + 2 < 0 \Rightarrow$ **a < ?**

A) $a < 12$ B) $a < 3$ C) $a < -2$

D) $a > -12$ E) $a < -3$

ÇÖZÜM

Yanıt **E**

$(a-1)x^2 - 2(a+1)x + a + 2 < 0$

$\Rightarrow \Delta = 4(a+1)^2 - 4(a+2)(a-1)$

$= 4a^2 + 8a + 4 - 4a^2 - 4a + 8$

$= 4a + 12 < 0$

$a < -3$

ÖRNEK 5

$x \in Z$,

$\frac{(x^3+1)\cdot(x^2-4)}{|x-5|} > 0 \Rightarrow$ **min(x) = ?**

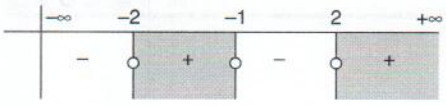
A) -2 B) -1 C) 1 D) 2 E) 3

ÇÖZÜM

Yanıt **E**

$x^3 + 1 = 0 \Rightarrow x = -1$

$x^2 - 4 = 0 \Rightarrow x = \mp 2$



$x \in Z \Rightarrow ÇK = x > 2 \Rightarrow \min(x) = 3$

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NOTE

$\forall x \in R$,

✓ $a > 0, \Delta < 0 \Rightarrow ax^2 + bx + c > 0$

✓ $a < 0, \Delta < 0 \Rightarrow ax^2 + bx + c < 0$

iii. $\Delta < 0 \Rightarrow$

x	$-\infty$		$+\infty$
$ax^2 + bx + c$	Same sign with a		

EXAMPLE 4

$\forall x \in R$,

$(a-1)x^2 - 2(a+1)x + a + 2 < 0 \Rightarrow$ **a < ?**

A) $a < 12$ B) $a < 3$ C) $a < -2$

D) $a > -12$ E) $a < -3$

SOLUTION

The answer is **E**

$(a-1)x^2 - 2(a+1)x + a + 2 < 0$

$\Rightarrow \Delta = 4(a+1)^2 - 4(a+2)(a-1)$

$= 4a^2 + 8a + 4 - 4a^2 - 4a + 8$

$= 4a + 12 < 0$

$a < -3$

EXAMPLE 5

$x \in Z$,

$\frac{(x^3+1)\cdot(x^2-4)}{|x-5|} > 0 \Rightarrow$ **min(x) = ?**

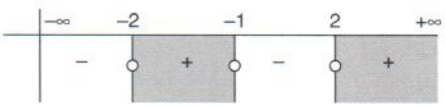
A) -2 B) -1 C) 1 D) 2 E) 3

SOLUTION

The answer is **E**

$x^3 + 1 = 0 \Rightarrow x = -1$

$x^2 - 4 = 0 \Rightarrow x = \mp 2$



$x \in Z \Rightarrow SS = x > 2 \Rightarrow \min(x) = 3$

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ÖRNEK 6

$-6x^2 + 51x - 105 \le 0 \Rightarrow$ **ÇK = ?**

A) $\left(-\infty, \frac{7}{2}\right] \cup [4, \infty)$ B) $\left(-\infty, -\frac{7}{2}\right] \cup [5, \infty)$ C) $\left(-\infty, \frac{7}{2}\right] \cup [-5, +\infty)$

D) $\left(-\infty, \frac{7}{2}\right] \cup [5, +\infty)$ E) $\left(-\infty, -\frac{7}{2}\right] \cup [4, +\infty)$

ÇÖZÜM

Yanıt **D**

$-6x^2 + 51x - 105 = 0$

$6x^2 - 51x + 105 = 0$

$2x \quad -7$

$3x \quad -15$

$6x^2 - 51x + 105 = 0$

$(2x - 7)(3x - 15) = 0$

$2x - 7 = 0 \qquad 3x - 15 = 0$

$x = \frac{7}{2} \qquad x = 5$

x	$-\infty$		$\frac{7}{2}$		5		$+\infty$
$-6x^2 + 51x - 105$		$-$	0	$+$	0	$-$	

$ÇK = \left\{ x \mid -\infty < x \le \frac{7}{2} \text{ veya } 5 \le x < +\infty,\ x \in R \right\}$

$ÇK = \left(-\infty, \frac{7}{2}\right] \cup [5, +\infty)$

PEKİŞTİRME

$x \in Z$,

$3x^2 - 13x + 4 \le 0 \Rightarrow \max(x) = ?$

YANIT : 4

ÖRNEK 7

$\frac{x^2 - 8x + 7}{(x+2)^2} < 0, x \in Z \Rightarrow$ **ÇK = ?**

A) {2, 3, 4, 5, 6} B) {3, 4, 5, 6, 7, 8} C) {– 12, 0, 1, 2, 3}

D) {1, 2, 3, 4, 5} E) {5, 6, 7, 8}

ÇÖZÜM

Yanıt **A**

$x^2 - 8x + 7 = 0$

$(x - 7)(x - 1) = 0 \Rightarrow x = 7$ veya $x = 1$

$(x + 2)^2 = 0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

x	$-\infty$		-2		1		7		$+\infty$
$x^2 - 8x + 7$	+		+		0	$-$	0	$+$	
$(x + 2)^2$	+		0	+		+		+	
$\frac{x^2 - 8x + 7}{(x + 2)^2}$	+		+		0	$-$	0	$+$	

$1 < x < 7, x \in Z \Rightarrow ÇK = \{2, 3, 4, 5, 6\}$

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REINFORCE

$x \in Z$,

$3x^2 - 13x + 4 \le 0 \Rightarrow \max(x) = ?$

ANSWER : 4

EXAMPLE 6

$-6x^2 + 51x - 105 \le 0 \Rightarrow$ **SS = ?**

A) $\left(\infty, \frac{7}{2}\right] \cup [5, -\infty)$ B) $\left(-\infty, \frac{7}{2}\right] \cup [5, -\infty)$ C) $\left(\infty, \frac{7}{2}\right] \cup [-5, +\infty)$

D) $\left(-\infty, \frac{7}{2}\right] \cup [5, +\infty)$ E) $\left(\infty, -\frac{7}{2}\right] \cup [5, +\infty)$

SOLUTION

The answer is **D**

$-6x^2 + 51x - 105 = 0$

$6x^2 - 51x + 105 = 0$

$2x \quad -7$

$3x \quad -15$

$6x^2 - 51x + 105 = 0$

$(2x - 7)(3x - 15) = 0$

$2x - 7 = 0 \qquad 3x - 15 = 0$

$x = \frac{7}{2} \qquad x = 5$

x	$-\infty$		$\frac{7}{2}$		5		$+\infty$
$-6x^2 + 51x - 105$		$-$	0	$+$	0	$-$	

$SS = \left\{ x \mid -\infty < x \le \frac{7}{2} \text{ or } 5 \le x < +\infty, \ x \in R \right\}$

$SS = \left(-\infty, \frac{7}{2}\right] \cup [5, +\infty)$

EXAMPLE 7

$\frac{x^2 - 8x + 7}{(x+2)^2} < 0, x \in Z \Rightarrow$ **SS = ?**

A) {2, 3, 4, 5, 6} B) {3, 4, 5, 6, 7, 8} C) {– 12, 0, 1, 2, 3}

D) {1, 2, 3, 4, 5} E) {5, 6, 7, 8}

SOLUTION

The answer is **A**

$x^2 - 8x + 7 = 0$

$(x - 7)(x - 1) = 0 \Rightarrow x = 7$ or $x = 1$

$(x + 2)^2 = 0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

x	$-\infty$		-2		1		7	$+\infty$
$x^2 - 8x + 7$	+		+		0	$-$	0	+
$(x + 2)^2$	+		0	+	+		+	+
$\frac{x^2 - 8x + 7}{(x + 2)^2}$	+		+		0	$-$	0	+

$1 < x < 7, x \in Z \Rightarrow SS = \{2, 3, 4, 5, 6\}$

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EŞİTSİZLİK SİSTEMİ

Birden fazla eşitsizliğin oluşturduğu sisteme *eşitsizlik sistemi* denir. Sistemi oluşturan eşitsizliklerin çözüm kümelerinin kesişimine ise, *eşitsizlik sisteminin çözüm kümesi* denir.

ÖRNEK 8

$$\left.\begin{array}{l}\dfrac{3+x}{2-x}<0\\ x^2-6x+8>0\end{array}\right\} \Rightarrow \text{ÇK} = ?$$

A) (2, 4) B) $(-\infty, -3) \cup (4, \infty)$ C) (–3, 2)
D) (–3, 4) E) $(-\infty, -3)$

ÇÖZÜM

Yanıt **B**

$3 + x = 0 \Rightarrow x = -3$

$2 - x = 0 \Rightarrow x = 2$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4, \quad x = 2$

x		–3		2		4	
$\frac{3+x}{2-x}$	–	○	+	○	–		–
x^2-6x+8	+		+	○	–	○	+

$\Rightarrow$ ÇK = $(-\infty, -3) \cup (4, \infty)$

ÖRNEK 9

$$\left.\begin{array}{l}x^2-5x+6>0\\ x^2-2x-3<0\end{array}\right\} \Rightarrow x \in ?$$

A) (2, 4) B) (– 1, 2) C) $(-\infty, -1)$
D) [– 2, 3) E) (– 3, 3)

ÇÖZÜM

Yanıt **B**

$x^2 - 5x + 6 = 0 \Rightarrow (x - 3)\cdot(x - 2) = 0$

$x = 2, \quad x = 3$

$x^2 - 2x - 3 = 0 \Rightarrow (x - 3)\cdot(x + 1) = 0$

$x = -1, \quad x = 3$

x		$-\infty$		–1		2		3		$+\infty$
x^2-5x+6	+		+		–	○	+			
x^2-2x-3	+	○	–		–	○	+			

ÇK = (–1, 2)

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SYSTEM of INEQUALITY

The system of more than one inequalities is called a *system of inequality*. The intersection of the solution sets of the inequalities is called *a solution set of inequality system*.

EXAMPLE 8

$$\left.\begin{array}{l}\dfrac{3+x}{2-x}<0\\ x^2-6x+8>0\end{array}\right\} \Rightarrow SS = ?$$

A) (2, 4) B) $(-\infty, -3) \cup (4, \infty)$ C) (–3, 2)

D) (–3, 4) E) $(-\infty, -3)$

SOLUTION

The answer is B

$3 + x = 0 \Rightarrow x = -3$

$2 - x = 0 \Rightarrow x = 2$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4, \quad x = 2$

x		–3		2		4	
$\frac{3+x}{2-x}$	–	○	+	○	–		–
x^2-6x+8	+		+	○	–	○	+

$\Rightarrow SS = (-\infty, -3) \cup (4, \infty)$

EXAMPLE 9

$$\left.\begin{array}{l}x^2-5x+6>0\\ x^2-2x-3<0\end{array}\right\} \Rightarrow x \in ?$$

A) (2, 4) B) (– 1, 2) C) $(-\infty, -1)$

D) [– 2, 3) E) (– 3, 3)

SOLUTION

The answer is B

$x^2 - 5x + 6 = 0 \Rightarrow (x - 3)\cdot(x - 2) = 0$

$x = 2, \quad x = 3$

$x^2 - 2x - 3 = 0 \Rightarrow (x - 3)\cdot(x + 1) = 0$

$x = -1, \quad x = 3$

x	$-\infty$		–1		2		3		$+\infty$
x^2-5x+6	+		+	○	–	○	+		
x^2-2x-3	+	○	–		–	○	+		

SS = (–1, 2)

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1. $(x+3)(2-x)>0 \Rightarrow ?<x<?$

A) $-3<x<2$

B) $-2<x<3$

C) $-1<x<4$

D) $0<x<5$

E) $1<x<6$

2. $x^2-9<0 \Rightarrow$ ÇK(SS) = ?

A) $(-9, 9)$

B) $(-3, 3)$

C) $(-3, 4)$

D) $(-4, 3)$

E) $(3, 9)$

3. $3x-1<x^2-x+2 \Rightarrow$ ÇK(SS) = ?

A) $(-\infty, -1) \cup (3, \infty)$

B) $(-\infty, 1) \cup (3, \infty)$

C) $(-\infty, 1) \cup (4, \infty)$

D) $(-\infty, -1) \cup (4, \infty)$

E) $(-\infty, -2) \cup (5, \infty)$

4. $x^2-x+4>0 \quad ?<x<?$

A) $-\infty<x<0$

B) $-\infty<x<\infty$

C) $0<x<\infty$

D) $-\infty<x<-4$

E) $-\infty<x<4$

5. $\frac{(2-x)(x^2+4x-5)}{x+2} \geq 0 \Rightarrow$ ÇK(SS) = ?

A) $(-\infty, -5) \cup (-2, 1)$

B) $[-5, -2) \cup (1, 2)$

C) $[-5, -2] \cup [1, 2]$

D) $[-5, -2) \cup [1, 2]$

E) $[1, 2)$

6. $\sqrt{x^2+x} < \sqrt{x+2} \Rightarrow x \in ?$

A) $(-\sqrt{2} \cup -1] \cup [0, \sqrt{2})$

B) $-1<x<1$

C) $\sqrt{3} < x$

D) $-\sqrt{2} < x < \sqrt{2}$

E) $-\sqrt{5} < x < \sqrt{5}$

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7. $\frac{(x-2)(x^2+4x+4)}{x^2+2x+1} \le 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-\infty, -1] \cup (2, \infty)$

B) $(2, \infty)$

C) $(-\infty, -1) \cup [2, \infty)$

D) $(-\infty, -1) \cup (1, 2]$

E) $(-\infty, 2] - \{-1\}$

8. $a \in R^-$

$\frac{(x+1)^4(x^2-a)}{x^3-2x^2-3x} \le 0 \Rightarrow$ **ÇK(SS) = ?**

A) $R^- - (-3, -1)$

B) $R^- - [-3, -1]$

C) $(-\infty, -1) \cup (0, 3)$

D) $(-1, 0) \cup (3, +\infty)$

E) $R - (0, 3)$

9. $\frac{(x+5)^{2012}|x-13|}{x^2-x+1} \ge 0 \Rightarrow$ **ÇK(SS) = ?**

A) R

B) $\varnothing$

C) $R - \{-5, 13\}$

D) $(-5, 13)$

E) $[-5, 13]$

10. $3x^2 - 13x - 10 \le 0 \Rightarrow$ **ÇK (SS) = ?**

A) $\left(-\infty, -\frac{2}{3}\right)$

B) $\left(-\frac{2}{3}, 5\right)$

C) $\left[-\frac{2}{3}, 5\right]$

D) $(5, +\infty)$

E) $\left(-\infty, -\frac{2}{3}\right) \cup (5, +\infty)$

11. $x \in Z$

$\frac{-(x+2)(x+3)^2}{x} \ge 0 \Rightarrow$ **min(x) = ?**

A) –3

B) –2

C) –1

D) 1

E) 2

12. $\left.\begin{array}{l} \frac{3x}{(x+2)^2} > 0 \\ \frac{x(x-2)}{x+3} < 0 \end{array}\right\} \Rightarrow x \in ?$

A) (0, 2)

B) (0, 2]

C) $(-\infty, 0) \cup (2, \infty)$

D) R

E) $\varnothing$

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1. A	2. B	3. B	4. B	5. D	6. A	7. E	8. C
9. A	10. C	11. A	12. A				

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1. $(x+3)\cdot(2-x)>0$

$x+3=0 \Rightarrow x=-3$

$2-x=0 \Rightarrow x=2$

x	$-\infty$	-3		2	$+\infty$
$x+3$	$-$	0	$+$		$+$
$2-x$	$+$		$+$	0	$-$
$(x+3)\cdot(2-x)$	$-$		$+$		$-$

$-3<x<2$

Yanıt / The Answer is A

2. $x^2-9<0$, $x^2-9=0$

$(x-3)(x+3)=0$

$x-3=0$, $x+3=0$

$x=3$ $\quad$ $x=-3$

x	$-\infty$	-3		3	$+\infty$
x^2-9	$+$	0	$-$	0	$+$

ÇK(SS) = { x | $-3<x<3$, $x\in R$ }

= $(-3, 3)$

Yanıt / The Answer is B

3. $3x-1<x^2-x+2$

$0<x^2-4x+3$

$x^2-4x+3=0$

$x-3=0$, $x-1=0$

$x=3$ $\quad$ $x=1$

x	$-\infty$	1		3	$+\infty$
x^2-4x+3	$+$	0	$-$	0	$+$

ÇK(SS) = { x | $-\infty<x<1$ veya (or) $3<x<\infty$, $x\in R$ }

= $(-\infty, 1)\cup(3, \infty)$

Yanıt / The Answer is B

4. $x^2-x+4>0$

$x^2-x+4=0$

$\Delta = b^2-4\cdot a\cdot c$

$\Delta = (-1)^2-4\cdot 1\cdot 4$

$\Delta = 1-16=-15$

$x^2-x+4=0 \Rightarrow$ ÇK (SS) = Ø

x	$-\infty$		$+\infty$
x^2-x+4	$+$	$+$	$+$

$-\infty<x<\infty$

Yanıt / The Answer is B

5. $2-x=0$ $\quad$ $x^2+4x-5=0$

$x=2$ $\quad$ $(x+5)(x-1)=0$

$x=-5, x=1$

$x+2=0 \Rightarrow x=-2$

x	$-\infty$	-5	-2	1	2	$+\infty$
$\frac{(2-x)\cdot(x^2+4x-5)}{(x+2)}$	$-$	0	$+$	0	$+$	0 $-$

ÇK(SS) = $[-5, -2)\cup[1, 2]$

Yanıt / The Answer is D

6. $\left(\sqrt{x^2+x}\right)^2 < \left(\sqrt{x+2}\right)^2$

$x^2+x<x+2$

$x^2-2<0$

$-\sqrt{2}<x<\sqrt{2}$

$x^2+x\geq 0$, $\quad x+2\geq 0$

$x(x+1)\geq 0$

x	$-\infty$	-2		-1		0	$+\infty$
$x(x+1)$	$+$	0	$+$	0	$-$	0	$+$
$x+2$	$-$	0	$+$		$+$		$+$

$[-2, -1]\cup[0, \infty]$

$\Rightarrow$ ÇK(SS) = $[-\sqrt{2}, -1]\cup[0, \sqrt{2})$

Yanıt / The Answer is A

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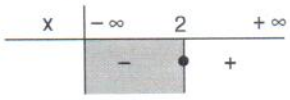
7. $x - 2 = 0$

$x = 2$



$\Rightarrow$ ÇK(SS) = $(-\infty, 2] - \{-1\}$

Yanıt / The Answer is E

8. $x^2 - a = 0 \Rightarrow x^2 = a$

$x = \mp\sqrt{a}$

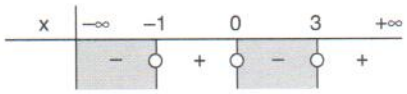
$a > 0 \Rightarrow$ ÇK(SS) = $\varnothing$

$x^3 - 2x^2 - 3x = 0$

$x(x^2 - 2x - 3) = 0$

$x = 0$, $(x - 3)(x + 1) = 0$

$x = 3$, $x = -1$



$\Rightarrow$ ÇK(SS) = $(-\infty, -1) \cup (0, 3)$

Yanıt / The Answer is C

9. $x^2 - x + 1 = 0$

$\Delta = 1^2 - 4 \cdot 1 < 0 \Rightarrow$ ÇK(SS) = $\varnothing$

$(x + 5)^{2012} \geq 0$

$|x - 13| \geq 0$

$\Rightarrow$ ÇK(SS) = R

Yanıt / The Answer is A

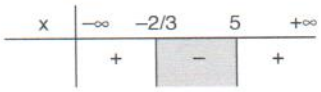
10. $3x^2 - 13x - 10 = 0$

$3x \quad +2$

$x \quad -5$

$(3x + 2)(x - 5) = 0$

$x = -\frac{2}{3}$, $x = 5$

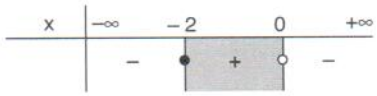


ÇK(SS) = $\left[-\frac{2}{3}, 5\right]$

Yanıt / The Answer is C

11. $x + 2 = 0$

$x = -2$



$\Rightarrow$ ÇK(SS) = $[-2, 0) \cup \{-3\}$

min(x) = –3

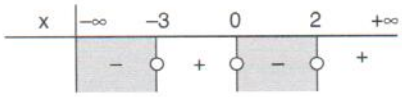
Yanıt / The Answer is A

12. $\frac{3x}{(x+2)^2} > 0$, $(x+2)^2 > 0$

$\Rightarrow x > 0$

$x = 0$, $x - 2 = 0$, $x + 3 = 0$

$x = 2$ $x = -3$



ÇK(SS) = $(-\infty, -3) \cup (0, 2)$

$\Rightarrow x \in (0, 2)$

Yanıt / The Answer is A

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1. $a < \frac{-4}{a+4} \Rightarrow a \in ?$

Yanıt / The Answer is $a < -4$

2. $a - \frac{a+4}{3} < 4 \Rightarrow a \in ?$

Yanıt / The Answer is $a < 8$

3. $2x + 4 < 1 \Rightarrow x \in ?$

Yanıt / The Answer is $x < -\frac{3}{2}$

4. $x^2 < 12 - 4x \Rightarrow$ ÇK(SS) = ?

Yanıt / The Answer is $(-6, 2)$

5. $-6x^2 + 51x - 105 \leq 0 \Rightarrow$ ÇK(SS) = ?

Yanıt / The Answer is $R - (\frac{7}{2}, 5)$

6. $\frac{(x-2)\cdot(x+3)^2}{x^2-2x-3} \leq 0 \Rightarrow$ ÇK(SS) = ?

Yanıt / The Answer is $(-\infty, -1) \cup [2, 3)$

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1. $4(x+1)-2>\frac{1}{2}(x-1) \Rightarrow x \in ?$

A) $x<0$

B) $x>\frac{1}{2}$

C) $x>-\frac{5}{7}$

D) $x<\frac{1}{3}$

E) $x<-1$

2. $x^2-2x-3<0 \Rightarrow x \in ?$

A) $-3<x<1$

B) $-1<x<3$

C) $0<x<2$

D) $-2<x<-1$

E) $-2<x<3$

3. $x\cdot(x^2-2x)>0 \Rightarrow$ ÇK(SS) = ?

A) $(2, +\infty)$

B) $(-\infty, 0)$

C) $(0, 2)$

D) $(0, +\infty)$

E) $(-\infty, 0)\cup(2, +\infty)$

4. $\frac{x^2-2}{x^2-2x-3}<0 \Rightarrow$ ÇK (SS) = ?

A) $(-3, -2)$

B) $(-\sqrt{2}, 3)$

C) $(-\sqrt{2}, \sqrt{2})\cup(\sqrt{2}, 3)$

D) $(-\sqrt{2}, \sqrt{2})$

E) $(-\sqrt{2}, -1)\cup(\sqrt{2}, 3)$

5. $x \in Z$,

$\frac{x^2-3x+4}{x+1}>0 \Rightarrow$ min(x) = ?

A) -3

B) -2

C) -1

D) 0

E) 1

6. $\frac{3-x}{2x-4}\geq -1 \Rightarrow x \in ?$

A) $(1, 2)$

B) $(-\infty, 2)$

C) $(-2, 2)$

D) $(1, +\infty)$

E) $(-\infty, 1]\cup(2, +\infty)$

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7. $x \in Z$,

$(x+3)^2 - 16x^2 \geq 0 \Rightarrow$ **min(x) = ?**

A) –2

B) –1

C) 0

D) 1

E) 2

8. $x \in Z$,

$|x+2| < |x-5| \Rightarrow$ **max(x) = ?**

A) 0

B) 1

C) 2

D) 3

E) 4

9. $\dfrac{(x^2+2x)\cdot(9-x^2)}{x^2-2x+1} \leq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -3] \cup [-2, 0] \cup [3, \infty)$

B) $(-\infty, 3] \cup [2, 0) \cup [1, \infty)$

C) $(-\infty, -3] \cup [-2, 0) \cup [1, \infty)$

D) $(-\infty, -3] \cup [2, 0) \cup [1, \infty)$

E) $(-\infty, 3] \cup [-2, 0) \cup [1, \infty)$

10. $\dfrac{(3-x)^2 \cdot (x^2+4x-21)}{(16-x^2)\cdot(x^2+3x+7)} \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-7, -4) \cup [3, 4)$

B) $(-4, 3)$

C) $[-4, 3)$

D) $[-4, 3]$

E) $[-7, -4] \cup [3, 4)$

11. $\left.\begin{matrix} x^2-7x+6<0 \\ x^2-8x+15\geq 0 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $(1, 2] \cup [4, 5)$

B) $(1, 3] \cup [4, 5)$

C) $(1, 3] \cup [4, 6)$

D) $(1, 2] \cup [5, 6)$

E) $(1, 3] \cup [5, 6)$

12. $\dfrac{(x^2-2x-8)(x-1)^4}{(x-2)^{10}} \leq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-2, 4] - \{2\}$

B) $[2, 4]$

C) $[-4, -2]$

D) $[-2, 3]$

E) $[-3, -2]$

YANITLAR / ANSWERS

1. C	2. B	3. A	4. E	5. D	6. E	7. C	8. B
9. A	10. A	11. E	12. A				

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1. $\left.\begin{array}{l} a-5 \geq 2a+3 \\ 3a-2 < a-5 \end{array}\right\} \Rightarrow a \in ?$

A) $a < -8$

B) $a \geq 3$

C) $a \leq -8$

D) $-8 < a < 3$

E) $1 < a < 4$

2. $\frac{x-2}{x+2} \leq 0 \Rightarrow x \in ?$

A) $-2 < x < 2$

B) $-2 < x \leq 2$

C) $-\infty < x < -2$

D) $-2 \leq x < 2$

E) $-2 \leq x \leq 2$

3. $ax^2 - 3ax + 4a + 8 = 0$,
ÇK(SS) = $\{x_1, x_2\}$,
$x_1 \cdot x_2 < 0 \Rightarrow a \in ?$

A) $0 < a < +\infty$

B) $-4 < a < 0$

C) $-3 < a < 0$

D) $-2 < a < 0$

E) $-2 \leq a \leq 2$

4. $x \in Z$,
$4 \leq x^2 - 4x + 4 \leq 25 \Rightarrow \sum x = ?$

A) 16

B) 18

C) 22

D) 25

E) 30

5. $\sqrt{x-1} < 3-x \Rightarrow$ ÇK (SS) = ?

A) $-1 < x < 2$

B) $x < -1$

C) $4 \leq x < 5$

D) $1 \leq x < 2$

E) $x > 5$

6. $\frac{x}{x-2} - \frac{3}{x} > 2 \Rightarrow$ ÇK(SS) = ?

A) $(-2, 0) \cup (2, 4)$

B) $(-3, 0) \cup (2, 3)$

C) $(-\infty, -2)$

D) $(-2, 0) \cup (2, 3)$

E) $(3, +\infty)$

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7. $x \in Z$,

$9 \le x^2 - 4x + 4 \le 16 \Rightarrow \sum x = ?$

A) 8

B) 9

C) 10

D) 11

E) 12

8. $\frac{1}{x^2-2x-15} > \frac{1}{x^2-x-2} \Rightarrow$ **ÇK(SS) = ?**

A) $(-13, -3) \cup (-1, 2) \cup (5, +\infty)$

B) $(-3, -1) \cup (2, 5)$

C) $[-3, -1] \cup [2, 5]$

D) $[-3, 1) \cup [2, 5)$

E) $[-13, -3) \cup [-1, 2) \cup (5, \infty)$

9. $\frac{x}{x-1} - \frac{1}{x} \le 0 \Rightarrow$ **ÇK(SS) = ?**

A) $R - (0, 1)$

B) $R - [0, 1]$

C) $\{0, 1\}$

D) $(0, 1)$

E) $[0, 1]$

10. $x^2 - 3mx + 3m > -3 \Rightarrow m \in ?$

A) $(-7, -3)$

B) $(-\frac{2}{3}, 3)$

C) $(-\frac{2}{3}, 2)$

D) $(1, +\infty)$

E) $(-\infty, 1) \cup (2, +\infty)$

11. $|2x + 1| \le |2x + 5| \Rightarrow$ **ÇK (SS) = ?**

A) $\left[-\frac{3}{2}, +\infty\right)$

B) $[-2, +\infty)$

C) $\left[-\frac{5}{2}, +\infty\right)$

D) $[-3, +\infty)$

E) $\left[-\frac{7}{2}, +\infty\right)$

12. $x \in Z$,

$\frac{3-x}{x} > \frac{6}{x-3} \Rightarrow \sum x = ?$

A) –9

B) –6

C) –3

D) 0

E) 3

YANITLAR / ANSWERS

1. C	2. B	3. D	4. A	5. D	6. D	7. A	8. A
9. D	10. C	11. A	12. E				

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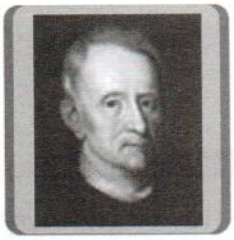
BÖLÜM 4
CHAPTER 4

TARİHSEL NOT / HISTORICAL NOTE

Argand; Jean Robert [1768-1822]



İsveç doğumlu matematikçi karmaşık sayıların geometrik gösterimi üzerinde çalışmıştır. Bu gösterim Argand diagramı olarak adlandırılır.

Swiss- born mathematician who was one of several people, including Gauss, who invented a geometrical representation for complex numbers. This explains the name Argand diagram.

İKİNCİ DERECEDEN FONKSİYONLAR
QUADRATIC FUNCTIONS

- ✔ Parabol Çizimi . . . . . 112 – 118
- ✔ Parabolün Denkleminin Yazılması . . . . . 120 – 126
- ✔ Bir Parabolle Bir Doğru Arasındaki İlişki . . . . . 128 – 130
- ✔ Çözümlü Test . . . . . 132 – 135
- ✔ YÖS Soruları ve Çözümleri . . . . . 136 – 139
- ✔ Alıştırmalar . . . . . 140 – 142
- ✔ Konu Testleri . . . . . 143 – 146

- ✔ Drawing a Parabola . . . . . 113 – 119
- ✔ Writing a Parabola Equation . . . . . 121 – 127
- ✔ Relationship Between a Parobola and a Line . . . . . 129 – 131
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İKİNCİ DERECEDEN FONKSİYONLAR
QUADRATIC FUNCTIONS

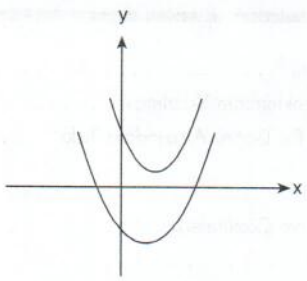
BÖLÜM 4
CHAPTER

Tanım: a, b, c ∈ R ve a ≠ 0 olmak üzere,
$y = f(x) = ax^2 + bx + c$ şeklindeki ikinci dereceden fonksiyonların grafiğine *parabol* denir.

PARABOL ÇİZİMİ

$y = f(x) = ax^2 + bx + c$ fonksiyonlarının grafiği çizilirken aşağıdaki sıra izlenir:

1 a.



a > 0 ise grafiğin kolları yukarı doğrudur.

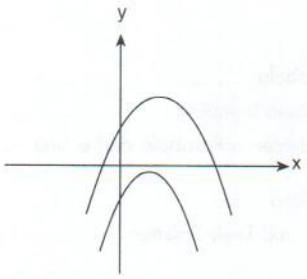
NOT

a, ikinci dereceden fonksiyonun baş katsayısıdır.

NOT

$y = ax^2 + bx + c$ denkleminde c = 0 ise, parabol orijinden geçer.

b.



a < 0 ise grafiğin kolları aşağı doğrudur.

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Definition: Provided that a, b, c ∈ R and a ≠ 0, the graph of a quadratic function $y = f(x) = ax^2 + bx + c$ is called a *parabola*.

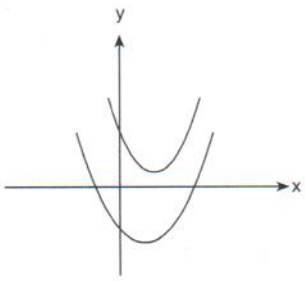
NOTE

a is called leading coefficient of a quadratic function.

DRAWING A PARABOLA

To draw the graph of a quadratic function, follow the steps below:

1 **a.**

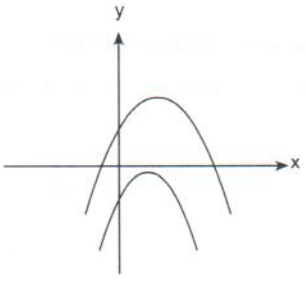


If a > 0, then the curve opens upward.

NOTE

In equation $y = ax^2 + bx + c$,
if c = 0, then parabola is passes through the origin.

b.



If a < 0, then the curve opens downward.

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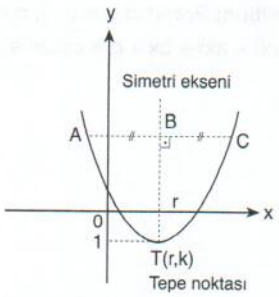
2 **Tepe Noktası:**

$$(r,k)=\left(-\frac{b}{2a},\frac{4ac-b^2}{4a}\right)$$



IABI = IBCI

NOT

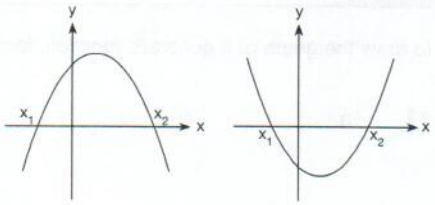
Tüm paraboller, simetri ekseni olarak adlandırılan doğruya göre simetriktir.

3 **Parabolün x Eksenini Kestiği Noktalar**

$y = f(x) = ax^2 + bx + c = 0$

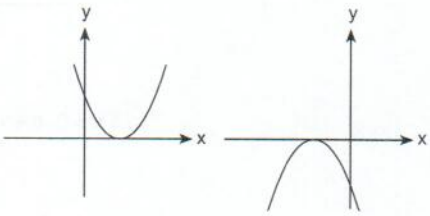
a. $\Delta > 0 \Rightarrow$

$ÇK = \{x_1, x_2\}$



b. $\Delta = 0 \Rightarrow$

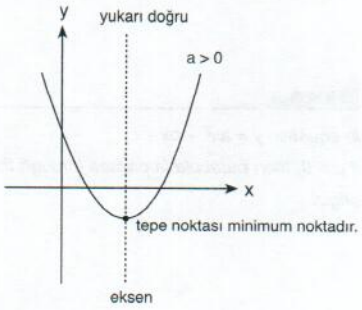
$ÇK = \{x_1, x_2\},\ x_1 = x_2$



NOT

Grafik, $x=-\frac{b}{2a}$ doğrusuna göre simetrik olduğundan $x=-\frac{b}{2a}$ doğrusuna parabolün *simetri ekseni* denir. k değerine; a > 0 ise parabolün *en küçük değeri*, a < 0 ise parabolün *en büyük değeri* denir.

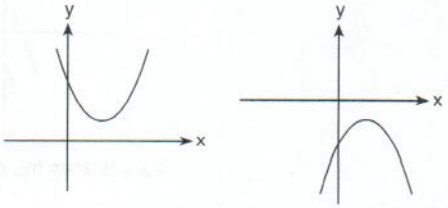
NOT

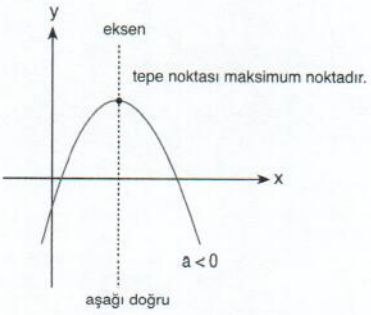


4 **Parabolün y Eksenini Kestiği Noktalar**

$f(x) = ax^2 + bx + c$ parabolünün y eksenini kestiği nokta x = 0 için f(0) = c dir.

Δ < 0





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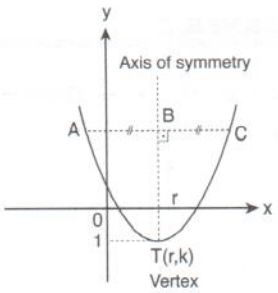
NOTE

All parabolas are symmetric with respect to a line called the axis of symmetry.

2 **Vertex:**

$$(r,k)=\left(-\frac{b}{2a},\frac{4ac-b^2}{4a}\right)$$



|ABI = |BCI

NOTE

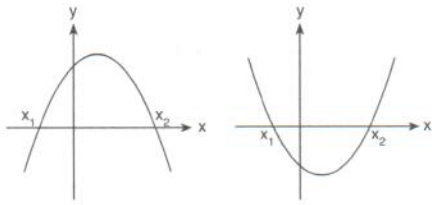
Since the graph is symmetric with respect to the line $x=-\frac{b}{2a}$, then line $x=-\frac{b}{2a}$ is called *symmetry axis*. If $a>0$, then k is called *the minimum value of the parabola*, and if $a<0$, then k is called *the maximum value of the parabola*.

3 **x – Intercepts of a Parabola**

$y = f(x) = ax^2 + bx + c = 0$

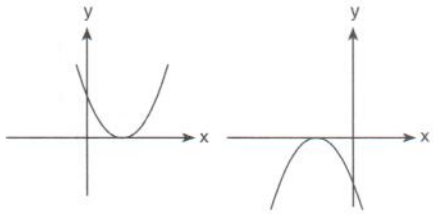
a. $\Delta > 0 \Rightarrow$

$SS = \{x_1, x_2\}$

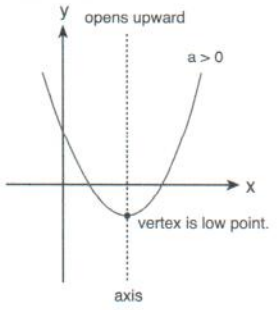


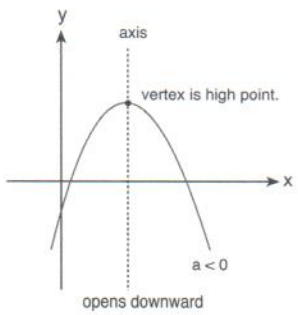
b. $\Delta = 0 \Rightarrow$

$SS = \{x_1, x_2\}, \ x_1 = x_2$



NOTE

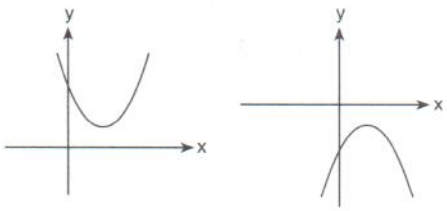




4 **y – Intercept of a Parabola**

y intercept of the parabola $f(x) = ax^2 + bx + c$, for $x = 0$, $f(0) = c$.

$\Delta < 0$



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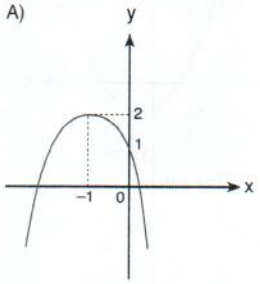
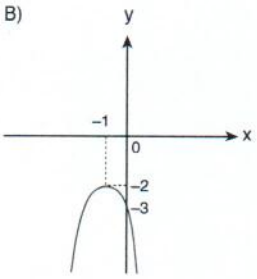
ÖRNEK 1

$y = -(x+1)^2 - 2 \Rightarrow$ Grafik = ?

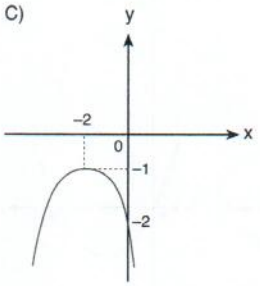
A)



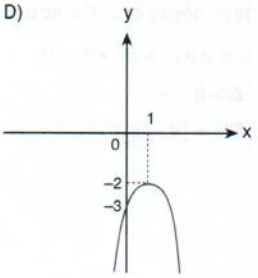
B)



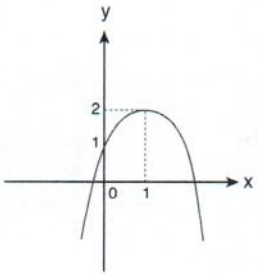
C)



D)



E)



ÇÖZÜM

Yanıt **B**

$y = -(x+1)^2 - 2 \Rightarrow T(-1, -2)$

$x = 0 \Rightarrow y = -(0+1)^2 - 2$

$y = -1 - 2$

$y = -3$

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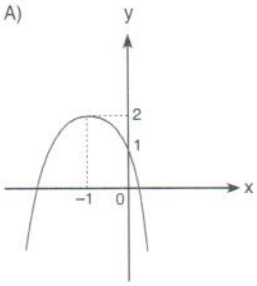
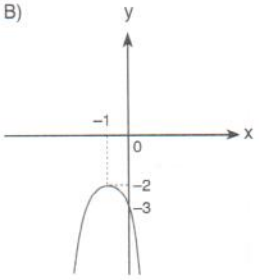
EXAMPLE 1

$y = -(x + 1)^2 - 2 \Rightarrow$ Graph = ?

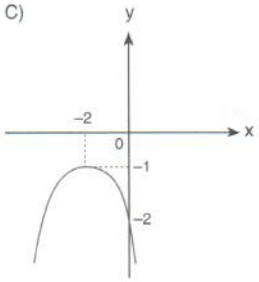
A)



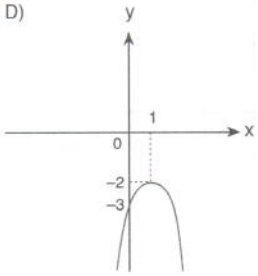
B)



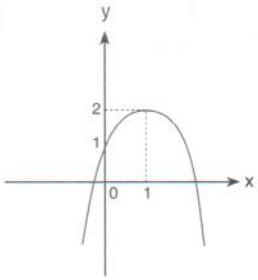
C)



D)



E)



SOLUTION

The answer is **B**

$y = -(x + 1)^2 - 2 \Rightarrow T(-1, -2)$

$x = 0 \Rightarrow y = -(0 + 1)^2 - 2$

$y = -1 - 2$

$y = -3$

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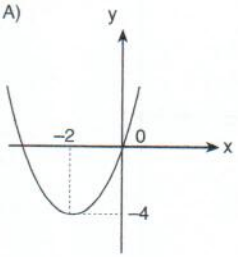
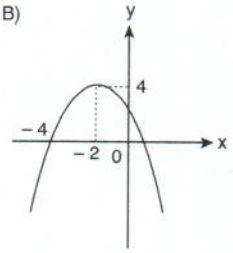
ÖRNEK 2

$y = -x^2 + 4x \Rightarrow$ Grafik = ?

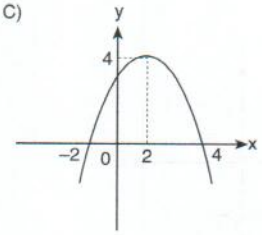
A)



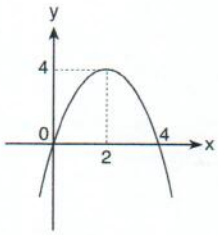
B)



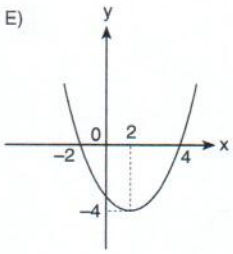
C)



D)



E)



ÇÖZÜM

Yanıt **D**

$y = -x^2 + 4x \Rightarrow a = -1\ ,\ b = 4\ ,\ c = 0$

$$T(r, k) = \left(\frac{-4}{2 \cdot (-1)} , \frac{4 \cdot (-1) \cdot 0 - 4^2}{4 \cdot (-1)} \right)$$

$$= \left(2, \frac{-16}{-4} \right)$$

$T(r, k) = (2\ ,\ 4)$

$x = 0 \Rightarrow\ y = -0^2 + 4 \cdot 0$

$y = 0$

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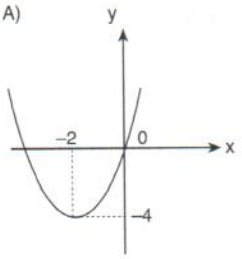
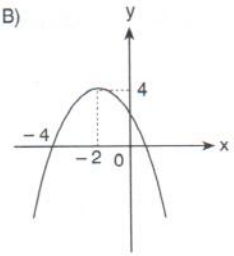
EXAMPLE 2

$y = -x^2 + 4x \Rightarrow$ **Graph = ?**

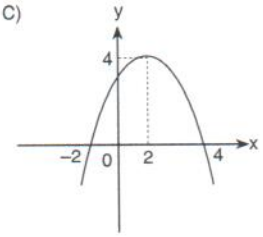
A)



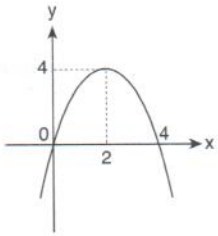
B)



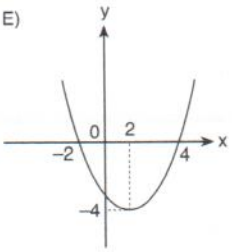
C)



D)



E)



SOLUTION

The answer is **D**

$y = -x^2 + 4x \Rightarrow a = -1 \ , \ b = 4 \ , \ c = 0$

$$T(r, k) = \left(\frac{-4}{2 \cdot (-1)} , \frac{4 \cdot (-1) \cdot 0 - 4^2}{4 \cdot (-1)} \right)$$

$$= \left(2, \frac{-16}{-4} \right)$$

$T(r, k) = (2 , 4)$

$x = 0 \Rightarrow \ y = -0^2 + 4 \cdot 0$

$y = 0$

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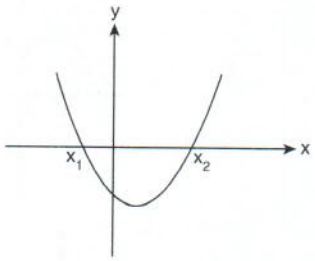
PARABOL DENKLEMİNİN YAZILMASI

Eksenleri kestiği noktalar ile bir noktası bilinen parabolün denkleminin bulunması

a. x eksenini $(x_1, 0)$, $(x_2, 0)$ noktalarında kesen parabolün denklemi:



$$y = a\cdot(x - x_1)\cdot(x - x_2)$$

ÖRNEK 3

$f(x) = (ax + b)(x + 1) \Rightarrow$ **a = ?**

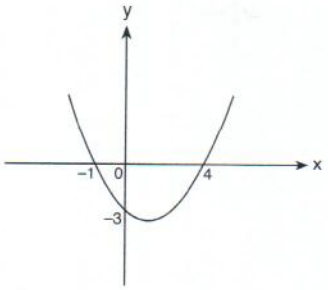
A) $\frac{3}{4}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) 3

E) 4



ÇÖZÜM

Yanıt **A**

$y = a \cdot (x + 1) \cdot (x - 4)$

$x = 0 \Rightarrow y = -3 \Rightarrow -3 = a \cdot 1 \cdot (-4)$

$$a = \frac{3}{4}$$

$$f(x) = \frac{3}{4}(x+1)(x-4) = (ax+b)(x+1)$$

$$\frac{3}{4}(x-4) = ax+b$$

$$a = \frac{3}{4}$$

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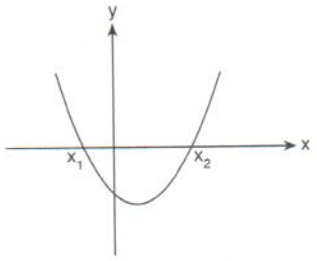
WRITING of a PARABOLA EQUATION

Finding the Equation of the Parabola with one point and the x and y intercepts which are given

a. The equation of the parabola with x – intercepts; $(x_1, 0)$ and $(x_2, 0)$:



$$y = a\cdot(x - x_1)\cdot(x - x_2)$$

EXAMPLE 3

$f(x) = (ax + b)(x + 1) \Rightarrow$ **a = ?**

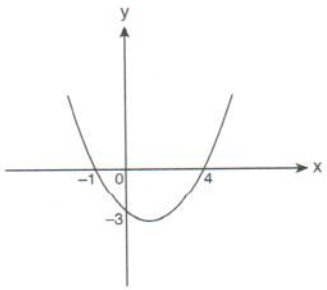
A) $\frac{3}{4}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) 3

E) 4



SOLUTION

The answer is **A**

$y = a \cdot (x + 1) \cdot (x - 4)$

$x = 0 \Rightarrow y = -3 \Rightarrow -3 = a\cdot 1\cdot(-4)$

$$a = \frac{3}{4}$$

$$f(x) = \frac{3}{4}(x+1)(x-4) = (ax+b)(x+1)$$

$$\frac{3}{4}(x-4) = ax + b$$

$$a = \frac{3}{4}$$

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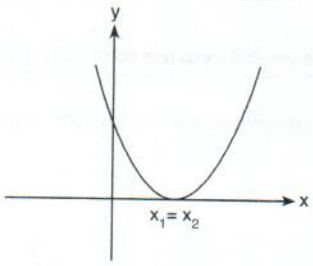
b. x eksenine $(x_1, 0) = (x_2, 0)$ noktasında teğet olan parabolün denklemi:



$$y = a\cdot(x - x_1)^2$$

ÖRNEK 4

$f(x) = (x - 2)(ax + b)$

$\Rightarrow$ **(a, b) = ?**

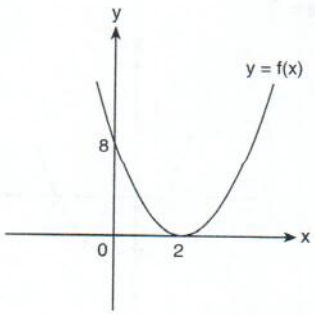
A) (–1, 2)

B) (2, –3)

C) (2, –4)

D) (2, –5)

E) (2, 4)



ÇÖZÜM

Yanıt **C**

$f(x) = a(x - 2)^2$

$x = 0 \Rightarrow y = 8 \Rightarrow 8 = 4a$

$a = 2$

$f(x) = 2(x - 2)^2 = (x - 2)(ax + b)$

$2(x - 2) = ax + b$

$2x - 4 = ax + b$

$a = 2,\ b = -4$

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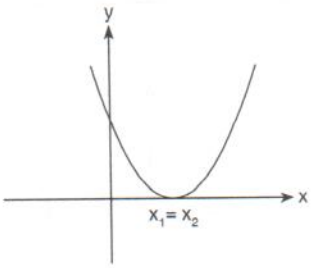
b. The equation of the parabola that is tangent to the x – axis at the point $(x_1, 0) = (x_2, 0)$:



$$y = a \cdot (x - x_1)^2$$

EXAMPLE 4

$f(x) = (x - 2)(ax + b)$

$\Rightarrow$ **(a, b) = ?**

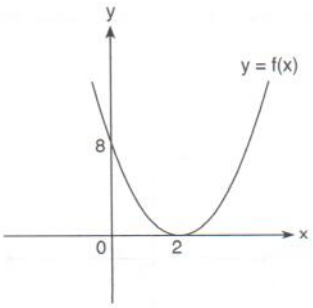
A) (–1, 2)

B) (2, –3)

C) (2, –4)

D) (2, –5)

E) (2, 4)



SOLUTION

The answer is **C**

$f(x) = a(x - 2)^2$

$x = 0 \Rightarrow y = 8 \Rightarrow 8 = 4a$

$a = 2$

$f(x) = 2(x - 2)^2 = (x - 2)(ax + b)$

$2(x - 2) = ax + b$

$2x - 4 = ax + b$

$a = 2, \; b = -4$

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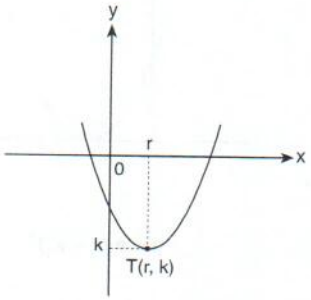
Tepe noktası T(r, k) olan parabolün denklemi



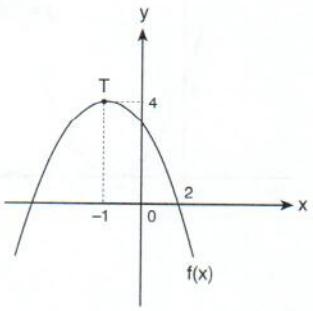
$$y = a(x - r)^2 + k$$

ÖRNEK 5

$f(x) = ax^2 + bx + c$

$\Rightarrow$ **f(–4) = ?**

A) –6
B) –4
C) 0
D) 4
E) 5



ÇÖZÜM

Yanıt **C**

$y = a(x+1)^2 + 4$

$0 = a(2+1)^2 + 4$

$-4 = 9a \Rightarrow a = \frac{-4}{9}$

$f(x) = -\frac{4}{9}(x+1)^2 + 4$

$f(-4) = -\frac{4}{9} \cdot 9 + 4 = 0$

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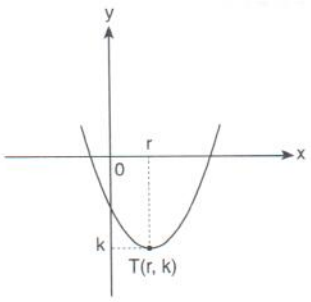
The equation of the parabola with a vertex T(r, k)



$$y = a(x - r)^2 + k$$

EXAMPLE 5

$f(x) = ax^2 + bx + c$

$\Rightarrow$ **f(–4) = ?**

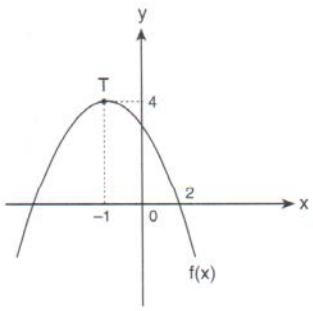
A) –6

B) –4

C) 0

D) 4

E) 5



SOLUTION

The answer is C

$y = a(x+1)^2 + 4$

$0 = a(2+1)^2 + 4$

$-4 = 9a \Rightarrow a = \frac{-4}{9}$

$f(x) = -\frac{4}{9}(x+1)^2 + 4$

$f(-4) = -\frac{4}{9} \cdot 9 + 4 = 0$

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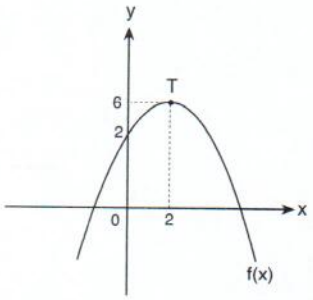
ÖRNEK 6

$f(x) = ax^2 + bx + c \Rightarrow$ **5a + b + c = ?**

A) –2
B) –1
C) 0
D) 1
E) 2



ÇÖZÜM

Yanıt **D**

$y = a(x - 2)^2 + 6$

$2 = a(0 - 2)^2 + 6$

$-4 = 4a$

$a = -1$

$y = -(x - 2)^2 + 6$

$= -x^2 + 4x + 2 = ax^2 + bx + c$

$a = -1,\ b = 4,\ c = 2$

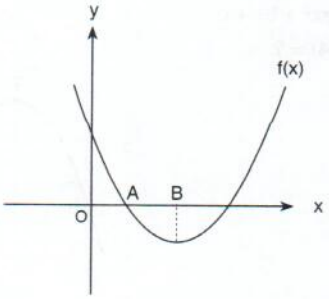
$5a + b + c = -5 + 4 + 2 = 1$

ÖRNEK 7

$f(x) = x^2 - 12x + m + 8$

$|AB| = 2 \cdot |AO| \Rightarrow$ **m = ?**

A) –8
B) –6
C) 18
D) 12
E) 19



ÇÖZÜM

Yanıt **D**

$|AB| = 2|AO|$

$|AO| = k \Rightarrow |AB| = 2k$

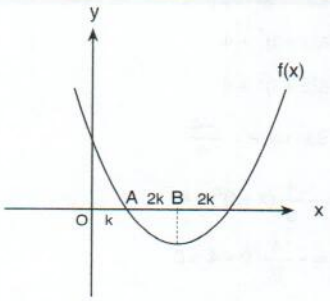
$x_1 = k,\ x_2 = 5k$

$x_1 + x_2 = 6k = 12$

$k = 2 \Rightarrow x_1 = 2,\ x_2 = 10$

$x_1 \cdot x_2 = 2 \cdot 10 = 20 = m + 8$

$m = 12$



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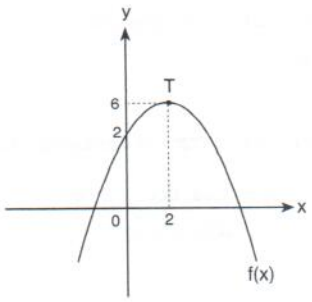
EXAMPLE 6

$f(x) = ax^2 + bx + c \Rightarrow$ **5a + b + c = ?**

A) –2
B) –1
C) 0
D) 1
E) 2



SOLUTION

The answer is **D**

$y = a(x - 2)^2 + 6$

$2 = a(0 - 2)^2 + 6$

$-4 = 4a$

$a = -1$

$y = -(x - 2)^2 + 6$

$= -x^2 + 4x + 2 = ax^2 + bx + c$

$a = -1, b = 4, c = 2$

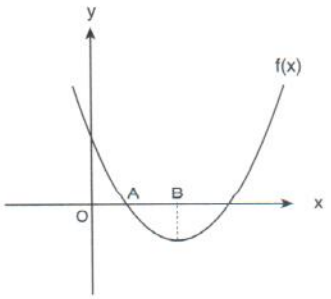
$5a + b + c = -5 + 4 + 2 = 1$

EXAMPLE 7

$f(x) = x^2 - 12x + m + 8$

$|AB| = 2 \cdot |AO| \Rightarrow$ **m = ?**

A) –8
B) –6
C) 18
D) 12
E) 19



SOLUTION

The answer is **D**

$|AB| = 2|AO|$

$|AO| = k \Rightarrow |AB| = 2k$

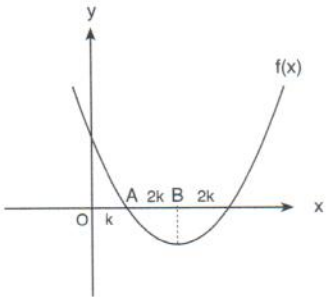
$x_1 = k, \ x_2 = 5k$

$x_1 + x_2 = 6k = 12$

$k = 2 \Rightarrow x_1 = 2, \ x_2 = 10$

$x_1 \cdot x_2 = 2 \cdot 10 = 20 = m + 8$

$m = 12$



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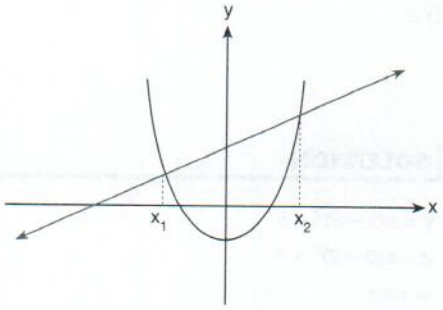
BİR PARABOLLE BİR DOĞRU ARASINDAKİ İLİŞKİ

$y = ax^2 + bx + c$ parabolü ile $y = mx + n$ doğrusunun durumu belirtmek için ortak çözümden yararlanılır.

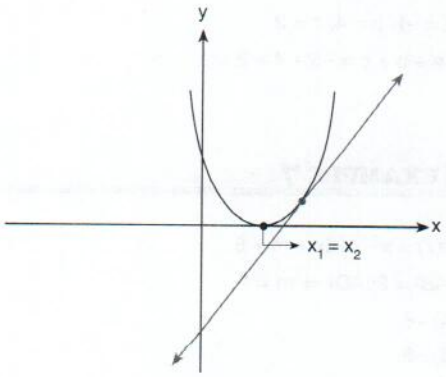
$ax^2 + bx + c = mx + n$ denkleminde,

1 $\Delta > 0$ ise, doğru parabolü iki noktada keser.



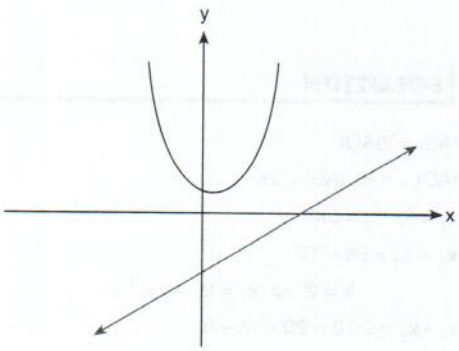
$\Delta > 0$

2 $\Delta = 0$ ise, doğru parabole bir noktada teğettir.



$\Delta = 0$

3 $\Delta < 0$ ise, parabol ile doğru kesişmez.



$\Delta < 0$

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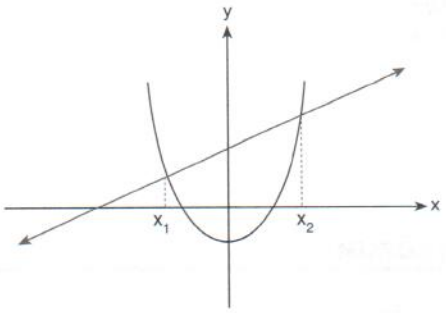
RELATIONSHIP BETWEEN a PARABOLA and a LINE

We use common solution to the relationship between the parabola $y = ax^2 + bx + c$ and the line $y = mx + n$.

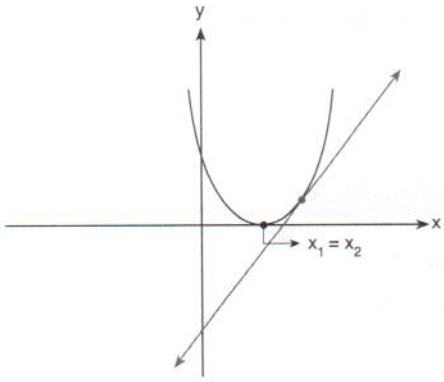
In equation $ax^2 + bx + c = mx + n$,

1 If $\Delta > 0$, then line and parabola is cut by two points.



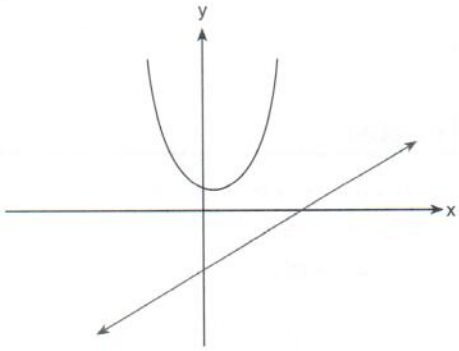
$\Delta > 0$

2 If $\Delta = 0$, line is tangent to the parabola at a point.



$\Delta = 0$

3 If $\Delta < 0$, parabola and line is not intersect.



$\Delta < 0$

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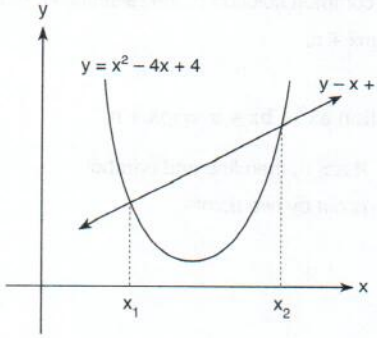
ÖRNEK 8

$x_1 + 2x_2 = ?$

A) 2
B) 4
C) 5
D) 6
E) 8



ÇÖZÜM

Yanıt E

$x^2 - 4x + 4 = x - 2$

$x^2 - 5x + 6 = 0$

$(x - 3)\cdot(x - 2) = 0$

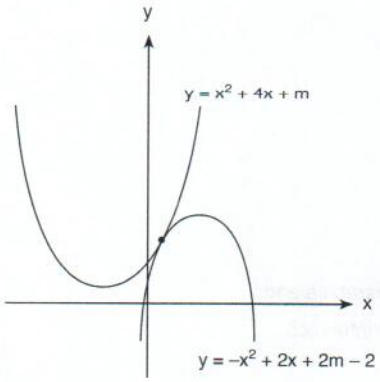
$x_1 = 2,\ x_2 = 3$

$\Rightarrow x_1 + 2x_2 = 2 + 6 = 8$

ÖRNEK 9

$m = ?$

A) $-\frac{3}{2}$
B) $-\frac{1}{2}$
C) $\frac{1}{2}$
D) $\frac{3}{2}$
E) $\frac{4}{3}$



ÇÖZÜM

Yanıt D

$x^2 + 4x + m = -x^2 + 2x + 2m - 2$

$2x^2 + 2x - m + 2 = 0$

$\Delta = 4 - 4\cdot2\cdot(2 - m) = 0$

$\frac{1}{2} = 2 - m$

$m = \frac{3}{2}$

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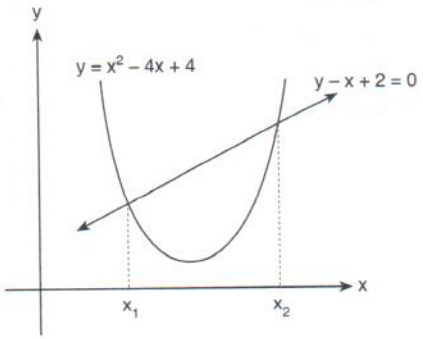
EXAMPLE 8

$x_1 + 2x_2 = ?$

A) 2
B) 4
C) 5
D) 6
E) 8



SOLUTION

The answer is E

$x^2 - 4x + 4 = x - 2$

$x^2 - 5x + 6 = 0$

$(x - 3)\cdot(x - 2) = 0$

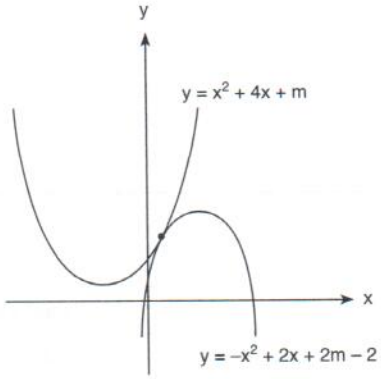
$x_1 = 2,\ x_2 = 3$

$\Rightarrow x_1 + 2x_2 = 2 + 6 = 8$

EXAMPLE 9

$m = ?$

A) $-\frac{3}{2}$
B) $-\frac{1}{2}$
C) $\frac{1}{2}$
D) $\frac{3}{2}$
E) $\frac{4}{3}$



SOLUTION

The answer is D

$x^2 + 4x + m = -x^2 + 2x + 2m - 2$

$2x^2 + 2x - m + 2 = 0$

$\Delta = 4 - 4\cdot2\cdot(2 - m) = 0$

$$\frac{1}{2} = 2 - m$$

$$m = \frac{3}{2}$$

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1. **y = ?**

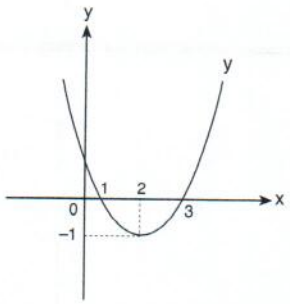
A) $(x-2)^2-1$

B) $(x+2)^2-1$

C) $(x-2)^2-4$

D) $(x-2)^2+4$

E) $(x+2)^2+1$



2. T (2,k) ,

$y = (m+1)x^2 - 2mx + 4$

$\Rightarrow$ **m = ?**

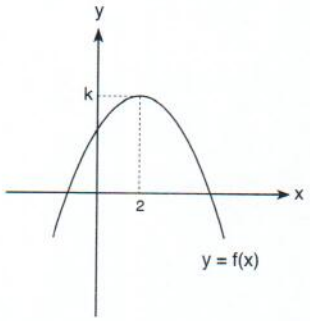
A) – 6

B) – 4

C) – 2

D) 2

E) 4



3. **f(4) = ?**

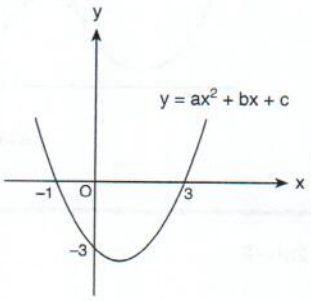
A) 1

B) 2

C) 3

D) 4

E) 5



4. **m = ?**

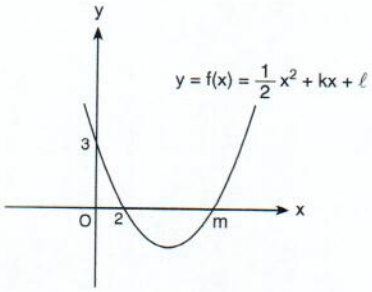
A) 1

B) 2

C) 3

D) 4

E) 5



5. **m·k = ?**

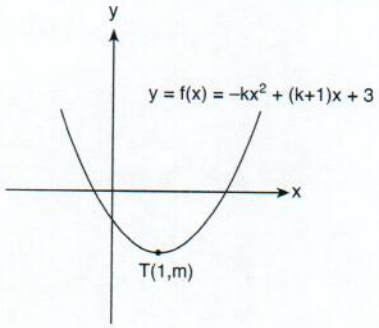
A) 5

B) 4

C) 3

D) 2

E) 1



6. 6·|OA| = |OB|

$\Rightarrow$ **k = ?**

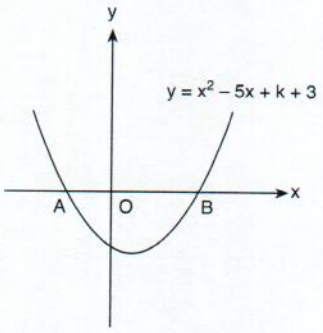
A) –4

B) –5

C) –6

D) –8

E) –9



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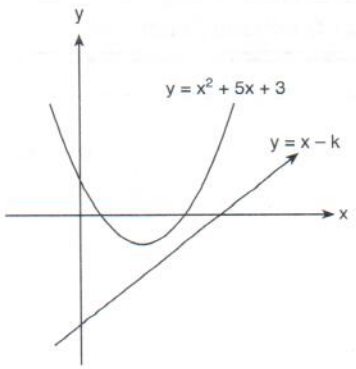
7. $k \in Z \Rightarrow$ **min(k) = ?**

A) 1
B) 2
C) 3
D) 4
E) 5

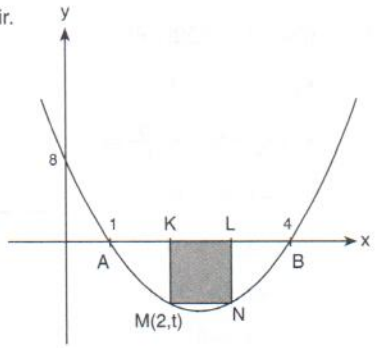


8. $y = f(x) = ax^2 + 2ax + 5$,
$\min(f(x)) = 1 \Rightarrow$ **a = ?**

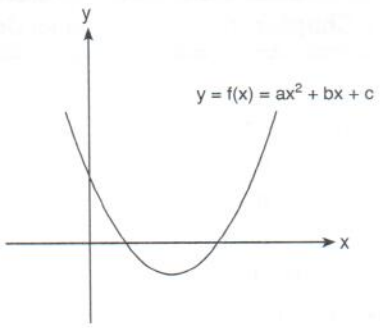
A) 3
B) 4
C) 5
D) 6
E) 7

9. KLMN bir dikdörtgendir.
KLMN is a rectangle.
A(KLMN) = ? br^2(u^2)

A) 12
B) 9
C) 8
D) 6
E) 4



10.

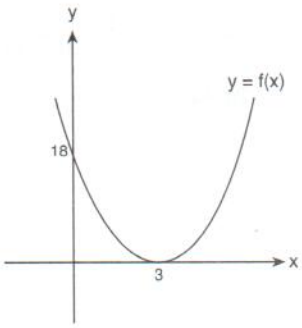


Aşağıdakilerden hangisi yanlıştır?
Which of the following is not true?

A) $a > 0$ B) $c > 0$ C) $\Delta > 0$
D) $b > 0$ E) $f(0) > 0$

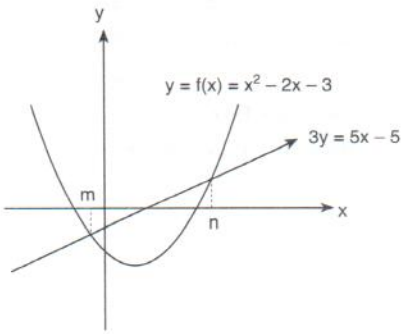
11. **f(7) = ?**

A) 14
B) 18
C) 20
D) 23
E) 32



12. **m + n = ?**

A) 3
B) $\frac{10}{3}$
C) $\frac{11}{3}$
D) 4
E) $\frac{13}{3}$



YANITLAR / ANSWERS

1. A	2. C	3. E	4. C	5. B	6. E	7. B	8. B
9. E	10. D	11. E	12. C				

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1. $T(2,-1)$

$y = a\cdot(x-2)^2 - 1$

$(1,0) \in y \Rightarrow 0 = a\cdot(1-2)^2 - 1$

$0 = a - 1$

$a = 1$

$y = 1\cdot(x-2)^2 - 1$

$y = (x-2)^2 - 1$

Yanıt / The Answer is A

2. $T = (2, k)$

$$2 = \frac{-b}{2a} = \frac{2m}{2\cdot(m+1)} = \frac{m}{m+1}$$

$2m + 2 = m \Rightarrow m = -2$

Yanıt / The Answer is C

3. $x_1 = -1,\ x_2 = 3 \Rightarrow y = a\cdot(x+1)(x-3)$

$x = 0,\ y = -3 \Rightarrow -3 = a\cdot(0+1)(0-3)$

$1 = a$

$f(x) = (x+1)\cdot(x-3)$

$f(4) = (4+1)\cdot(4-3) = 5$

Yanıt / The Answer is E

4. $\ell = 3,\ x_1 = 2,\ x_2 = m$

$$x_1\cdot x_2 = 2\cdot m = \frac{\ell}{\frac{1}{2}} = 6 \Rightarrow m = 3$$

Yanıt / The Answer is C

5. $$r = \frac{-b}{2a} = -\frac{(k+1)}{-2k} = 1$$

$k + 1 = 2k$

$k = 1$

$y = -x^2 + 2x + 3$

$m = f(1) = -1^2 + 2 + 3 = 4$

$m\cdot k = 4\cdot 1 = 4$

Yanıt / The Answer is B

6. $|OA| = t \Rightarrow |OB| = 6t$

$x_1 = -t,\ x_2 = 6t$

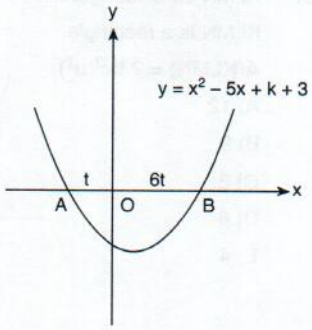
$$x_1 + x_2 = 5t = -\frac{-5}{1} \Rightarrow t = 1$$

$x_1 = -1,\ x_2 = 6$

$\Rightarrow x_1\cdot x_2 = k + 3$

$-6 = k + 3$

$k = -9$



Yanıt / The Answer is E

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7. $x^2 + 5x + 3 = x - k$

$x^2 + 4x + 3 + k = 0$

$\Delta = 4^2 - 4(3 + k) < 0$

$16 - 12 - 4k < 0$

$4 < 4k$

$1 < k$

$\min(k) = 2$

Yanıt / The Answer is B

8. $r = \frac{-b}{2a} = \frac{-2a}{2a} = -1$

$k = 1 \Rightarrow a \cdot (-1)^2 + 2a \cdot (-1) + 5 = 1$

$-a + 5 = 1$

$a = 4$

Yanıt / The Answer is B

9. $\left.\begin{matrix} x_1 = 1 \\ x_2 = 4 \end{matrix}\right\} \Rightarrow y = a(x-1)(x-4)$

$x = 0, y = 8 \Rightarrow 8 = a(0-1)(0-4)$

$a = 2$

$f(x) = 2(x - 1)(x - 4)$

$M(2, t) \Rightarrow t = 2 \cdot (2 - 1) \cdot (2 - 4)$

$t = -4$

$A(KLMN) = t^2 = 16 \text{ br}^2 (u^2)$

Yanıt / The Answer is E

10. $a > 0$

$c > 0 \Rightarrow x_1 \cdot x_2 > 0$

$x_1, x_2 > 0 \Rightarrow x_1 + x_2 = \frac{-b}{a} > 0$

$\Rightarrow b < 0$

Yanıt / The Answer is D

11. $f(x) = a(x - 3)^2$

$18 = a(0 - 3)^2$

$2 = a$

$f(x) = 2(x - 3)^2$

$f(7) = 2(7 - 3)^2$

$= 32$

Yanıt / The Answer is E

12. $x^2 - 2x - 3 = \frac{5x - 5}{3}$

$3x^2 - 11x - 4 = 0$

$(3x + 1)(x - 4) = 0$

$m = -\frac{1}{3}, \; n = 4 \Rightarrow m + n = \frac{11}{3}$

Yanıt / The Answer is C

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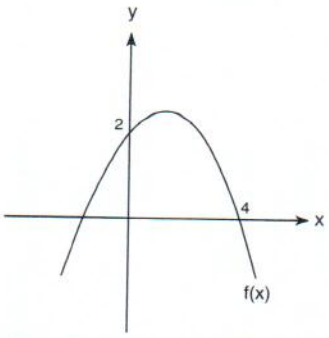
1.

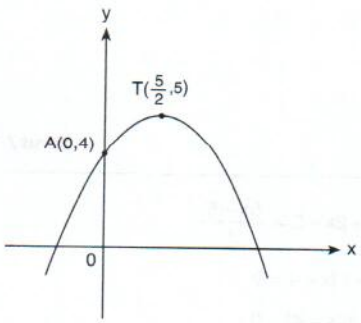


$f(x) = -x^2 + kx + m \Rightarrow$ **f(2) = ?**

A) –1 B) 0 C) 1 D) 5 E) 9

[ANKARA ÜNİV. YÖS 2013]

2.

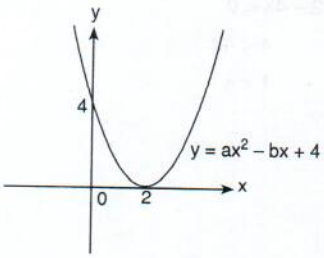


$y = ax^2 + bx + c \Rightarrow$ **b = ?**

A) $-\frac{4}{25}$ B) $-\frac{4}{5}$ C) $\frac{4}{25}$ D) $\frac{4}{5}$ E) $\frac{5}{4}$

[İSTANBUL ÜNİV. YÖS 2013]

3.

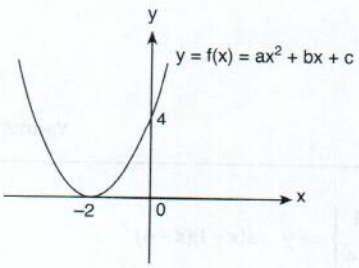


$\Rightarrow$ **a + b = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

[YÖS 2003]

4.



$\Rightarrow$ **f(–3) = ?**

A) – 2 B) – 1 C) 0 D) 1 E) 2

[YÖS 2002]

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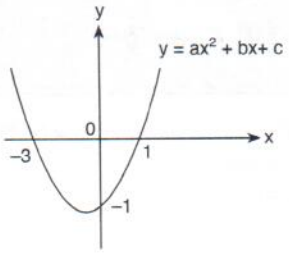
5.



$\Rightarrow$ **3a + 6b + 2c = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

[YÖS 2001]

YANITLAR / ANSWERS

1. D	2. D	3. B	4. D	5. C

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1. $f(x) = -x^2 + kx + 2$

$f(4) = -16 + kx + 2 = 0$

$-14 = -4k$

$k = \frac{7}{2}$

$f(2) = -4 + \frac{7}{2} \cdot 2 + 2$

$= -4 + 7 + 2$

$= 5$

Yanıt / The Answer is D

2. $f(x) = a(x - r)^2 + k$

$f(x) = a\left(x - \frac{5}{2}\right)^2 + 5$

$4 = a\left(\frac{5}{2}\right)^2 + 5$

$\frac{25}{4}a = -1 \Rightarrow a = -\frac{4}{25}$

$\Rightarrow f(x) = -\frac{4}{25} \cdot \left(x^2 - 5x + \frac{25}{4}\right) + 5$

$= -\frac{4}{25}x^2 + \frac{4}{5}x + 4 \Rightarrow b = \frac{4}{5}$

Yanıt / The Answer is D

3. Tepe noktasının koordinatları: (r, k) = (2, 0)

Coordinates of vertex

$y = a(x - 2)^2 + 0$

$x = 0 \Rightarrow 4 = a \cdot (0 - 2)^2 \Rightarrow 4 = 4a$

$a = 1$

$y = 1(x - 2)^2 = x^2 - 4x + 4$

$x^2 - 4x + 4 = ax^2 - bx + 4$

$\Rightarrow a = 1$ ve (and) $b = 4$

$a + b = 5$

Yanıt / The Answer is B

4. Tepe noktasının koordinatları: (r, k) = (– 2, 0)

Coordinates of vertex

$y = a(x - r)^2 + k = a(x + 2)^2 + 0$

$x = 0 \Rightarrow 4 = a \cdot 4$

$a = 1$

$\Rightarrow y = f(x) = (x + 2)^2$

$f(-3) = (-3 + 2)^2 = 1$

Yanıt / The Answer is D

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5. $y = a(x+3)(x-1)$,

$x = 0 \Rightarrow -1 = a\cdot(3)\cdot(-1)$

$$a = \frac{1}{3}$$

$$y = \frac{1}{3}(x+3)(x-1) = \frac{x^2}{3} + \frac{2}{3}x - 1$$

$$a = \frac{1}{3}\ ,\ b = \frac{2}{3}\ ,\ c = -1$$

$$\Rightarrow 3a + 6b + 2c = 3\cdot\frac{1}{3} + 6\cdot\frac{2}{3} - 2\cdot 1$$

$$= 1 + 4 - 2$$

$$= 3$$

Yanıt / The Answer is C

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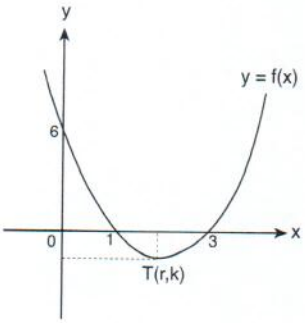
1. $f(x) = ax^2 + bx + c$

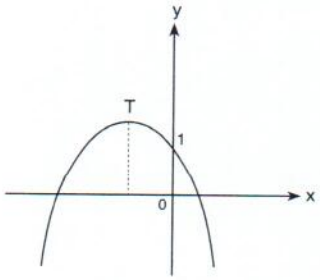
$\Rightarrow$ **k = ?**



Yanıt / The Answer is **–2**

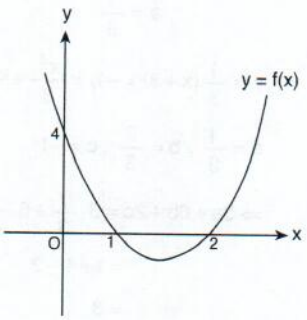
2. $f(x) = ax^2 + bx + c$

$\Rightarrow$ **c = ?**



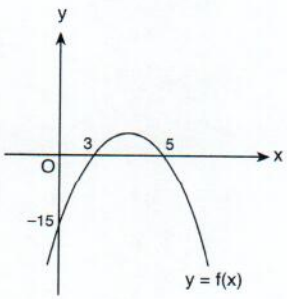
Yanıt / The Answer is **1**

3. **f(4) = ?**



Yanıt / The Answer is **12**

4. **f(1) = ?**



Yanıt / The Answer is **–8**

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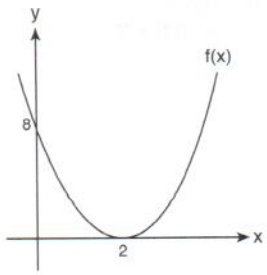
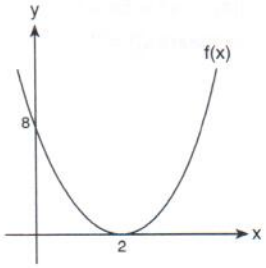
5. $f(x) = (x - 2)\cdot(ax + b)$

$\Rightarrow$ **a + b = ?**



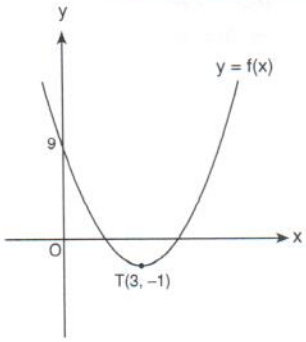
Yanıt / The Answer is **–2**

6. **f(1) = ?**



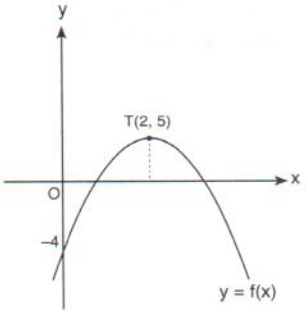
Yanıt / The Answer is **2**

7. **f(–1) = ?**



Yanıt / The Answer is $\frac{151}{9}$

8. **f(1) = ?**



Yanıt / The Answer is $\frac{11}{4}$

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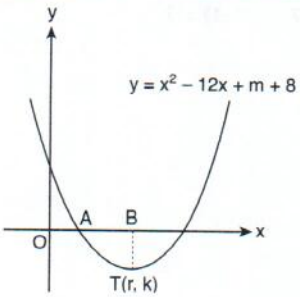
9. $|AB| = 2|AO|$

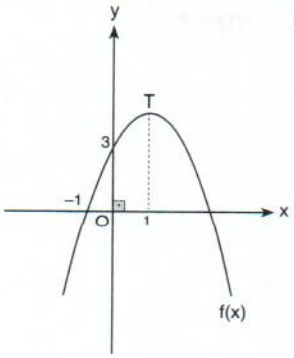
$\Rightarrow$ **m = ?**



Yanıt / The Answer is **12**

10. $f(x) = ax^2 + bx + c$

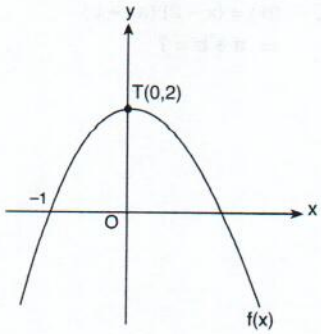
$\Rightarrow$ **a – b + c = ?**



Yanıt / The Answer is **0**

11. T(0, 2)

$\Rightarrow$ **f(3) = ?**



Yanıt / The Answer is **–16**

12. $f(x) = -x^2 + 6x + 5$

$\Rightarrow$ **max(f(x)) = ?**

Yanıt / The Answer is **14**

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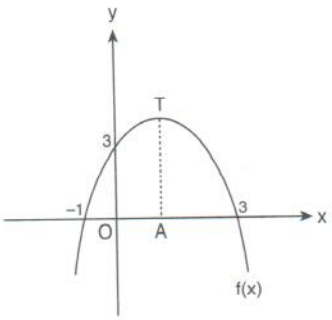
1. $y = ax^2 + bx + c$

$\Rightarrow$ **|TAI = ? br(u)**

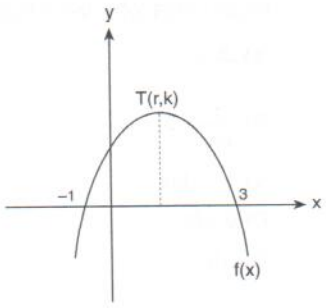
A) 4
B) 5
C) 6
D) 7
E) 8



2. $y = ax^2 + bx + c$

$\Rightarrow$ **r = ?**

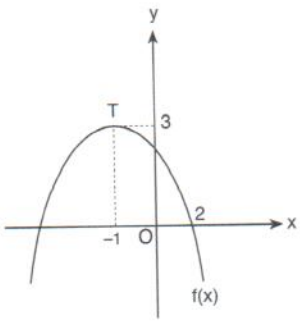
A) $\frac{1}{3}$
B) $\frac{1}{2}$
C) 1
D) $\frac{3}{2}$
E) 2



3. $f(x) = ax^2 + bx + c$

$\Rightarrow$ **f(5) = ?**

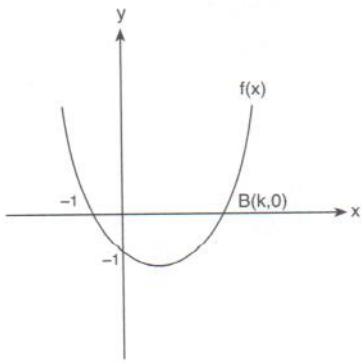
A) –24
B) –9
C) –6
D) –3
E) 0



4. $f(x) = x^2 + ax + b$

$\Rightarrow$ **k = ?**

A) $\frac{1}{2}$
B) 1
C) $\frac{3}{2}$
D) 2
E) $\frac{5}{2}$



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5. $y = x^2 - (b-1)x + b - 4$

$\Rightarrow$ **IABI = ? br(u)**

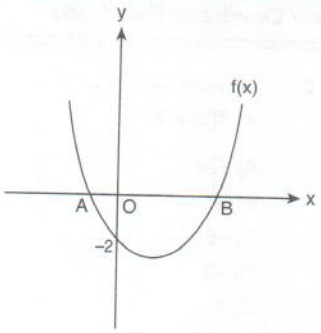
A) $\sqrt{13}$

B) $\sqrt{17}$

C) 2

D) 3

E) 5



6. $f(x) = 2x^2 - 16x + k - 3$,

$2 \cdot IOAI = 3 \cdot IABI$

$\Rightarrow$ **k = ?**

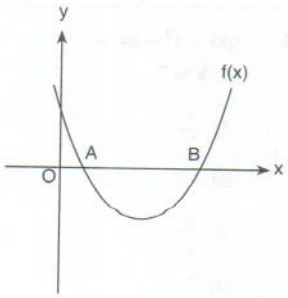
A) 26

B) 27

C) 29

D) 30

E) 33



7. **min(f(x)) = ?**

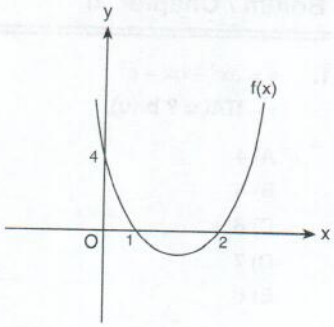
A) -1

B) $-\frac{1}{2}$

C) -2

D) $-\frac{3}{2}$

E) -3



8. **Hangisi doğrudur?**

Which of the following is true?

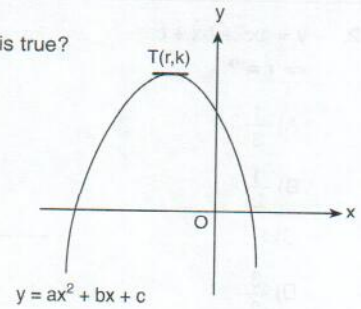
A) $\frac{a}{c} > 0$

B) $\frac{b}{2a} < 0$

C) $b^2 - 4ac < 0$

D) $b > 0$

E) $a \cdot b \cdot c > 0$



YANITLAR / ANSWERS							
1. A	2. C	3. B	4. B	5. D	6. E	7. B	8. E

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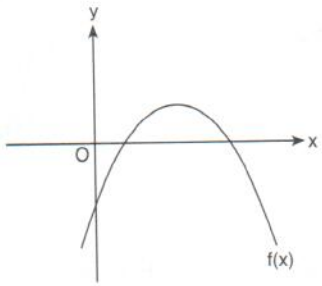
1.



$f(x) = ax^2 + bx + c$

Buna göre aşağıdakilerden hangisi doğrudur?

Which of the following is true?

A) $a \cdot c < 0$ B) $b^2 - 4ac < 0$ C) $a \cdot b > 0$

D) $b \cdot c > 0$ E) $b > 0$

2. $f(x) = x^2 + 8x + a + 3$

$|AB| = 6$ br(u)

$\Rightarrow$ **$|OC| = ?$ br(u)**

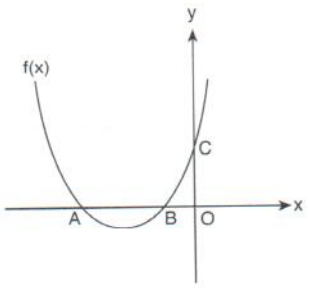
A) 4

B) 5

C) 6

D) 7

E) 8



3. $f(x) = x^2 + mx + n$

$A(n, 0)$, $B(m, 0)$

$\Rightarrow$ **$m + n = ?$**

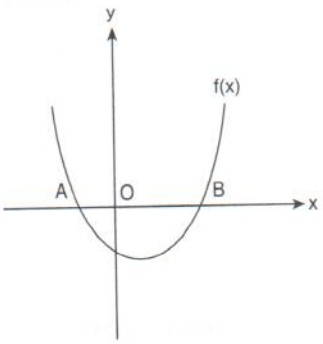
A) $-\frac{1}{2}$

B) $-\frac{3}{4}$

C) -1

D) -2

E) -3



4. $f(x) = x^2 - 8x + 3 - 3a$,

$|AB| = 2$ br(u) $\Rightarrow$ **$a = ?$**

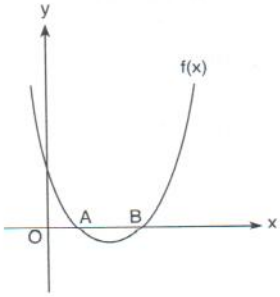
A) 2

B) –4

C) 4

D) 3

E) –3



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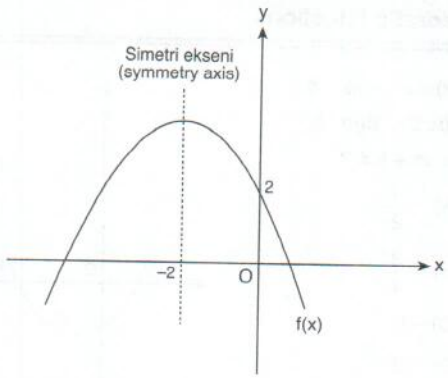
5.



$f(x) = (m-1)x^2 - (2m+1)x + k \Rightarrow$ **m + k = ?**

A) $\frac{5}{2}$ B) $-\frac{3}{2}$ C) $\frac{9}{4}$

D) $-\frac{9}{4}$ E) $-\frac{7}{4}$

6. $f(x) = -mx^2 - 2nx + k$

$\Rightarrow$ m + n + k = ?

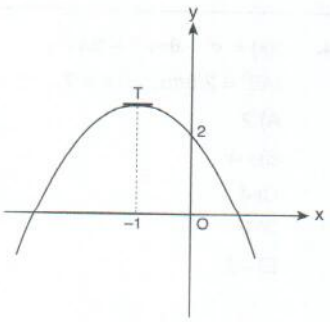
A) n – 2

B) 2 + n

C) 5n + 2

D) 2 – 5n

E) 2n + 2

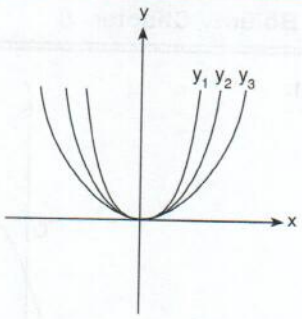


7. $y_1 = ax^2$

$y_2 = bx^2$

$y_3 = cx^2$

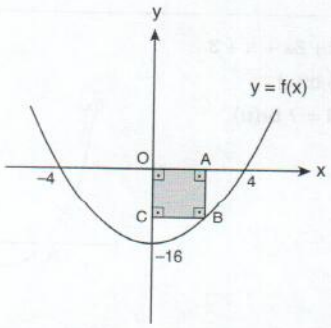
$\Rightarrow$? < ? < ?



A) c < b < a B) b < c < a C) a < b < c

D) c < a < b E) a < c < b

8.



A(2, a) $\Rightarrow$ **taralı alan / the shaded area = ? $br^2(u^2)$**

A) 12 B) 24 C) 30

D) 36 E) 40

YANITLAR / ANSWERS							
1. E	2. D	3. C	4. B	5. A	6. E	7. A	8. B

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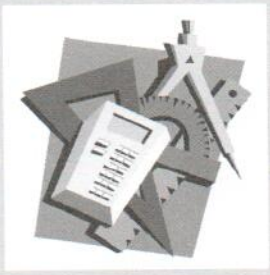
ÜNİTE 2

UNIT 2

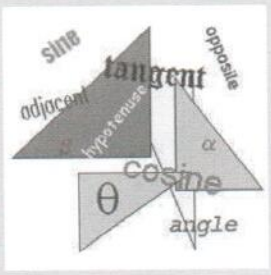
Bu ünitede, trigonometri ayrıntılı olarak anlatılmıştır. Trigonometri konusu karmaşık olduğundan iki bölüme ayrılmıştır. Birinci bölümde temel trigonometri kuralları, ikinci bölümde trigonometrik formüller anlatılmıştır.

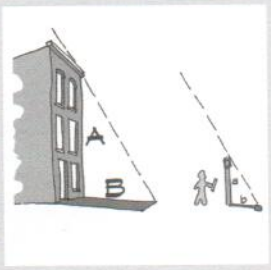
Bu ünite; 118 çözümlü örnek, 60 alıştırma ve 72 test sorusundan oluşmaktadır.

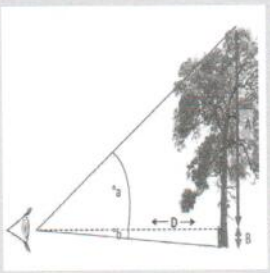


In this unit, trigonometry is explained in detailed. Trigonometry is consist of two chapters since this subject is complex. The first chapter is consist of the basic rules of trigonometry and the second chapter is consist of trigonometric formulas.

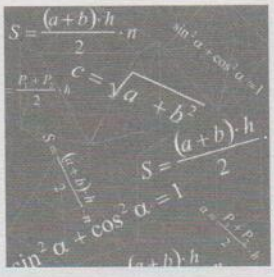
This unit includes; 118 examples, 60 exercises, and 72 test questions.







- **TRİGONOMETRİ I** ..... 149 – 210
- **TRİGONOMETRİ II** ..... 211 – 272



- **TRIGONOMETRY I** ..... 149 – 210
- **TRIGONOMETRY II** ..... 211 – 272

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BÖLÜM 1
CHAPTER

TARİHSEL NOT / HISTORICAL NOTE

Bolyai, Janos [1802 – 1860]

Macar matematikçi. Öklid-dışı geometrik yaklaşımı bulduğunu 1832 yılında yaptığı çalışmalarla duyurmuştur. Lobachevsky'den bağımsız olarak çalışmıştır.

Hungarian mathematician who in a work published in 1832 but probably dating from 1823, announced his discovery of non- Euclidean geometry. His work was independent of Lobachevsky.

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TRIGONOMETRY I

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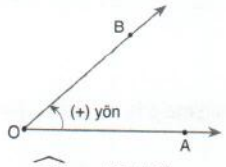
TRİGONOMETRİ I
TRIGONOMETRY I

BÖLÜM 1
CHAPTER

YÖNLÜ AÇILAR

Tanım: Bir açının kenarlarından birini başlangıç kenarı; diğerini bitim kenarı olarak alırsak elde edilen açıya *yönlü açı* denir.

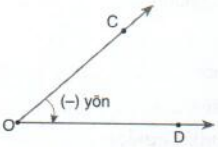
I.



$\widehat{AOB}$ pozitif yönlü açı

[OA, başlangıç kenarı; [OB, bitim kenarı olmak üzere saatin dönme yönünün tersi yönde alınan AOB açısına *pozitif yönlü açı* denir.

II.



$\widehat{COD}$ negatif yönlü açı

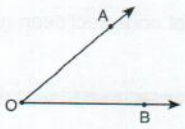
[OC, başlangıç kenarı; [OD, bitim kenarı olmak üzere saatin dönme yönünde alınan COD açısına *negatif yönlü açı* denir.

NOT

Açının köşesi etrafında, başlangıç kenarından bitim kenarına iki türlü gidilebilir. Biri saatin dönme yönünün tersi, diğeri saatin dönme yönünün aynısıdır.

HATIRLATMA

Başlangıç noktaları ortak olan iki açının birleşim kümesine açı denir.



AOB açısı, $\widehat{AOB}$ ile gösterilir.

NOTLAR

- ✓ *Bir açının ölçüsünün büyüklüğünü veya küçüklüğünü tanımlamak için ölçü birimi kullanılır.*
- ✓ *Açıyı ölçmek demek, açının kolları arasındaki açıklığı belirlemek demektir.*
- ✓ *Derece günlük yaşamda, radyan ise bilimsel çalışmalarda kullanılır.*

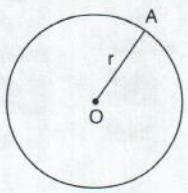
AÇI ÖLÇÜ BİRİMLERİ

Derece: Tüm çember yayının 360 da birine 1 derecelik yay, bu yayı gören merkez açısının ölçüsüne de *1 derece (1°)* denir. Tüm çember yayının ölçüsü 360° dir.

Radyan: Yarıçap uzunluğundaki çember yayına bir radyanlık yay, bu yayı gören merkez açısının ölçüsüne de *1 radyan* denir.

$$\frac{D}{180} = \frac{R}{\pi}$$

NOTLAR



- ✓ *Çevre uzunluğu = $2\pi r$*
- ✓ *Tüm çember yayının ölçüsü 2π radyandır.*

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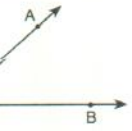
NOTE

We can going to the terminal side from the intial side with two kinds over the angle. Ones ckwise anticlockwise direction, other cl direction.

angles that intial side is angle.



B is denoted by $\widehat{AOB}$.

REMINDER

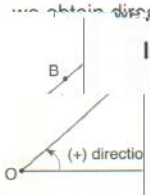
The union set of tw congruent is calle

Angle AC

DIRECTED ANGLES

Definition: If we take one of the side of an angle is initial side and other side is terminal side, we obtain directed angle.

I.



$\widehat{AOB}$ is positive dire

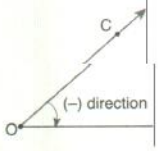
ın a counter – clockw
de, [OB is the termina

If the angle AOB rotates e
where [OA is the initial si

ctional angle.

ise direction, the rotation called *positiv*
al side.

II.



$\widehat{COD}$ is negative dire

the angle COD rotates in a clockwise directi
)C is the initial side, [OD is the terminal side

ctional angle.

If called *negative* where
[C

on, the rotation is

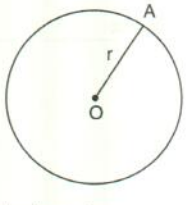
NOTES

leasure unit is used to the definition of calar of an angle.

we said that measure of an angle, we ay the range between the corner of an ngle.

egree is use to the daily life, radian is se to the scientific calculations.

NOTES



The perimeter = 2πr

The measure of a circular arc is 2π radii.

NOTES of ANGLE

: Arc with a degree is $\frac{1}{360}$ of the arc and t
e. One revolution is called 360°.

: If the length of the arc is equal to radius, th
angle is called a *radian*.

$$\frac{D}{180} = \frac{R}{\pi}$$

UNI

Degree central angle is called
a degr

Radian and the measure of the
central

he measure of th

is arc is a radian

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ÖRNEK 1

$\frac{3\pi}{5}$ = ? D

A) E)140° D) 135°

ÇÖZÜM

Yanıt B

$\frac{D}{180^\circ} = \frac{3\pi}{5\pi} \Rightarrow \frac{D}{180^\circ} = \frac{3}{5}$

$D = \frac{3 \cdot 180^\circ}{5} = 108^\circ$

ÖRNEK 2

225° = ? R

A) $\frac{2\pi}{3}$ B) $\frac{11\pi}{15}$ C) $\frac{4\pi}{5}$ D) $\frac{5\pi}{4}$ E) $\frac{7\pi}{4}$

ÇÖZÜM

Yanıt D

$\frac{D}{180^\circ} = \frac{R}{\pi} \Rightarrow \frac{225^\circ}{180^\circ} = \frac{R}{\pi}$

$\frac{5}{4} = \frac{R}{\pi}$

$R = \frac{5\pi}{4}$

PEKİŞTİRME

$275^\circ = x \cdot R \Rightarrow x = ?$

YANIT : $55/36\pi$

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EXAMPLE 1

$\frac{3\pi}{5} = ?$ D

A) 72° B) 108° C) 124° D) 135° E)140°

SOLUTION

The answer is B

$$\frac{R}{\pi} = \frac{D}{180^\circ} \Rightarrow \frac{3\pi}{5\pi} = \frac{D}{180^\circ}$$

$$D = \frac{3 \cdot 180^\circ}{5} = 108^\circ$$

REINFORCE

275° = x·R ⇒ x = ?

ANSWER : 55/36π

EXAMPLE 2

225° = ? R

A) $\frac{2\pi}{3}$ B) $\frac{11\pi}{15}$ C) $\frac{4\pi}{5}$ D) $\frac{5\pi}{4}$ E) $\frac{7\pi}{4}$

ÇÖZÜM

Yanıt D

$$\frac{D}{180^\circ} = \frac{R}{\pi} \Rightarrow \frac{225^\circ}{180^\circ} = \frac{R}{\pi}$$

$$\frac{5}{4} = \frac{R}{\pi}$$

$$R = \frac{5\pi}{4}$$

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ESAS ÖLÇÜ

Tanım: Verilen ölçülerin aşağıdaki denkliklerle hesaplanan pozitif yöndeki değerlerine *esas ölçü* denir.

I. $2\pi \cdot k + \theta \equiv \theta \pmod{2\pi}$ $(k \in Z)$

II. $360° \cdot k + \theta \equiv \theta \pmod{360°}$ $(k \in Z)$

ÖRNEK 3

$750° = \alpha + 360° \cdot k \Rightarrow \alpha = ?$

A) 20° B) 30° C) 40° D) 60° E)120°

ÇÖZÜM

Yanıt **B**

Çözüm 1

$750° = \alpha + 360° \cdot k$

$750° = 30° + 360° \cdot 2$

$\alpha = 30°$

Çözüm 2

$750° \mid 360°$

$720° \mid 2$

$30° \rightarrow \alpha$

▼ Derece cinsinden verilen negatif yönlü açıların esas ölçüsünü bulurken, açı pozitif yönlü açı gibi düşünülüp esas ölçü bulunur. Bulunan değer 360° den çıkarılır.

ÖRNEK 4

$-1060° = \alpha + 360° \cdot k \Rightarrow \alpha = ?$

A) 10° B) 20° C) 30° D) 40° E) 50°

ÇÖZÜM

Yanıt **B**

$1060° \mid 360$

$720° \mid 2$

$340° \rightarrow \alpha = 360° - 340° = 20°$

NOTLAR

- ✓ *Derece cinsinden verilen bir açının 360°'ye bölümünden kalan, derece cinsinden esas ölçüdür.*
- ✓ *Verilen ölçü derece cinsinden ise 360° nin tam katları, radyan cinsinden ise 2π nin tam katları olabilir.*
- ✓ *Radyan cinsinden verilen kesirli ifadede esas ölçü aranırken paydanın çift katları paydan çıkarılır.*
- ✓ *Bir açının esas ölçüsü kesinlikle negatif(–) olamaz. Daima gördüğü yayın bitim noktası ile x ekseninin pozitif yönü arasındaki açı esas ölçü olarak alınır.*

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NOTES

- ✓ *Reference measure in terms of degree, given angle is divided by 360°.*
- ✓ *The given angle can multiple of 360° in degrees or multiple of 2π in radii.*
- ✓ *In expression involving fraction reference measure is found sul numerator by the even mu*
- ✓ *The cen nega termi*

in radii, ract the tiples of ..

al angle of an angle is not ve. Central angle is between al side and positive direction of the

REFERENCE MEASURE

Definition: Values of the positive direction that calculated following equivalent is called *reference measure* of the given measure.

I. $2\pi \cdot k + \theta \equiv \theta \pmod{2\pi}$ $(k \in Z)$

II. $360° \cdot k + \theta \equiv \theta \pmod{360°}$ $(k \in Z)$

EXAMPLE

0°·k ⇒ α = ?

A) 20° B) 30° C) 40° D) 60° E)120°

750° = α + 36

SOLUTION

The answer is B

750° = α + 360°·k

750° 720°

α = 30°

30 → α

angle that negative di angle. Finding value i

▼ To find the reference ve directional angle of given

rectional angle we think as positi s subtracted by 360°.

EXAMPLE 4

–1060° = α + 360°·k ⇒ α =

A) 10° B) 20° C) 30° D) 40° E) 50°

SOLUTION

The answer is B

1060° | 360
720° | 2
340°

340° → α = 360° – 340° =

20°

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▼ a, b ∈ Z⁺ olmak üzere, $\frac{a\cdot\pi}{b}$ 'nin esas ölçüsünü bulmak için a, 2b'ye bölünür. Kalan paya, payda aynen yazılır. a'nın, 2b'ye bölümünden kalan k ise, $\frac{a\pi}{b}$ 'nin, esas ölçüsü, $\frac{k\pi}{b}$ 'dir.

ÖRNEK 5

$\frac{17\pi}{5} = \alpha + 2k\pi \Rightarrow \alpha = ?$

A) $\frac{2\pi}{5}$ B) $\frac{4\pi}{5}$ C) π D) $\frac{6\pi}{5}$ E) $\frac{7\pi}{5}$

ÇÖZÜM Yanıt E

Çözüm 1

$\frac{17\pi}{5} = \alpha + 2k\pi$

$\frac{7\pi}{5} = \alpha + 2k\pi$

$\alpha = \frac{7\pi}{5}$

Çözüm 2

$\frac{10\pi + }{5}$

$2\pi +$

Çöz

10 | 2

7

$\Rightarrow \alpha = \frac{7\pi}{5}$

adyan cinsinden verilen negati
itif yönlü gibi düşünülerek esas

▼ unurken, verilen açı po-
2π den çıkarılır.

f yönlü açıların esas ölçüsü bul
ölçü bulunur. Bulunan değer,

NEK 6

$= \alpha + 2k\pi \Rightarrow \alpha = ?$

B) $\frac{3\pi}{2}$

ÖRN

$\frac{-13\pi}{2}$

A) $\frac{\pi}{2}$ E) $\frac{3\pi}{4}$ C) π D) $\frac{5\pi}{2}$

ÜM

$\frac{4}{3}$

$\Rightarrow 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

ÇÖZ Yanıt B

13 | 12

1

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▼ To find the reference angle of $\frac{a\cdot\pi}{b}$ where a, b ∈ Z^+, a is divided by 2b. Remainder write to numerator, denominator is b. If remainder of a to 2b is k, the reference angle of $\frac{a\pi}{b}$ is $\frac{k\pi}{b}$.

EXAMPLE 5

$\frac{17\pi}{5} = \alpha + 2k\pi \Rightarrow \alpha = ?$

A) $\frac{2\pi}{5}$ B) $\frac{4\pi}{5}$ C) π D) $\frac{6\pi}{5}$ E) $\frac{7\pi}{5}$

SOLUTION

The answer is E

Solution 1

$$\frac{17\pi}{5} = \alpha + 2k\pi$$

$$\frac{10\pi + 7\pi}{5} = \alpha + 2k\pi$$

$$2\pi + \frac{7\pi}{5} = \alpha + 2k\pi$$

$$\alpha = \frac{7\pi}{5}$$

Solution 2

$$\begin{array}{r|l} 17 & 10 \\ -\,10 & 2 \\ \hline 7 & \end{array}$$

$$\Rightarrow \alpha = \frac{7\pi}{5}$$

▼ To find the reference angle that in terms of radian and negative directional angle, we think as positive directional angle of the given angle. The finding value is subtracted by 2π.

EXAMPLE 6

$\frac{-13\pi}{2} = \alpha + 2k\pi \Rightarrow \alpha = ?$

A) $\frac{\pi}{2}$ B) $\frac{3\pi}{2}$ C) π D) $\frac{5\pi}{2}$ E) $\frac{3\pi}{4}$

SOLUTION

The answer is B

$$\begin{array}{r|l} 13 & 4 \\ -\,12 & 3 \\ \hline 1 & \end{array} \quad \Rightarrow 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

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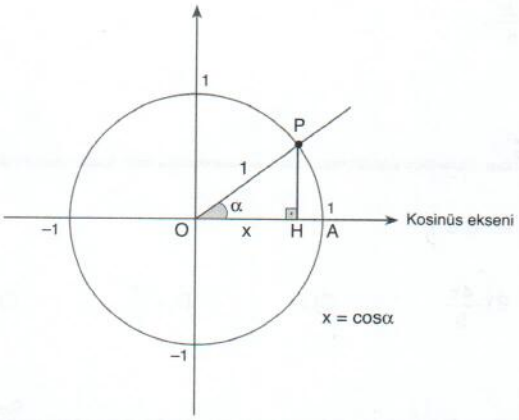
TRİGONOMETRİK FONKSİYONLAR

Kosinüs Fonksiyonu



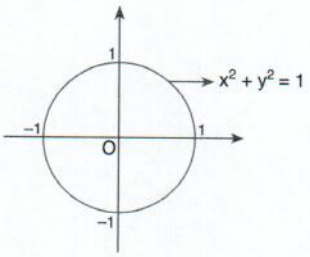
Bir α açısına birim çember üzerinde karşılık gelen P noktasının apsisine *α açısının kosinüsü* denir.

$\cos(\widehat{AOP}) = |OH|$

$\cos: R \to [-1, 1]$

NOT

Analitik düzlemde merkezi (0,0) ve yarıçapı 1 birim olan çembere, birim (trigonometrik) çember denir.



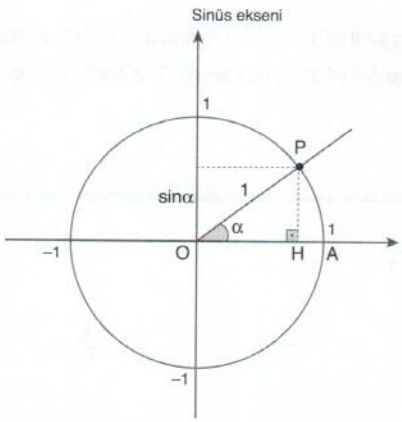
NOT

$-1 \le \cos\alpha \le 1$

Sinüs Fonksiyonu

NOT

$-1 \le \sin\alpha \le 1$



Bir α açısına birim çember üzerinde karşılık gelen P noktasının ordinatına *α açısının sinüsü* denir.

$\sin(\widehat{AOP}) = |PH|$

$\sin: R \to [-1, 1]$

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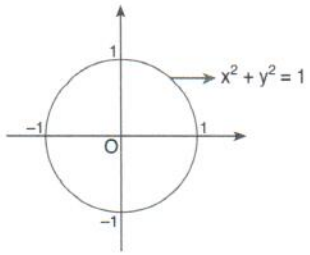
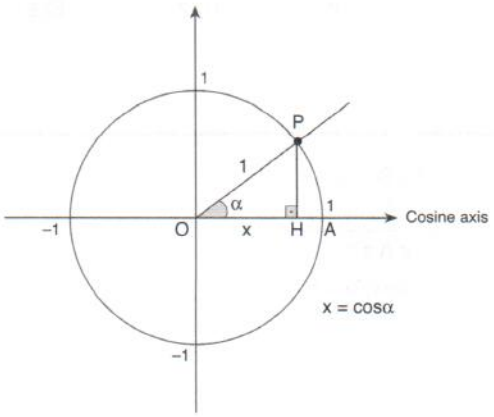
NOTE

The circle is unit (trigonometric) circle that with center (0,0) and radius 1 unit in analytic plane.



TRIGONOMETRIC FUNCTIONS

Cosine Function



NOTE

$-1 \le \cos\alpha \le 1$

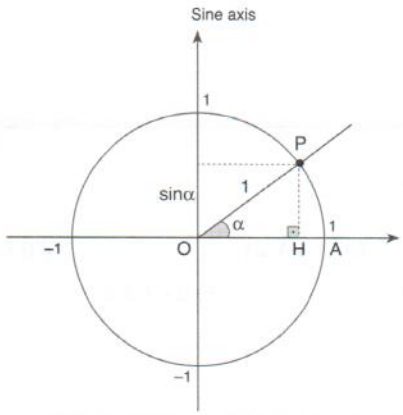
Abscissa of point P is called *cosine of angle* α that on the unit circle of the angle α.

$$\cos(\widehat{AOP}) = |OH|$$
$$\cos: R \to [-1, 1]$$

Sine Function

NOTE

$-1 \le \sin\alpha \le 1$



The ordinate of point P is called *sine of angle* α that on the unit circle of the angle α.

$$\sin(\widehat{AOP}) = |PH|$$
$$\sin: R \to [-1, 1]$$

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ÖRNEK 7

$A = 2\cos x + 3\sin y \Rightarrow \max(A) = ?$

A) -5 B) -1 C) 0 D) 2 E) 5

ÇÖZÜM

Yanıt **E**

$-1 \le \cos x \le 1 \Rightarrow -2 \le 2\cos x \le 2$

$-1 \le \sin y \le 1 \Rightarrow + \quad -3 \le 3\sin y \le 3$

$-5 \le A \le 5$

$\Rightarrow \max(A) = 5$

ÖRNEK 8

$A = \frac{1+\sin x}{3} \Rightarrow ? \le A \le ?$

A) $0 \le A \le 1$ B) $-1 \le A \le 1$ C) $0 \le A \le \frac{2}{3}$

D) $1 \le A \le 2$ E) $2 \le A \le 3$

ÇÖZÜM

Yanıt **C**

$-1 \le \sin x \le 1$

$0 \le 1 + \sin x \le 2$

$\Rightarrow 0 \le A \le \frac{2}{3}$

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EXAMPLE 7

$A = 2\cos x + 3\sin y \Rightarrow$ **max(A) = ?**

A) -5 B) -1 C) 0 D) 2 E) 5

SOLUTION

The answer is **E**

$$-1 \le \cos x \le 1 \Rightarrow \quad -2 \le 2\cos x \le 2$$
$$-1 \le \sin y \le 1 \Rightarrow + \quad -3 \le 3\sin y \le 3$$
$$-5 \le A \le 5$$
$$\Rightarrow \max(A) = 5$$

EXAMPLE 8

$A = \frac{1+\sin x}{3} \Rightarrow ? \le A \le ?$

A) $0 \le A \le 1$ B) $-1 \le A \le 1$ C) $0 \le A \le \frac{2}{3}$

D) $1 \le A \le 2$ E) $2 \le A \le 3$

SOLUTION

The answer is **C**

$-1 \le \sin x \le 1$

$0 \le 1 + \sin x \le 2$

$\Rightarrow 0 \le A \le \frac{2}{3}$

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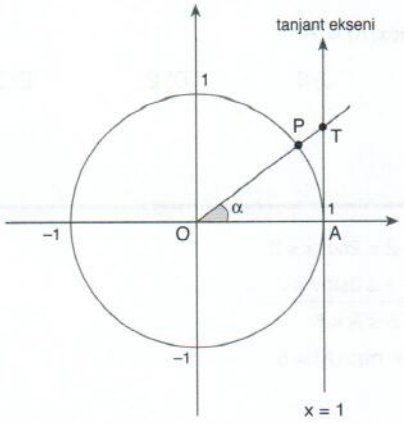
Tanjant Fonksiyonu

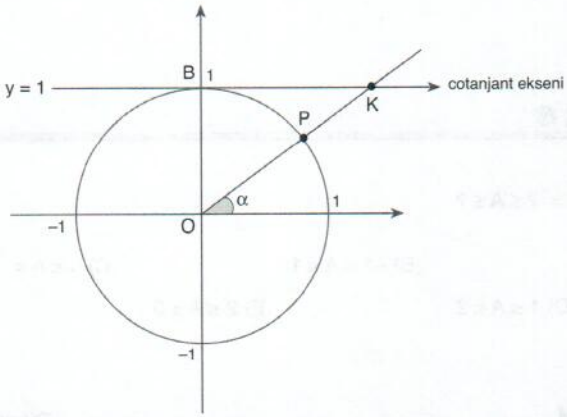


Bir α açısına birim çember üzerinde karşılık gelen P noktasından geçen [OT ışınının x = 1 doğrusunu kestiği T noktasının ordinatına *α açısının tanjantı* denir.

$$\tan(\widehat{AOP}) = |TA|$$

$$\tan : R - \left\{\frac{\pi}{2} + 2k\pi , \frac{3\pi}{2} + 2k\pi\right\} \to R$$

Kotanjant Fonksiyonu



Bir α açısına birim çember üzerinde karşılık gelen P noktasından geçen [OK ışınının y = 1 doğrusunu kestiği K noktasının apsisine *α açısının cotanjantı* denir.

$$\cot(\widehat{AOP}) = |BK|$$

cotanjant: $R - \{\pi + 2k\pi , 2\pi + + 2k\pi\} \to R$

NOT

Fonksiyon	Tanım kümesi	Görüntü kümesi
sin	R	[–1, 1]
cos	R	[–1, 1]
tan	$R - \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$	R
cot	$R - \{\pi, 2\pi\}$	R

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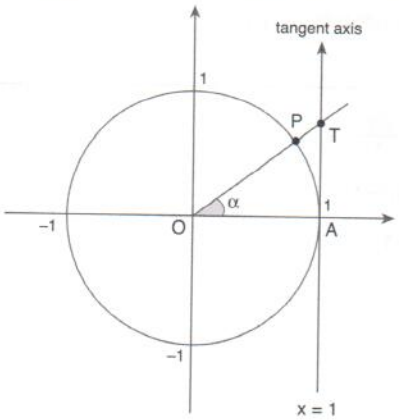
Tangent Function



Ordinate of point T is *tangent of the angle* α that cut by tangent axis of ray [OT the passes through P.

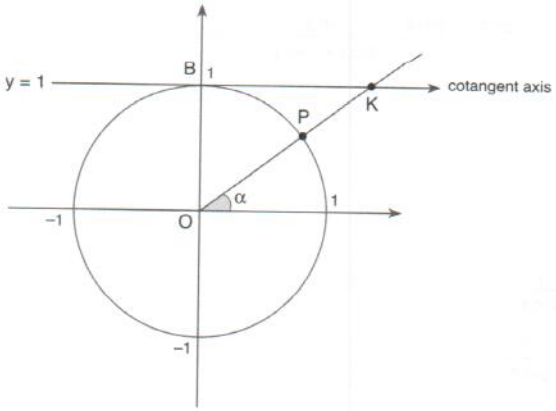
$$\tan(\widehat{AOP}) = |TA|$$

$$\tan : R - \left\{\frac{\pi}{2} + 2k\pi , \frac{3\pi}{2} + 2k\pi\right\} \to R$$

NOTE

Function	Domain	Range
sin	R	[–1, 1]
cos	R	[–1, 1]
tan	$R - \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$	R
cot	$R - \{\pi, 2\pi\}$	R

Cotangent Function



Abscissa of point K is *cotangent of the angle* α that cut by tangent axis of ray [OK the passes throung P.

$$\cot(\widehat{AOP}) = |BK|$$

cotanjant: $R - \{\pi + 2k\pi , 2\pi + + 2k\pi\} \to R$

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TRİGONOMETRİK ÖZDEŞLİKLER

i. $\sin^2x + \cos^2x = 1$ [Pisagor özdeşliği]

Etkinlik

✓ $\sin^2\frac{3\pi}{2} + \cos^2\frac{3\pi}{2} = 1$

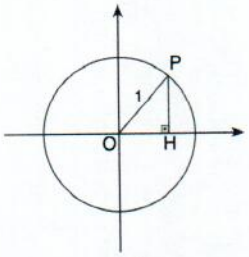
✓ $\sin^2 18° + \cos^2 18° = 1$

✓ $\sin^2 a + \cos^2 a = 1$

ii. $\tan x = \frac{\sin x}{\cos x}$

iii. $\cot x = \frac{\cos x}{\sin x}$

KANIT



Δ $\perp$ OHP:

$|OH|^2 + |PH|^2 = 1^2$ [Pisagor]

$\cos^2x + \sin^2x = 1$

ÖRNEK 9

$\frac{1+\tan x}{1+\cot x} : \frac{1}{\cos x} = ?$

A) 1 B) cosx C) sinx D) 1 + sinx E) 1 – sinx

ÇÖZÜM

Yanıt C

$$\frac{1+\tan x}{1+\cot x} : \frac{1}{\cos x} = \frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}} \cdot \cos x$$

$$= \frac{\cos x + \sin x}{\cos x} \cdot \frac{\sin x}{\sin x + \cos x} \cdot \cos x$$

$$= \sin x$$

iv. $\sec x = \frac{1}{\cos x}$

v. $\operatorname{cosec} x = \frac{1}{\sin x}$

vi. $\tan x \cdot \cot x = 1$

Etkinlik

✓ $\tan 28° \cdot \cot 28° = 1$

✓ $\cot 13° \cdot \tan 13° = 1$

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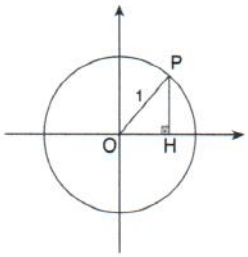
PROOF



Δ
$\perp$ OHP:
$|OH|^2 + |PH|^2 = 1^2$ [Pythagorean]
$\cos^2 x + \sin^2 x = 1$

TRIGONOMETRIC IDENTITIES

i. $\sin^2 x + \cos^2 x = 1$ [Pythagorean Identity]

Activity

✓ $\sin^2 \frac{3\pi}{2} + \cos^2 \frac{3\pi}{2} = 1$

✓ $\sin^2 18° + \cos^2 18° = 1$

✓ $\sin^2 a + \cos^2 a = 1$

ii. $\tan x = \frac{\sin x}{\cos x}$

iii. $\cot x = \frac{\cos x}{\sin x}$

EXAMPLE 9

$\frac{1+\tan x}{1+\cot x} : \frac{1}{\cos x} = ?$

A) 1 B) cosx C) sinx
D) 1 + sinx E) 1 – sinx

SOLUTION

The answer is C

$$\frac{1+\tan x}{1+\cot x} : \frac{1}{\cos x} = \frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}} \cdot \cos x$$

$$= \frac{\cos x + \sin x}{\cos x} \cdot \frac{\sin x}{\sin x + \cos x} \cdot \cos x$$

$$= \sin x$$

iv. $\sec x = \frac{1}{\cos x}$

v. $\operatorname{cosec} x = \frac{1}{\sin x}$

vi. $\tan x \cdot \cot x = 1$

Activity

✓ $\tan 28° \cdot \cot 28° = 1$

✓ $\cot 13° \cdot \tan 13° = 1$

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vii. $x+y=\frac{\pi}{2} \Rightarrow \sin x=\cos y \ , \ \tan x=\cot y$

Etkinlik

- ✓ $\sin 30° = \cos 60°$
- ✓ $\tan 13° = \cot 77°$
- ✓ $\sin 62° = \cos 28°$

viii. $0<x<y<\frac{\pi}{2} \Rightarrow \sin x<\sin y \ , \ \cos x>\cos y$

ÖRNEK 10

$2\sin 42° - 3\cos 48° + \frac{1}{\text{cosec}\, 42°} + 15 = ?$

A) cosx + 15 B) 15 C) sin42° + 15 D) 14 E) 13

ÇÖZÜM

Yanıt **B**

$2\sin 42° - 3\cos 48° + \frac{1}{\text{cosec}\, 42°} + 15 = 2\cdot\sin 42° - 3\sin 42° + \sin 42° + 15 = 15$

ÖRNEK 11

$\frac{\sin x + 2\cos x}{3\sin x - 4\cos x} = 2 \Rightarrow \tan x = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) $\frac{5}{3}$ E) 2

ÇÖZÜM

Yanıt **E**

$\frac{\sin x + 2\cos x}{3\sin x - 4\cos x} = 2$

$\sin x + 2\cos x = 6\sin x - 8\cos x$

$10\cos x = 5\sin x$

$\frac{\sin x}{\cos x} = \frac{10}{5} \Rightarrow \tan x = 2$

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vii. $x+y=\frac{\pi}{2} \Rightarrow \sin x=\cos y \ , \ \tan x=\cot y$

Activity

- ✓ $\sin 30° = \cos 60°$
- ✓ $\tan 13° = \cot 77°$
- ✓ $\sin 62° = \cos 28°$

viii. $0<x<y<\frac{\pi}{2} \Rightarrow \sin x<\sin y \ , \ \cos x>\cos y$

EXAMPLE 10

$$2\sin 42° - 3\cos 48° + \frac{1}{\text{cosec } 42°} + 15 = ?$$

A) cosx + 15 B) 15 C) sin42° + 15 D) 14 E) 13

SOLUTION

The answer is **B**

$$2\sin 42° - 3\cos 48° + \frac{1}{\text{cosec } 42°} + 15 = 2\cdot\sin 42° - 3\sin 42° + \sin 42° + 15 = 15$$

EXAMPLE 11

$$\frac{\sin x + 2\cos x}{3\sin x - 4\cos x} = 2 \Rightarrow \textbf{tanx = ?}$$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) $\frac{5}{3}$ E) 2

SOLUTION

The answer is **E**

$$\frac{\sin x + 2\cos x}{3\sin x - 4\cos x} = 2$$

$$\sin x + 2\cos x = 6\sin x - 8\cos x$$

$$10\cos x = 5\sin x$$

$$\frac{\sin x}{\cos x} = \frac{10}{5} \Rightarrow \tan x = 2$$

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ÖRNEK 12

$\tan x + \cot x = 3 \Rightarrow \tan^2 x + \cot^2 x = ?$

A) 7 B) 8 C) 9 D) 10 E) 11

ÇÖZÜM

Yanıt **A**

$(\tan x + \cot x)^2 = 3^2$ [Her iki tarafın karesini al.]

$\tan^2 x + 2\tan x\cot x + \cot^2 x = 9$ [Tam kare ifadeler]

$\tan^2 x + 2 + \cot^2 x = 9$ [Trigonometrik özdeşlik]

$\tan^2 x + \cot^2 x = 7$ [Sonuç]

Düşünün

$A = \cos x + \dfrac{\cos^2 a + \sin^2 a - 1}{\sin 15° + \cos 125°} + \sin y$

$\Rightarrow \max(A) = ?$

ÖRNEK 13

$\sin x + \cos x = \dfrac{1}{2} \Rightarrow \sin x \cdot \cos x = ?$

A) $\dfrac{-1}{8}$ B) $\dfrac{-1}{4}$ C) $\dfrac{-3}{8}$ D) $-\dfrac{1}{2}$ E) $-\dfrac{5}{8}$

ÇÖZÜM

Yanıt **C**

$\sin x + \cos x = \dfrac{1}{2}$ [Verilen eşitlik]

$(\sin x + \cos x)^2 = \left(\dfrac{1}{2}\right)^2$ [Her iki tarafın karesine al.]

$\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x = \dfrac{1}{4}$ [Tam kare]

$1 + 2\sin x \cdot \cos x = \dfrac{1}{4}$ [Trigonometrik özdeşlik]

$2\sin x \cdot \cos x = -\dfrac{3}{4}$ [Her iki taraftan 1 çıkar.]

$\sin x \cdot \cos x = -\dfrac{3}{8}$ [Sonuç]

Yanıt : 2

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Think about it

$A = \cos x + \frac{\cos^2 a + \sin^2 a - 1}{\sin 15° + \cos 125°} + \sin y$

$\Rightarrow$ max(A) = ?

EXAMPLE 12

$\tan x + \cot x = 3 \Rightarrow$ **$\tan^2 x + \cot^2 x = ?$**

A) 7 B) 8 C) 9 D) 10 E) 11

SOLUTION

The answer is **A**

$(\tan x + \cot x)^2 = 3^2$ [Squre the both sides]

$\tan^2 x + 2\tan x \cot x + \cot^2 x = 9$ [Perfect square expressions]

$\tan^2 x + 2 + \cot^2 x = 9$ [Trigonometric identity]

$\tan^2 x + \cot^2 x = 7$ [Conclusion]

EXAMPLE 13

$\sin x + \cos x = \frac{1}{2} \Rightarrow$ **$\sin x \cdot \cos x = ?$**

A) $\frac{-1}{8}$ B) $\frac{-1}{4}$ C) $\frac{-3}{8}$ D) $-\frac{1}{2}$ E) $-\frac{5}{8}$

SOLUTION

The answer is **C**

$\sin x + \cos x = \frac{1}{2}$ [Given equality]

$(\sin x + \cos x)^2 = \left(\frac{1}{2}\right)^2$ [Square both sides.]

$\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x = \frac{1}{4}$ [Perfect square]

$1 + 2\sin x \cdot \cos x = \frac{1}{4}$ [Trigonometric identity]

$2\sin x \cdot \cos x = -\frac{3}{4}$ [Subtract 1 from both sides.]

$\sin x \cdot \cos x = -\frac{3}{8}$ [Conclusion]

Answer : 2

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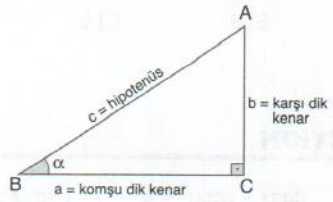
DİK ÜÇGENDE TRİGONOMETRİK ORANLAR

$$\sin\alpha = \frac{\text{karşı dik kenar}}{\text{hipotenüs}} = \frac{b}{c}$$

$$\cos\alpha = \frac{\text{komşu dik kenar}}{\text{hipotenüs}} = \frac{a}{c}$$

$$\tan\alpha = \frac{\text{karşı dik kenar}}{\text{komşu dik kenar}} = \frac{b}{a}$$

$$\cot\alpha = \frac{\text{komşu dik kenar}}{\text{karşı dik kenar}} = \frac{a}{b}$$



NOT

Hipotenüs, en uzun kenardır.

AÇIKLAMA

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{\frac{a}{c}} = \frac{c}{a}$$

$$\text{cosec}\alpha = \frac{1}{\sin\alpha} = \frac{1}{\frac{b}{c}} = \frac{c}{b}$$

▼ **Trigonometrik Oranlardan Biri Verildiğinde Diğerini Bulma:** Verilen trigonometrik oran için dik üçgen çizilir. Bu üçgen yardımıyla istenen oranın değeri mutlak değerce bulunur. İstenen açının bulunduğu bölgeye göre işaret belirlenir.

ÖRNEK 14

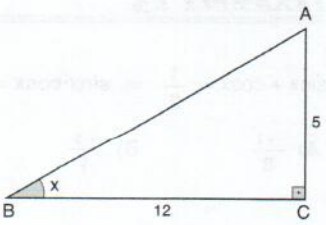
$[AC] \perp [BC]$,

$|AC| = 5$ cm,

$|BC| = 12$ cm

$m(\widehat{ABC}) = x$.

$\text{cosec}\,x \cdot \cos x \cdot \cot x = ?$



A) $\frac{25}{169}$ B) $\frac{169}{25}$ C) $\frac{25}{144}$ D) $\frac{144}{25}$ E) $\frac{144}{169}$

ÇÖZÜM

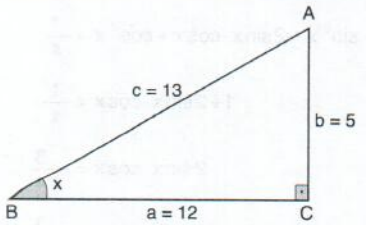
Yanıt **D**

$c^2 = 12^2 + 5^2 = 169$

$c = 13$

$$\text{cosec}\,x \cdot \cos x \cdot \cot x = \frac{13}{5} \cdot \frac{12}{13} \cdot \frac{12}{5}$$

$$= \frac{144}{25}$$



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NOTE

Hypotenuse is the longest side.

EXPLANATION

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{\frac{a}{c}} = \frac{c}{a}$$

$$\text{cosec}\alpha = \frac{1}{\sin\alpha} = \frac{1}{\frac{b}{c}} = \frac{c}{b}$$

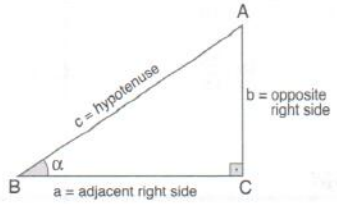
TRIGONOMETRIC RATIOS in a RIGHT TRIANGLE

$$\sin\alpha = \frac{\text{opposite right side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos\alpha = \frac{\text{adjacent right side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan\alpha = \frac{\text{opposite right side}}{\text{adjacent right side}} = \frac{b}{a}$$

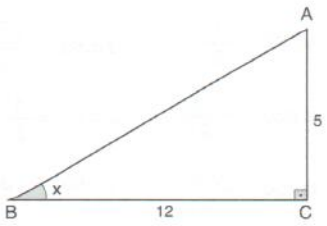
$$\cot\alpha = \frac{\text{adjacent right side}}{\text{opposite right side}} = \frac{a}{b}$$



▼ **Finding the Other Ratio That Given Ones the Trigonometric Ratios:** We draw the right triangle to the given trigonometric ratio. Value of finding ratio is found from this triangle with absolute value. According to quadrant is written sign.

EXAMPLE 14

[AC] ⊥ [BC],
|AC| = 5 cm,
|BC| = 12 cm
$m(\widehat{ABC}) = x$.
$\text{cosec}\,x \cdot \cos x \cdot \cot x$ = ?



A) $\frac{25}{169}$ B) $\frac{169}{25}$ C) $\frac{25}{144}$ D) $\frac{144}{25}$ E) $\frac{144}{169}$

SOLUTION

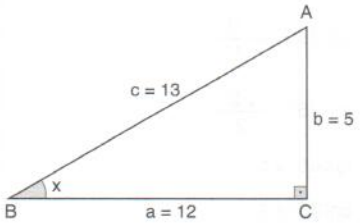
The answer is **D**

$c^2 = 12^2 + 5^2 = 169$

$c = 13$

$$\text{cosec}\,x \cdot \cos x \cdot \cot x = \frac{13}{5} \cdot \frac{12}{13} \cdot \frac{12}{5}$$

$$= \frac{144}{25}$$



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ÖRNEK 15

$0 < x < \frac{\pi}{2}$, $\tan x = \frac{5}{12} \Rightarrow \sin x + \cos x + \cot x = ?$

A) 3 B) $\frac{38}{13}$ C) 4 D) $\frac{241}{65}$ E) $\frac{72}{13}$

ÇÖZÜM

Yanıt **D**

$\tan x = \frac{b}{a} = \frac{5}{12}$

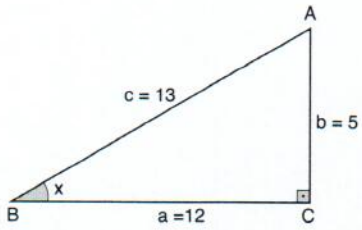
$c^2 = 25 + 144 = 169 \Rightarrow c = 13$

$\sin x = \frac{5}{13}$, $\cos x = \frac{12}{13}$, $\cot x = \frac{12}{5}$

$\sin x + \cos x + \cot x = \frac{5}{13} + \frac{12}{13} + \frac{12}{5}$

$= \frac{17}{13} + \frac{12}{5}$

$= \frac{85}{65} + \frac{156}{65} = \frac{241}{65}$



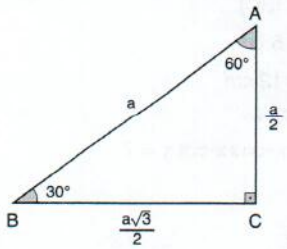
30° ve 60° nin Trigonometrik Değerleri

$\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{\sqrt{3}}{3}$ $\tan 60° = \sqrt{3}$

$\cot 30° = \sqrt{3}$ $\cot 60° = \frac{\sqrt{3}}{3}$



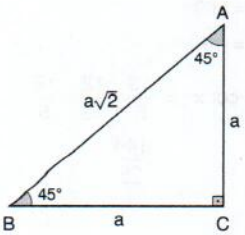
45° nin Trigonometrik Değerleri

$\sin 45° = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

$\cot 45° = 1$



KANIT

$|AB|^2 = |AC|^2 + |BC|^2$ [Pisagor]

$= a^2 + a^2$

$= 2a^2$

$\Rightarrow |AB| = \sqrt{2}a$

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EXAMPLE 15

$0 < x < \frac{\pi}{2}$, $\tan x = \frac{5}{12} \Rightarrow \sin x + \cos x + \cot x = ?$

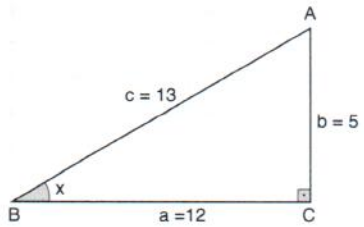
A) 3 B) $\frac{38}{13}$ C) 4 D) $\frac{241}{65}$ E) $\frac{72}{13}$

SOLUTION

The answer is D

$\tan x = \frac{b}{a} = \frac{5}{12}$

$c^2 = 25 + 144 = 169 \Rightarrow c = 13$



$\sin x = \frac{5}{13}$, $\cos x = \frac{12}{13}$, $\cot x = \frac{12}{5}$

$\sin x + \cos x + \cot x = \frac{5}{13} + \frac{12}{13} + \frac{12}{5}$

$= \frac{17}{13} + \frac{12}{5}$

$= \frac{85}{65} + \frac{156}{65} = \frac{241}{65}$

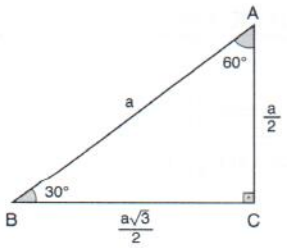
Trigonometric Values of 30° and 60°

$\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{\sqrt{3}}{3}$ $\tan 60° = \sqrt{3}$

$\cot 30° = \sqrt{3}$ $\cot 60° = \frac{\sqrt{3}}{3}$



PROOF

$|AB|^2 = |AC|^2 + |BC|^2$ [Pythagorean]

$= a^2 + a^2$

$= 2a^2$

$\Rightarrow |AB| = \sqrt{2}a$

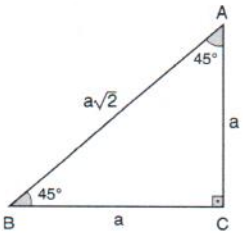
Trigonometric Values of 45°

$\sin 45° = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

$\cot 45° = 1$



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TEMEL AÇILARIN TRİGONOMETRİK DEĞERLERİ

x (radyan)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x (derece)	0°	30°	45°	60°	90°	180°	270°	360°
sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cosx	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
tanx	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Belirsiz	0	Belirsiz	0
cotx	Belirsiz	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Belirsiz	0	Belirsiz

Düşünün

$x = \cos 60°$,
$y = \sin 60°$,
$z = \tan 60°$
$\Rightarrow ? < ? < ?$

ÖRNEK 16

$$\left(\frac{\sin\pi + \cos\frac{\pi}{4}}{\cos\pi + \sin\frac{\pi}{4}}\right)^{-1} = ?$$

A) $\sqrt{2}$ B) $1 + \sqrt{2}$ C) $1 - \sqrt{2}$
D) $-\sqrt{2}$ E) $2\sqrt{2}$

ÇÖZÜM

Yanıt C

$$\left(\frac{\sin\pi + \cos\frac{\pi}{4}}{\cos\pi + \sin\frac{\pi}{4}}\right)^{-1} = \left(\frac{0 + \frac{\sqrt{2}}{2}}{-1 + \frac{\sqrt{2}}{2}}\right)^{-1}$$

$$= \left(\frac{\sqrt{2}}{\sqrt{2} - 2}\right)^{-1}$$

$$= \frac{\sqrt{2} - 2}{\sqrt{2}}$$

$$= 1 - \sqrt{2}$$

Yanıt : z > y > x

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Think about it

x = cos60°,
y = sin60°,
z = tan60°
⇒ ? < ? < ?

TRIGONOMETRIC VALUES of the BASIC ANGLES

x (radian)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x (degree)	0°	30°	45°	60°	90°	180°	270°	360°
sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cosx	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
tanx	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Indefinite	0	Indefinite	0
cotx	Belirsiz	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Indefinite	0	Indefinite

EXAMPLE 16

$$\left(\frac{\sin\pi+\cos\frac{\pi}{4}}{\cos\pi+\sin\frac{\pi}{4}}\right)^{-1} = ?$$

A) $\sqrt{2}$ B) $1+\sqrt{2}$ C) $1-\sqrt{2}$
D) $-\sqrt{2}$ E) $2\sqrt{2}$

SOLUTION

The answer is **C**

$$\left(\frac{\sin\pi+\cos\frac{\pi}{4}}{\cos\pi+\sin\frac{\pi}{4}}\right)^{-1} = \left(\frac{0+\frac{\sqrt{2}}{2}}{-1+\frac{\sqrt{2}}{2}}\right)^{-1}$$

$$= \left(\frac{\sqrt{2}}{\sqrt{2}-2}\right)^{-1}$$

$$= \frac{\sqrt{2}-2}{\sqrt{2}}$$

$$= 1-\sqrt{2}$$

Answer : z > y > x

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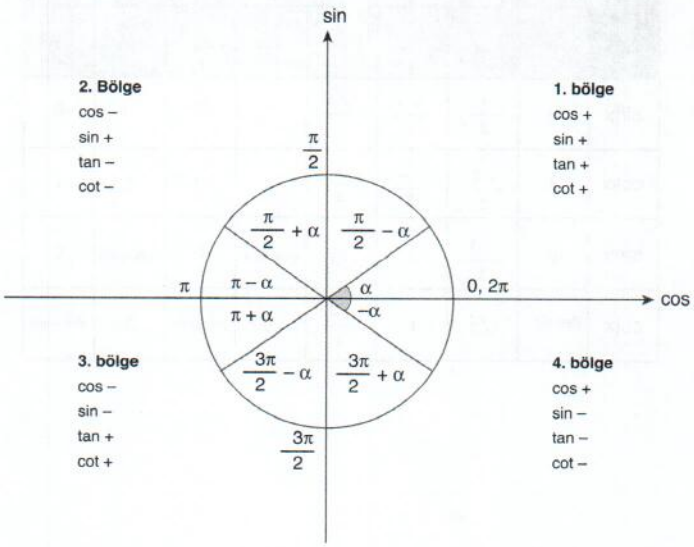
ÖLÇÜSÜ 90° den BÜYÜK OLAN AÇILARIN TRİGONOMETRİK DEĞERLERİ (İndirgeme Formülleri)

$0 < \alpha < \frac{\pi}{2}$



1. Bölge

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

i. $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

ii. $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

iii. $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$

iv. $\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$

2. Bölge

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$

$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$

$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$

$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\cos(\pi - \alpha) = -\cos\alpha$

$\tan(\pi - \alpha) = -\tan\alpha$

$\cot(\pi - \alpha) = -\cot\alpha$

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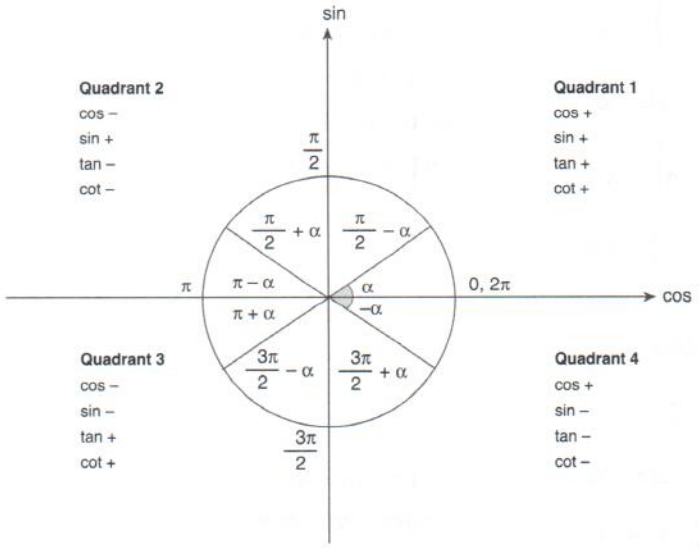
TRIGONOMETRIC VALUES of ANGLES with GREATER THAN 90° (Reduce Formulas)

$0 < \alpha < \frac{\pi}{2}$



Quadrant 1

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

i. $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

ii. $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

iii. $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$

iv. $\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$

Quadrant 2

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$

$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$

$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$

$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\cos(\pi - \alpha) = -\cos\alpha$

$\tan(\pi - \alpha) = -\tan\alpha$

$\cot(\pi - \alpha) = -\cot\alpha$

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3. Bölge

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{3\pi}{2}-\alpha\right) = -\cos\alpha$

$\cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha$

$\tan\left(\frac{3\pi}{2}-\alpha\right) = \cot\alpha$

$\cot\left(\frac{3\pi}{2}-\alpha\right) = \tan\alpha$

$\sin(\pi+\alpha) = -\sin\alpha$

$\cos(\pi+\alpha) = -\cos\alpha$

$\tan(\pi+\alpha) = \tan\alpha$

$\cot(\pi+\alpha) = \cot\alpha$

4. Bölge

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{3\pi}{2}+\alpha\right) = -\cos\alpha$

$\cos\left(\frac{3\pi}{2}+\alpha\right) = \sin\alpha$

$\tan\left(\frac{3\pi}{2}+\alpha\right) = -\cot\alpha$

$\cot\left(\frac{3\pi}{2}+\alpha\right) = -\tan\alpha$

$\sin(2\pi-\alpha) = -\sin\alpha$

$\cos(2\pi-\alpha) = \cos\alpha$

$\tan(2\pi-\alpha) = -\tan\alpha$

$\cot(2\pi-\alpha) = -\cot\alpha$

Etkinlikler

225° nin trigonometrik oranları nedir?

✓ $\sin 225° = \sin\left(\pi+\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\sin 45° = -\frac{\sqrt{2}}{2}$

✓ $\cos 225° = \cos\left(\pi+\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\cos 45° = -\frac{\sqrt{2}}{2}$

✓ $\tan 225° = \tan\left(\pi+\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = \tan 45° = 1$

✓ $\cot 225° = \cot\left(\pi+\frac{\pi}{4}\right) = \cot\frac{\pi}{4} = \cot 45° = 1$

Etkinlikler

120° nin trigonometrik oranları nedir?

✓ $\sin 120° = \sin\left(\pi-\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$

✓ $\cos 120° = \cos\left(\pi-\frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\cos 60° = -\frac{1}{2}$

✓ $\tan 120° = \tan\left(\pi-\frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\tan 60° = -\sqrt{3}$

✓ $\cot 120° = \cot\left(\pi-\frac{\pi}{3}\right) = -\cot\frac{\pi}{3} = -\cot 60° = -\frac{\sqrt{3}}{3}$

Düşünün

$x = \cos 150°$,
$y = \sin 150°$,
$z = \tan 150°$
$\Rightarrow ? < ? < ?$

Yanıt : x < z < y

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Think about it

x = cos150°,
y = sin150°,
z = tan150°
⇒ ? < ? < ?

Quadrant 3

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos\alpha$

$\cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin\alpha$

$\tan\left(\frac{3\pi}{2} - \alpha\right) = \cot\alpha$

$\cot\left(\frac{3\pi}{2} - \alpha\right) = \tan\alpha$

$\sin(\pi + \alpha) = -\sin\alpha$

$\cos(\pi + \alpha) = -\cos\alpha$

$\tan(\pi + \alpha) = \tan\alpha$

$\cot(\pi + \alpha) = \cot\alpha$

Quadrant 4

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos\alpha$

$\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin\alpha$

$\tan\left(\frac{3\pi}{2} + \alpha\right) = -\cot\alpha$

$\cot\left(\frac{3\pi}{2} + \alpha\right) = -\tan\alpha$

$\sin(2\pi - \alpha) = -\sin\alpha$

$\cos(2\pi - \alpha) = \cos\alpha$

$\tan(2\pi - \alpha) = -\tan\alpha$

$\cot(2\pi - \alpha) = -\cot\alpha$

Activities

What are the trigonometric values of 225°?

✓ $\sin 225° = \sin\left(\pi + \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\sin 45° = -\frac{\sqrt{2}}{2}$

✓ $\cos 225° = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\cos 45° = -\frac{\sqrt{2}}{2}$

✓ $\tan 225° = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = \tan 45° = 1$

✓ $\cot 225° = \cot\left(\pi + \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = \cot 45° = 1$

Activities

What are the trigonometric values of 120°?

✓ $\sin 120° = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$

✓ $\cos 120° = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\cos 60° = -\frac{1}{2}$

✓ $\tan 120° = \tan\left(\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\tan 60° = -\sqrt{3}$

✓ $\cot 120° = \cot\left(\pi - \frac{\pi}{3}\right) = -\cot\frac{\pi}{3} = -\cot 60° = -\frac{\sqrt{3}}{3}$

Answer : x < z < y

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ÖRNEK 17

a = sin 108°,
b = cos 316°,
c = tan 286°,
d = cot 183° ⇒ ?, ?, ?, ?

A) –, –, +, – B) –, +, –, + C) +, +, –, +
D) +, –, +, – E) +, +, –, –

ÇÖZÜM

Yanıt C

Açı	Bölge	İşaret
108°	II	+
316°	IV	+
286°	IV	–
183°	III	+

Düşünün

a = cos143°,
b = sin174°,
c = tan58°
d = cot42°
⇒ ? < ? < ? < ?

NEGATİF AÇILARIN TRİGONOMETRİK DEĞERLERİ

- ✓ sin (– α) = – sin α
- ✓ cos (– α) = cos α
- ✓ tan (– α) = – tan α
- ✓ cot (– α) = – cot α

ÖRNEK 18

sin(– 300°) = ?

A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

ÇÖZÜM

Yanıt E

$\sin(-300°) = -\sin 300°$

$= -\sin\left(2\pi - \frac{\pi}{3}\right) = -\left(-\sin\frac{\pi}{3}\right) = \sin 60 = \frac{\sqrt{3}}{2}$

Yanıt: c > d > b > a

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Think about it

a = cos143°,
b = sin174°,
c = tan58°
d = cot42°
⇒ ? < ? < ? < ?

EXAMPLE 17

a = sin 108°,
b = cos 316°,
c = tan 286°,
d = cot 183° ⇒ ? , ? , ?, ?

A) –, –, +, – B) –, +, –, + C) +, +, –, +
D) +, –, +, – E) +, +, –, –

SOLUTION

The answer is C

Angle	Quadrant	Sign
108°	II	+
316°	IV	+
286°	IV	–
183°	III	+

TRIGONOMETRIC VALUES of NEGATIVE ANGLES

✓ $\sin(-\alpha) = -\sin\alpha$
✓ $\cos(-\alpha) = \cos\alpha$
✓ $\tan(-\alpha) = -\tan\alpha$
✓ $\cot(-\alpha) = -\cot\alpha$

EXAMPLE 18

sin(– 300°) = ?

A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

SOLUTION

The answer is E

$\sin(-300°) = -\sin 300°$

$= -\sin\left(2\pi - \frac{\pi}{3}\right) = -\left(-\sin\frac{\pi}{3}\right) = \sin 60 = \frac{\sqrt{3}}{2}$

Answer: c > d > b > a

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ÖRNEK 19

$\tan(-870°) = ?$

A) $-\frac{1}{2}$ B) $\frac{\sqrt{3}}{3}$ C) $-\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

ÇÖZÜM

Yanıt **B**

$\begin{array}{r|l} 870° & 360° \\ 720° & 2 \\ \hline 150° & \end{array}$

$\tan(-870°) = -\tan 870° = -\tan 150°$

$= -\tan\left(\pi - \frac{\pi}{6}\right) = -\left(-\tan\frac{\pi}{6}\right) = \tan 30° = \frac{\sqrt{3}}{3}$

ÖRNEK 20

$x \in \left(\frac{3\pi}{2}, 2\pi\right)$, $\tan x = -2 \Rightarrow \cos x \cdot \sin^2 x - \sin x \cdot \cos^2 x = ?$

A) $-\frac{2\sqrt{5}}{5}$ B) $-\frac{6\sqrt{5}}{25}$ C) $\frac{2\sqrt{5}}{25}$ D) $\frac{6\sqrt{5}}{25}$ E) $\frac{\sqrt{5}}{5}$

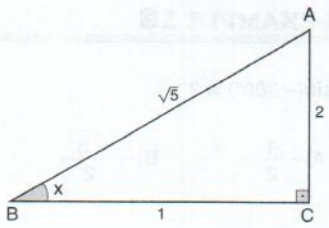
ÇÖZÜM

Yanıt **D**

$x \in \left(\frac{3\pi}{2}, 2\pi\right) \Rightarrow$ IV. Bölge

$\cos x > 0, \sin x < 0$

$\sin x = -\frac{2}{\sqrt{5}}, \cos x = \frac{1}{\sqrt{5}}$



$\cos x \cdot \sin^2 x - \sin x \cdot \cos^2 x = \frac{1}{\sqrt{5}} \cdot \frac{4}{5} + \frac{2}{\sqrt{5}} \cdot \frac{1}{5}$

$= \frac{6}{5\sqrt{5}} = \frac{6\sqrt{5}}{25}$

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EXAMPLE 19

tan(– 870°) = ?

A) $-\frac{1}{2}$ B) $\frac{\sqrt{3}}{3}$ C) $-\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

SOLUTION

The answer is **B**

$$\begin{array}{r|l} 870^\circ & 360^\circ \\ 720^\circ & 2 \\ \hline 150^\circ & \end{array}$$

$$\tan(-870^\circ) = -\tan 870^\circ = -\tan 150^\circ$$

$$= -\tan\left(\pi - \frac{\pi}{6}\right) = -\left(-\tan\frac{\pi}{6}\right) = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

EXAMPLE 20

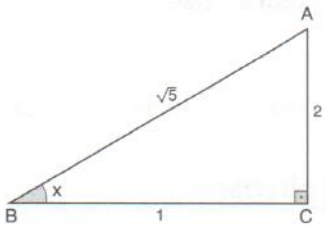
$x \in \left(\frac{3\pi}{2}, 2\pi\right)$, $\tan x = -2 \Rightarrow \cos x \cdot \sin^2 x - \sin x \cdot \cos^2 x = ?$

A) $-\frac{2\sqrt{5}}{5}$ B) $-\frac{6\sqrt{5}}{25}$ C) $\frac{2\sqrt{5}}{25}$ D) $\frac{6\sqrt{5}}{25}$ E) $\frac{\sqrt{5}}{5}$

SOLUTION

The answer is **D**

$x \in \left(\frac{3\pi}{2}, 2\pi\right) \Rightarrow 4^{th}$ Quadrant



$\cos x > 0$, $\sin x < 0$

$\sin x = -\frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

$$\cos x \cdot \sin^2 x - \sin x \cdot \cos^2 x = \frac{1}{\sqrt{5}} \cdot \frac{4}{5} + \frac{2}{\sqrt{5}} \cdot \frac{1}{5}$$

$$= \frac{6}{5\sqrt{5}} = \frac{6\sqrt{5}}{25}$$

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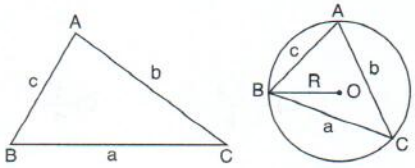
SİNÜS TEOREMİ



$$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}} = 2R$$

R : Çevrel çemberin yarıçapı

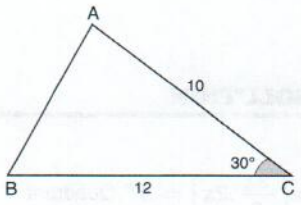
$$A(\overset{\Delta}{ABC}) = \frac{1}{2}\cdot b\cdot c \sin\hat{A} = \frac{1}{2}\cdot a\cdot c\cdot \sin\hat{B} = \frac{1}{2}\cdot a\cdot b\cdot \sin\hat{C}$$

AÇIKLAMA

Bir üçgenin kenar uzunlukları ile kenar uzunluklarının karşısındaki açıların sinüslerinin oranı, üçgenin çevrel çemberinin çapına eşittir.

ÖRNEK 21

|AC| = 10 cm,
|BC| = 12 cm,
$m(\widehat{ACB}) = 30°$.
$A(\overset{\Delta}{ABC}) = ?\ cm^2$



A) 120 B) 30 C) $30\sqrt{3}$ D) 40 E) $15\sqrt{2}$

ÇÖZÜM

Yanıt **B**

$$A(\overset{\Delta}{ABC}) = \frac{1}{2}\cdot 10\cdot 12\cdot \sin 30°$$

$$= \frac{1}{2}\cdot 10\cdot 12\cdot \frac{1}{2}$$

$$= 30\,cm^2$$

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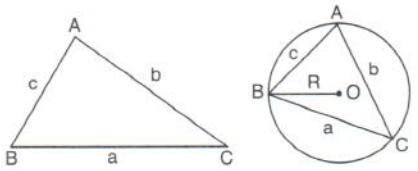
EXPLANATION

The ratio of the lengths of side to the angles that opposite the sides is equal to radius of circumcircle.

THE SINE THEOREM



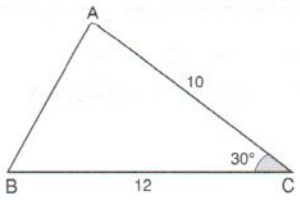
$$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}} = 2R$$

R : Radius of circumcircle

$$A(\overset{\Delta}{ABC}) = \frac{1}{2}\cdot b\cdot c\sin\hat{A} = \frac{1}{2}\cdot a\cdot c\cdot\sin\hat{B} = \frac{1}{2}\cdot a\cdot b\cdot\sin\hat{C}$$

EXAMPLE 21

$|AC| = 10$ cm,
$|BC| = 12$ cm,
$m(\widehat{ACB}) = 30°$.
$A(\overset{\Delta}{ABC}) = ?\ cm^2$



A) 120 B) 30 C) $30\sqrt{3}$ D) 40 E) $15\sqrt{2}$

SOLUTION

The answer is **B**

$$A(\overset{\Delta}{ABC}) = \frac{1}{2}\cdot 10\cdot 12\cdot\sin 30°$$

$$= \frac{1}{2}\cdot 10\cdot 12\cdot\frac{1}{2}$$

$$= 30\,cm^2$$

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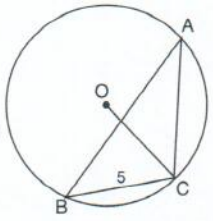
ÖRNEK 22

|CO| = 6 cm,
|BC| = 5 cm
⇒ **sinA = ?**



A) $\frac{5}{6}$ B) $\frac{5}{3}$ C) $\frac{3}{4}$ D) $\frac{2}{5}$ E) $\frac{5}{12}$

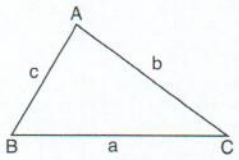
ÇÖZÜM

Yanıt **E**

$$\frac{|BC|}{\sin A} = 2 \cdot R \Rightarrow \sin A = \frac{5}{2 \cdot 6} = \frac{5}{12}$$

KOSİNÜS TEOREMİ

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \hat{A}$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \hat{B}$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \hat{C}$

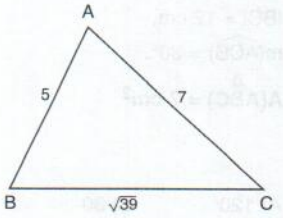


AÇIKLAMA

Üçgenin kenar uzunlukları biliniyorken herhangi bir açının ölçüsünü bulmak ya da iki kenar uzunluğu ve bu iki kenar arasındaki açısı verilen üçgenin diğer kenarının uzunluğunu bulmak için kullanılır.

ÖRNEK 23

|AB| = 5 cm, |AC| = 7 cm,
|BC| = $\sqrt{39}$ cm ⇒ **m(BAC) = ?**



A) 30° B) 45° C) 60° D) 75° E) 90°

ÇÖZÜM

Yanıt **C**

$$(\sqrt{39})^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos\alpha$$

$$39 = 25 + 49 - 70 \cdot \cos\alpha$$

$$\cos\alpha = \frac{35}{70} = \frac{1}{2} \Rightarrow \alpha = 60°$$

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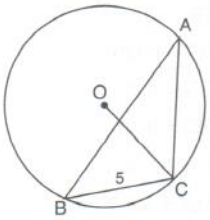
EXAMPLE 22

$|CO| = 6$ cm,
$|BC| = 5$ cm
$\Rightarrow$ **sinA = ?**



A) $\frac{5}{6}$ B) $\frac{5}{3}$ C) $\frac{3}{4}$ D) $\frac{2}{5}$ E) $\frac{5}{12}$

SOLUTION

The answer is **E**

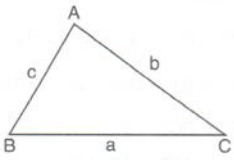
$$\frac{|BC|}{\sin A} = 2 \cdot R \Rightarrow \sin A = \frac{5}{2 \cdot 6} = \frac{5}{12}$$

EXPLANATION

The cosine theorem use to finding the measure of any angle or other length given that two lengths and between the angle.

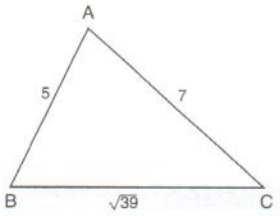
THE COSINE THEOREM

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \hat{A}$$
$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \hat{B}$$
$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \hat{C}$$



EXAMPLE 23

$|AB| = 5$ cm, $|AC| = 7$ cm,
$|BC| = \sqrt{39}$ cm $\Rightarrow$ **$m(\widehat{BAC}) = ?$**



A) 30° B) 45° C) 60° D) 75° E) 90°

SOLUTION

The answer is **C**

$$(\sqrt{39})^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos\alpha$$
$$39 = 25 + 49 - 70 \cdot \cos\alpha$$
$$\cos\alpha = \frac{35}{70} = \frac{1}{2} \Rightarrow \alpha = 60°$$

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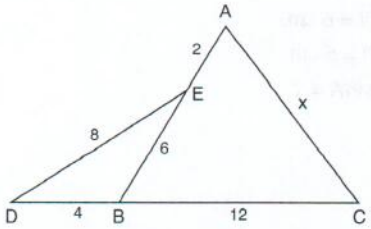
ÖRNEK 24

|AE| = 2 cm,
|EB| = 6 cm,
|DB| = 4 cm,
|DE| = 8 cm,
|BC| =12 cm
|AC| = x = ? cm



A) $6\sqrt{3}$ B) $8\sqrt{2}$ C) $4\sqrt{5}$ D) $4\sqrt{10}$ E) $5\sqrt{6}$

ÇÖZÜM

Yanıt **D**

ΔEBD:

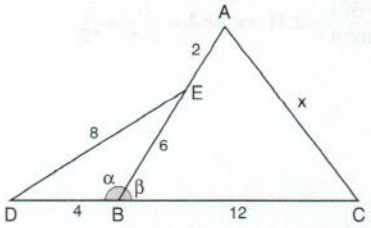
$64 = 16 + 36 - 2\cdot4\cdot6\cos\alpha$

$48\cdot\cos\alpha = 52 - 64$

$48\cdot\cos\alpha = -12 \Rightarrow \cos\alpha = -\frac{1}{4}$

$\alpha + \beta = 180°$

$\cos\beta = -\cos\alpha = -\left(-\frac{1}{4}\right) = \frac{1}{4}$



ΔABC:

$x^2 = 8^2 + 12^2 - 2\cdot8\cdot12\cdot\cos\beta$

$x^2 = 64 + 144 - 2\cdot8\cdot12\cdot\frac{1}{4}$

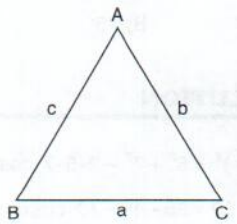
$x^2 = 208 - 48$

$x^2 = 160 = 16\cdot10 \Rightarrow x = 4\sqrt{10}$ cm

TANJANT TEOREMİ

|BC| = a, |AC| = b, |AB| = c

$$\Rightarrow \frac{b+c}{b-c} = \frac{\tan\frac{m(\widehat{B})+m(\widehat{C})}{2}}{\tan\frac{m(\widehat{B})-m(\widehat{C})}{2}}$$



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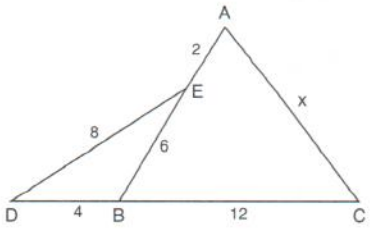
EXAMPLE 24

|AE| = 2 cm,
|EB| = 6 cm,
|DB| = 4 cm,
|DE| = 8 cm,
|BC| =12 cm
|AC| = x = ? cm



A) $6\sqrt{3}$ B) $8\sqrt{2}$ C) $4\sqrt{5}$ D) $4\sqrt{10}$ E) $5\sqrt{6}$

SOLUTION

The answer is **D**

$\overset{\Delta}{EBD}$:

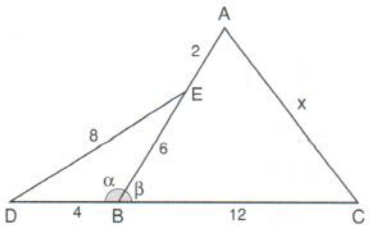
$64 = 16 + 36 - 2\cdot4\cdot6\cos\alpha$

$48\cdot\cos\alpha = 52 - 64$

$48\cdot\cos\alpha = -12 \Rightarrow \cos\alpha = -\frac{1}{4}$

$\alpha + \beta = 180°$

$\cos\beta = -\cos\alpha = -\left(-\frac{1}{4}\right) = \frac{1}{4}$



$\overset{\Delta}{ABC}$:

$x^2 = 8^2 + 12^2 - 2\cdot8\cdot12\cdot\cos\beta$

$x^2 = 64 + 144 - 2\cdot8\cdot12\cdot\frac{1}{4}$

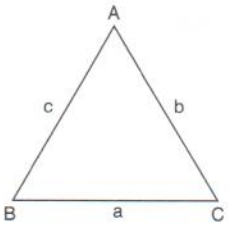
$x^2 = 208 - 48$

$x^2 = 160 = 16\cdot10 \Rightarrow x = 4\sqrt{10}$ cm

THE TANGENT THEOREM

|BC| = a, |AC| = b, |AB| = c

$$\Rightarrow \frac{b+c}{b-c} = \frac{\tan\frac{m(\widehat{B})+m(\widehat{C})}{2}}{\tan\frac{m(\widehat{B})-m(\widehat{C})}{2}}$$



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1. $210° = x\pi \Rightarrow x = ?$

A) $\frac{2}{3}$

B) $\frac{5}{6}$

C) 1

D) $\frac{7}{6}$

E) $\frac{4}{3}$

2. $\frac{6\pi}{5} = x° \Rightarrow x = ?$

A) 200

B) 208

C) 216

D) 224

E) 232

3. $2813° = x° + k\cdot360° \Rightarrow x = ?$

A) 293

B) 300

C) 307

D) 314

E) 320

4. $1905° = x° + k\cdot360° \Rightarrow x = ?$

A) 100

B) 105

C) 110

D) 115

E) 120

5. $-1572° = x° + k\cdot360° \Rightarrow x = ?$

A) 212

B) 220

C) 224

D) 228

E) 232

6. $\frac{24\pi}{5} = x + 2k\pi \Rightarrow x = ?$

A) $\frac{\pi}{5}$

B) $\frac{2\pi}{5}$

C) $\frac{3\pi}{5}$

D) $\frac{4\pi}{5}$

E) π

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7. $\frac{37\pi}{4} = x + 2k\pi \Rightarrow x = ?°$

A) 45

B) 90

C) 135

D) 180

E) 225

8. $-\frac{42\pi}{5} = x + 2k\pi \Rightarrow x = ?$

A) $\frac{4\pi}{5}$

B) π

C) $\frac{5\pi}{4}$

D) $\frac{8\pi}{5}$

E) $\frac{9\pi}{5}$

9. $\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1}{3} \Rightarrow \tan x = ?$

A) 2

B) $\frac{1}{3}$

C) $\frac{1}{2}$

D) $\frac{2}{3}$

E) $\frac{3}{4}$

10. $\left(\frac{1}{1-\cos x} + \frac{1}{1+\cos x}\right) \cdot \tan x = ?$

A) 2tanx

B) 2secx

C) 2cosecx

D) 2secx·cosecx

E) 2cotx

11. $(\tan x + \cot x) \cdot \left(\frac{1}{\operatorname{cosec} x \cdot \sec x}\right) = ?$

A) 1

B) sinx

C) cosx

D) sinx + cosx

E) 0

12. $\frac{\sin^3 x - \cos^3 x}{\sin^2 x - \cos^2 x} - \frac{1}{\sin x + \cos x} = ?$

A) $\frac{\sin x \cdot \cos x}{\sin x + \cos x}$

B) $\frac{\cos x}{\sin x + \cos x}$

C) 1

D) $\frac{\sin x - \cos x}{\sin x + \cos x}$

E) $\frac{\sin x + \cos x}{\sin x - \cos x}$

YANITLAR / ANSWERS

1. D	2. C	3. A	4. B	5. D	6. D	7. E	8. D
9. A	10. D	11. A	12. A				

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1. $\frac{D}{180^\circ} = \frac{R}{\pi}$

$\frac{210^\circ}{180^\circ} = \frac{R}{\pi}$

$R = \frac{210^\circ}{180^\circ}\pi$

$= \frac{7\pi}{6}$

Yanıt / The Answer is D

2. $\frac{D}{180^\circ} = \frac{R}{\pi}$

$\frac{D}{180^\circ} = \frac{6\pi}{5\pi}$

$5D = 180^\circ \cdot 6$

$D = 216^\circ$

Yanıt / The Answer is C

3.

$$\begin{array}{r|l} 2813^\circ & 360^\circ \\ \underline{2520^\circ} & \hline 7 \\ 293^\circ \rightarrow x & \end{array}$$

Yanıt / The Answer is A

4.

$$\begin{array}{r|l} 1905^\circ & 360^\circ \\ \underline{1800^\circ} & \hline 5 \\ 105^\circ \rightarrow x & \end{array}$$

Yanıt / The Answer is B

5.

$$\begin{array}{r|l} 1572^\circ & 360^\circ \\ \underline{1440^\circ} & \hline 4 \\ 132^\circ & \end{array}$$

$x = 360^\circ - 132^\circ = 228^\circ$

Yanıt / The Answer is D

6.

$$\begin{array}{r|l} 24 & 10 \\ \underline{20} & \hline 2 \\ 4 & \end{array} \quad \Rightarrow x = \frac{4\pi}{5}$$

Yanıt / The Answer is D

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7. $\begin{array}{r|l} 37 & 8 \\ \hline 32 & 4 \\ \hline 5 & \end{array}$ $\Rightarrow x = \frac{5\pi}{4} = \frac{5\cdot 180^\circ}{4} = 225^\circ$

Yanıt / The Answer is E

8. $\begin{array}{r|l} 42 & 10 \\ \hline 40 & 4 \\ \hline 2 & \end{array}$ $\Rightarrow 2\pi - \frac{2\pi}{5} = \frac{8\pi}{5}$

Yanıt / The Answer is D

9. $\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1}{3}$

$3\sin x - 3\cos x = \sin x + \cos x$

$2\sin x = 4\cos x$

$\tan x = \frac{4}{2} = 2$

Yanıt / The Answer is A

10. $\left(\underbrace{\frac{1}{1-\cos x}}_{(1+\cos x)} + \underbrace{\frac{1}{1+\cos x}}_{(1-\cos x)}\right)\cdot \tan x = \frac{1+\cos x+1-\cos x}{1-\cos^2 x}\cdot \tan x$

$= \frac{2}{\sin^2 x}\cdot\frac{\sin x}{\cos x}$

$= \frac{2}{\sin x\cdot\cos x}$

$= 2\sec x\cdot \operatorname{cosec} x$

Yanıt / The Answer is D

11. $(\tan x + \cos x)\cdot\left(\frac{1}{\operatorname{cosec} x\cdot \sec x}\right)$

$= \left(\underbrace{\frac{\sin x}{\cos x}}_{(\sin x)} + \underbrace{\frac{\cos x}{\sin x}}_{(\cos x)}\right)\cdot(\sin x\cos x)$

$= \frac{1}{\sin x\cos x}\cdot \sin x\cos x$

$= 1$

Yanıt / The Answer is A

12. $\frac{\sin^3 x - \cos^3 x}{\sin^2 x - \cos^2 x} - \frac{1}{\sin x + \cos x}$

$= \frac{(\sin x - \cos x)\cdot(\sin^2 x + \sin x\cos x + \cos^2 x)}{(\sin x - \cos x)(\sin x + \cos x)} - \frac{1}{\sin x + \cos x}$

$= \frac{1+\sin x\cos x - 1}{\sin x + \cos x}$

$= \frac{\sin x\cos x}{\sin x + \cos x}$

Yanıt / The Answer is A

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1. $\frac{\sin x}{1-\cos x} - \cos x = ?$

A) $\operatorname{cosec} x + \cot x - \cos x$

B) $\sec x + \cot x - \cos x$

C) $\operatorname{cosec} x - \cot x - \cos x$

D) $\operatorname{cosec} x - \cot x + \cos x$

E) $\sec x + \cot x + \cos x$

2. $\left.\begin{array}{l} a = \sin 62^\circ \\ b = \cos 60^\circ \\ c = \tan 47^\circ \end{array}\right\} \Rightarrow ? < ? < ?$

A) $a < b < c$

B) $b < c < a$

C) $c < a < b$

D) $b < a < c$

E) $a < c < b$

3. $\sin x - \cos x = \frac{1}{4} \Rightarrow \sin x \cdot \cos x = ?$

A) $\frac{7}{16}$

B) $\frac{15}{16}$

C) $\frac{5}{3}$

D) $\frac{15}{32}$

E) $\frac{5}{12}$

4. $\tan x - \cot x = 2 \Rightarrow \tan^3 x - \cot^3 x = ?$

A) 12

B) 13

C) 14

D) 15

E) 16

5. $0 < x < 90^\circ$,

$\sin x = \frac{3}{7} \Rightarrow \tan x = ?$

A) $\frac{3}{\sqrt{10}}$

B) $\frac{1}{2\sqrt{10}}$

C) $\frac{3}{2\sqrt{10}}$

D) $\frac{2}{3\sqrt{10}}$

E) $\frac{7}{2\sqrt{10}}$

6. $\frac{3\pi}{2} < x < 2\pi$,

$\cos x = \frac{1}{3} \Rightarrow \sin x \cdot \tan x = ?$

A) $-\frac{8}{3}$

B) $-\frac{4}{3}$

C) $\frac{1}{3}$

D) $\frac{4}{3}$

E) $\frac{8}{3}$

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7. $x \in \left(\frac{\pi}{2}, \pi\right)$

$3\sin x = 1 \Rightarrow$ **tanx = ?**

A) $\frac{\sqrt{2}}{2}$

B) $\frac{\sqrt{2}}{4}$

C) $\sqrt{2}$

D) $-\frac{\sqrt{2}}{2}$

E) $-\frac{\sqrt{2}}{4}$

8. $\sin 20° = m \Rightarrow$ **cos110° = ?**

A) $-m$

B) $-\frac{m}{2}$

C) $\frac{1}{m}$

D) $-\frac{1}{m}$

E) m

9. $\frac{\sin 420° + \cos 240°}{\tan(-30°)} = ?$

A) $\frac{3-\sqrt{3}}{4}$

B) $\frac{3-\sqrt{3}}{2}$

C) $3-\sqrt{3}$

D) $\frac{\sqrt{3}-3}{2}$

E) $\sqrt{3}-3$

10. $x+y=\frac{\pi}{2}$,

$\cos(2x+y) = -\frac{1}{4} \Rightarrow$ **tanx = ?**

A) $\frac{3}{4}$

B) $\frac{1}{2}$

C) $\frac{1}{\sqrt{3}}$

D) $\frac{1}{\sqrt{5}}$

E) $\frac{1}{\sqrt{15}}$

11. $\sin\theta$ **= ?**

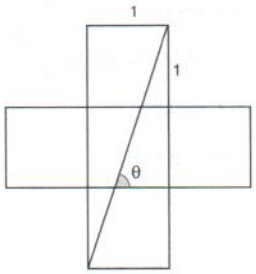
A) $\frac{\sqrt{10}}{3}$

B) $\frac{1}{3}$

C) $\frac{3}{\sqrt{10}}$

D) $\frac{3}{\sqrt{15}}$

E) $\frac{3}{5\sqrt{2}}$



12. $x+y+z=\pi \Rightarrow \cot\left(\frac{x+y}{2}\right) = ?$

A) $\tan z$

B) $\cot\frac{z}{2}$

C) $-\cot z$

D) $\tan\frac{z}{2}$

E) $-\tan\frac{z}{2}$

YANITLAR / ANSWERS							
1. A	2. D	3. D	4. C	5. C	6. E	7. E	8. A
9. D	10. E	11. C	12. D				

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1. $\frac{\sin x}{1-\cos x} - \cos x = \frac{\sin x(1+\cos x)}{1-\cos^2 x} - \cos x$

$(1+\cos x)$

$= \frac{1+\cos x}{\sin x} - \cos x$

$= \frac{1}{\sin x} + \frac{\cos x}{\sin x} - \cos x$

$= \operatorname{cosec} x + \cot x - \cos x$

Yanıt / The Answer is A

2. $\left.\begin{array}{l} a = \sin 62^\circ \\ b = \cos 60^\circ = \sin 30^\circ \end{array}\right\} \Rightarrow a > b$

$c = \tan 47^\circ$

$\Rightarrow c > a > b$

Yanıt / The Answer is D

3. $(\sin x - \cos x)^2 = \left(\frac{1}{4}\right)^2$

$\sin^2 x - 2\sin x\cos x + \cos^2 x = \frac{1}{16}$

$1 - 2\sin x\cos x = \frac{1}{16}$

$2\sin x\cos x = \frac{15}{16}$

$\sin x\cos x = \frac{15}{32}$

Yanıt / The Answer is D

4. $a^3 - b^3 = (a-b)^3 + 3a\cdot b(a-b)$

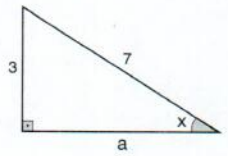
$\tan^3 x - \cot^3 x = (\tan x - \cot x)^3 + 3\tan x\cdot\cot x(\tan x - \cot x)$

$= 2^3 + 3\cdot 1\cdot 2$

$= 14$

Yanıt / The Answer is C

5.



$a^2 + 3^2 = 7^2$

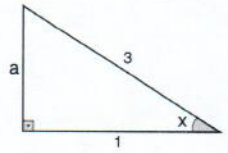
$a^2 + 9 = 49$

$a = 2\sqrt{10}$

$\tan x = \frac{3}{2\sqrt{10}}$

Yanıt / The Answer is C

6.



$a^2 + 1^2 = 3^2$

$a = 2\sqrt{2}$

$\frac{3\pi}{2} < x < 2\pi \Rightarrow \sin x < 0 \ , \ \tan x < 0$

$\sin x \cdot \tan x = \frac{-2\sqrt{2}}{3} \cdot (-2\sqrt{2}) = \frac{8}{3}$

Yanıt / The Answer is E

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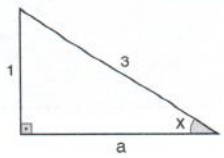
7.



$a^2 + 1^2 = 3^2 \Rightarrow a = 2\sqrt{2}$

$3\sin x = 1 \Rightarrow \sin x = \frac{1}{3}$

$\frac{\pi}{2} < x < \pi \Rightarrow \tan x < 0$

$\tan x = \frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$

Yanıt / The Answer is E

8. $\cos 110° = \cos(90° + 20°)$

$= -\sin 20°$

$= -m$

Yanıt / The Answer is A

9. $\sin 420° = \sin(360° + 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 240° = \cos(180° + 60°) = -\cos 60° = -\frac{1}{2}$

$\tan(-30°) = -\tan 30° = -\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\sin 420° + \cos 240°}{\tan(-30°)} = \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{-\frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3} - 1}{2} - \sqrt{3}$

$= \frac{\sqrt{3} - 3}{2}$

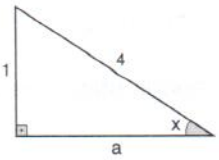
Yanıt / The Answer is D

10. $x + y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} - x$

$\cos(2x + y) = \cos\left(2x + \frac{\pi}{2} - x\right)$

$= \cos\left(x + \frac{\pi}{2}\right)$

$= -\sin x = -\frac{1}{4} \Rightarrow \sin x = \frac{1}{4}$



$a^2 + 1^2 = 4^2$

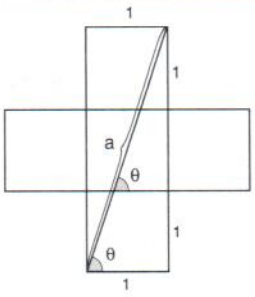
$a^2 = 15$

$a = \sqrt{15}$

$\Rightarrow \tan x = \frac{1}{\sqrt{15}}$

Yanıt / The Answer is E

11.



$a^2 = 1^2 + 3^2$

$a = \sqrt{10}$

$\sin\theta = \frac{3}{\sqrt{10}}$

Yanıt / The Answer is C

12. $x + y + z = \pi \Rightarrow x + y = \pi - z$

$\cot\left(\frac{x+y}{2}\right) = \cot\left(\frac{\pi - z}{2}\right)$

$= \cot\left(\frac{\pi}{2} - \frac{z}{2}\right)$

$= \tan\frac{z}{2}$

Yanıt / The Answer is D

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YÖS Soruları / YÖS Questions

1. $\cos(\alpha-3\pi)+\sin(\alpha-\frac{\pi}{2})=?$

A) $\cos(\alpha)$ B) $\cos(2\alpha)$ C) $\sin(\alpha)$

D) $-2\cos(\alpha)$ E) $-2\sin(\alpha)$

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

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2. $\tan x+\cot x=\frac{4}{\sqrt{3}} \Rightarrow \tan^2 x+\cot^2 x=?$

A) $\frac{4}{3}$ B) $\frac{10}{3}$ C) $\frac{25}{4}$

D) $\frac{4}{9}$ E) 1

[MARMARA ÜNİV. YÖS 2013]

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3. $\frac{1}{\tan x}\cdot\frac{\cos 2x-1}{\sin 2x}=?$

A) $-\sin x$ B) $-\cos x$ C) -1

D) $\sin x$ E) $\frac{1}{\cos x}$

[ANKARA ÜNİV. YÖS 2013]

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4. $\frac{\sin\frac{5\pi}{6}+\cos\frac{3\pi}{4}}{\sin\frac{11\pi}{6}}=?$

A) $\sqrt{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\sqrt{2}-1$

D) -1 E) $\frac{\sqrt{2}}{2}+1$

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

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5. $\frac{\sin x}{3}=\frac{\cos x}{4} \Rightarrow \tan x=?$

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) $\frac{4}{3}$

[MARMARA ÜNİV. YÖS 2013]

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6. $\frac{\cos 15^\circ}{\sin 75^\circ}=?$

A) 0 B) -1 C) 1

D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

(ANKARA ÜNİV. YÖS 2011)

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7. $\frac{\cos310^\circ \cdot \sin290^\circ}{\sin40^\circ \cdot \cos20^\circ} = ?$

A) 0 B) 1 C) −1

D) $\frac{1}{2}$ E) $\frac{\sqrt{2}}{2}$

(ANADOLU ÜNİV. YÖS 2011)

8. $\frac{\cos15^\circ}{\sin105^\circ} = ?$

A) 0 B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$ E) 1

(YÖS 2010)

9. $\sin x \cdot \cos x = \frac{1}{4}$

$\sin^4 x + \cos^4 x = ?$

A) $\frac{3}{4}$ B) $\frac{5}{8}$ C) $\frac{7}{8}$ D) $\frac{5}{12}$ E) $\frac{7}{12}$

(YÖS 2009)

10. $\frac{1+\tan50^\circ}{1+\tan40^\circ} = ?$

A) 1 + tan20° B) 1 + cot20° C) 1 + tan40°

D) tan40° E) cot40°

(YÖS 2008)

YANITLAR / ANSWERS							
1. D	2. B	3. C	4. C	5. C	6. C	7. C	8. E
9. C	10. E						

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1. $\cos(\alpha-3\pi)+\sin(\alpha-\frac{\pi}{2})=\cos(3\pi-\alpha)-\sin(\frac{\pi}{2}-\alpha)$

$=-\cos\alpha-\cos\alpha$

$=-2\cos\alpha$

Yanıt / The Answer is D

2. $\tan x+\cot x=\frac{4}{\sqrt{3}}$

$(\tan x+\cot x)^2=\left(\frac{4}{\sqrt{3}}\right)^2$

$\tan^2 x+2\cdot\tan x\cdot\cot x+\cot^2 x=\frac{16}{3}$

$\tan^2 x+2+\cot^2 x=\frac{16}{3}$

$\tan^2 x+\cot^2 x=\frac{16}{3}-2$

$\tan^2 x+\cot^2 x=\frac{16-6}{3}=\frac{10}{3}$

Yanıt / The Answer is B

3. $\frac{1}{\tan x}\cdot\frac{\cos 2x-1}{\sin 2x}=\frac{1}{\frac{\sin x}{\cos x}}\cdot\frac{1-2\sin^2 x-1}{2\sin x\cdot\cos x}$

$=\frac{\cos x}{\sin x}\cdot\frac{-2\sin^2 x}{2\sin x\cdot\cos x}=-1$

Yanıt / The Answer is C

4. $\frac{\sin\frac{5\pi}{6}+\cos\frac{3\pi}{4}}{\sin\frac{11\pi}{6}}=\frac{\frac{1}{2}-\frac{\sqrt{2}}{2}}{-\frac{1}{2}}$

$=\frac{1-\sqrt{2}}{2}\cdot(-2)$

$=\sqrt{2}-1$

Yanıt / The Answer is C

5. $\frac{\sin x}{3}=\frac{\cos x}{4}$

$4\sin x=3\cos x \Rightarrow \frac{\sin x}{\cos x}=\frac{3}{4}$

$\tan x=\frac{3}{4}$

Yanıt / The Answer is C

6. $\frac{\cos 15^\circ}{\sin 75^\circ}=\frac{\sin 75^\circ}{\sin 75^\circ}=1$

Yanıt / The Answer is C

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7. $\frac{\cos 310^\circ \cdot \sin 290^\circ}{\sin 40^\circ \cdot \cos 20^\circ} = \frac{\sin 40^\circ \cdot (-\cos 20^\circ)}{\sin 40^\circ \cdot \cos 20^\circ} = -1$

Yanıt / The Answer is C

8. $\frac{\cos 15^\circ}{\sin 105^\circ} = \frac{\cos 15^\circ}{\sin(90^\circ + 15^\circ)} = \frac{\cos 15^\circ}{\cos 15^\circ} = 1$

Yanıt / The Answer is E

9. $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cdot \cos^2 x$

$= 1 - 2 \cdot \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{8} = \frac{7}{8}$

Yanıt / The Answer is C

10. $\tan 40^\circ = \cot 50^\circ = \frac{1}{\tan 50^\circ}$

$\Rightarrow \frac{1 + \tan 50^\circ}{1 + \tan 40^\circ} = \frac{1 + \tan 50^\circ}{1 + \frac{1}{\tan 50^\circ}}$

$= \tan 50^\circ = \cot 40^\circ$

Yanıt / The Answer is E

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1. $225° = x\pi \Rightarrow x = ?$

Yanıt / The Answer is $\frac{5}{4}$

2. $\frac{5\pi}{6} = x° \Rightarrow x = ?$

Yanıt / The Answer is **150°**

3. $1470° = x + k\cdot360° \Rightarrow x = ?$

Yanıt / The Answer is **30°**

4. $\frac{29\pi}{3} = x + 2k\pi \Rightarrow x = ?$

Yanıt / The Answer is $\frac{5\pi}{3}$

5. $-1310° = x + k\cdot360° \Rightarrow x = ?$

Yanıt / The Answer is **130°**

6. $-\frac{45\pi}{7} = x + 2k\pi \Rightarrow x = ?$

Yanıt / The Answer is $\frac{11\pi}{7}$

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7. $A = 4\sin x + 3\cos y$

$\Rightarrow \max(A) + \min(A) = ?$

Yanıt / The Answer is **0**

8. $a = \sin 15°$

$b = \cos 165°$

$c = \sin 65°$

$d = \cos 25°$

$\Rightarrow ? < ? < ? < ?$

Yanıt / The Answer is **c = d > a > b**

9. $\frac{1+\cos x}{1+\sin x} \cdot \frac{1+\operatorname{cosec} x}{1+\sec x} = ?$

Yanıt / The Answer is **cotx**

10. $\frac{1}{\sin x \cdot \cos x} - \tan x - \cot x = ?$

Yanıt / The Answer is **0**

11. $\tan x + \frac{\cos x}{1+\sin x} = ?$

Yanıt / The Answer is **secx**

12. $\frac{1-\tan^2 x}{1+\tan^2 x} = ?$

Yanıt / The Answer is **$\cos^2 x - \sin^2 x$**

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13. $\frac{1}{1+\tan^2 x}+\frac{1}{1+\cot^2 x}=?$

Yanıt / The Answer is **1**

14. $4\sin x - 2m = 2 \Rightarrow ? \le m \le ?$

Yanıt / The Answer is $-3 \le m \le 1$

15. $\sin^2 15° + \sin^2 75° + \sin^2 45° = ?$

Yanıt / The Answer is $\frac{3}{2}$

16. $(0{,}25)^{\sin x} \cdot 2^{\cos x} = 1 \Rightarrow 3^{\cot x} = ?$

Yanıt / The Answer is **9**

17. $\frac{\sin 15° + \cos 75° + 1}{6 \cdot \cos 795° + 3} = ?$

Yanıt / The Answer is $\frac{1}{3}$

18. $x \in \left(0, \frac{\pi}{2}\right)$,

$\cot x = \frac{15}{8} \Rightarrow \sin^2 x + \cos x \cdot \sin x = ?$

Yanıt / The Answer is $\frac{184}{289}$

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19. $\dfrac{4\cdot\tan\frac{5\pi}{6}-\frac{2}{\cos\frac{11\pi}{6}}}{3\cdot\sin\frac{3\pi}{2}-5\cos\pi}=?$

Yanıt / The Answer is $\dfrac{-4}{\sqrt{3}}$

20. $f(x)=\cos 3x-\dfrac{1}{2}\sin 2x+1 \Rightarrow f\left(\dfrac{\pi}{6}\right)+f\left(\dfrac{\pi}{2}\right)=?$

Yanıt / The Answer is $\dfrac{8-\sqrt{3}}{4}$

21. $\dfrac{\sin220°+\sin150°+\sin40°}{\cos220°+\cos30°+\cos(-40°)}=?$

Yanıt / The Answer is $\dfrac{1}{\sqrt{3}}$

22. $\pi<x<\dfrac{3\pi}{2}$,

$2\sin x=-\sqrt{3} \Rightarrow \cot\dfrac{x}{2}=?$

Yanıt / The Answer is $-\dfrac{\sqrt{3}}{3}$

23. $\sin50° = a \Rightarrow$ **tan320° + cot230° + cos140° = ?**

Yanıt / The Answer is **–a**

24. $\dfrac{\sin\left(\frac{3\pi}{2}+x\right)+\cos(\pi-x)}{\cos\left(\frac{\pi}{2}+x\right)+\sin(-x)}=?$

Yanıt / The Answer is **cotx**

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25. $\cos(90° - \alpha) + \sin(270° - \alpha) = ?$

Yanıt / The Answer is $\sin\alpha - \cos\alpha$

26. $\tan 300° = ?$

Yanıt / The Answer is $-\sqrt{3}$

27. $\sin\frac{19\pi}{3} \cdot \cos\frac{17\pi}{3} = ?$

Yanıt / The Answer is $\frac{\sqrt{3}}{4}$

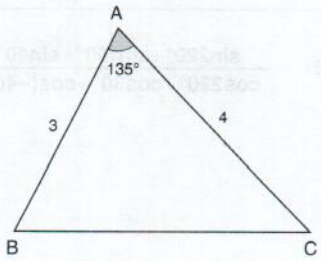
28. $\sin 150°, \cos 560°, \tan 342° \Rightarrow ?, ?, ?$

Yanıt / The Answer is $+, -, -$

29. $\sin 163°, \sec 115°, \operatorname{cosec} 220°, \tan 300° \Rightarrow ?, ?, ?, ?$

Yanıt / The Answer is $+, -, -, -$

30. $\overset{\Delta}{A}(ABC) = ?$



Yanıt / The Answer is $3\sqrt{2}$

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1. $6540° = x° + k\cdot 360° \Rightarrow x = ?$

A) 6

B) 15

C) 30

D) 60

E) 70

2. $-1260° = x° + k\cdot 360° \Rightarrow x = ?$

A) 120

B) 150

C) 180

D) 210

E) 240

3. $144° = x\pi \Rightarrow x = ?$

A) $\frac{3}{5}$

B) $\frac{4}{5}$

C) $\frac{7}{10}$

D) $\frac{9}{10}$

E) 1

4. $\frac{126\pi}{5} = x + 2k\pi \Rightarrow x = ?$

A) $\frac{4\pi}{5}$

B) π

C) $\frac{6\pi}{5}$

D) $\frac{7\pi}{5}$

E) $\frac{8\pi}{5}$

5. $\sin 215°$, $\cos 1750°$, $\tan 800° \Rightarrow ?, ?, ?$

A) $-$, $+$, $+$

B) $-$, $+$, $-$

C) $+$, $-$, $+$

D) $-$, $-$, $+$

E) $+$, $+$, $+$

6. $a = \sin 320°$

$b = \cos 220°$

$c = \tan 100°$

$\Rightarrow ? < ? < ?$

A) $a < b < c$

B) $c < b < a$

C) $c < a < b$

D) $b < c < a$

E) $a < c < b$

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7. $\sin(5\pi+x)+\cos\left(\frac{7\pi}{2}-x\right)=?$

A) 1

B) 2sinx

C) 0

D) –2sinx

E) –1

8. $\frac{\cos150^\circ-\sin120^\circ}{2\cdot\sin60^\circ+\cos30^\circ}=?$

A) $-\frac{2}{3}$

B) $\frac{2\sqrt{3}}{3}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{2}{3}$

E) 1

9. $\sin(-60°)+\cos(-45°)=?$

A) $\frac{\sqrt{2}+\sqrt{3}}{2}$

B) $\frac{\sqrt{3}-\sqrt{2}}{2}$

C) $\frac{\sqrt{3}+\sqrt{2}}{4}$

D) $\frac{\sqrt{2}-\sqrt{3}}{2}$

E) $\frac{\sqrt{3}}{4}$

10. $\sin30^\circ\cdot\cos45^\circ\cdot\tan60^\circ=?$

A) $\frac{\sqrt{2}}{2}$

B) $\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{6}}{2}$

D) $\frac{\sqrt{6}}{3}$

E) $\frac{\sqrt{6}}{4}$

11. $\tan25^\circ=k \Rightarrow \frac{\tan205^\circ+\tan115^\circ}{\tan245^\circ-\tan335^\circ}=?$

A) $\frac{k^2+1}{1-k^2}$

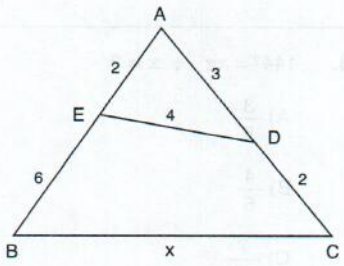
B) $\frac{k^2-1}{1+k^2}$

C) $\frac{k+1}{1+k^2}$

D) $\frac{k-1}{1+k^2}$

E) $\frac{k}{k-1}$

12. x = ?



A) $\sqrt{103}$ B) $\sqrt{105}$ C) $\sqrt{107}$ D) $\sqrt{109}$ E) $\sqrt{113}$

YANITLAR / ANSWERS							
1. D	2. C	3. B	4. C	5. A	6. B	7. D	8. A
9. D	10. E	11. B	12. D				

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1. $\frac{\cos^2 x - 1}{\sin x} = ?$

A) sinx
B) cosx
C) tanx
D) –sinx
E) –cosx

2. $\frac{3\cos^2 x + \sin^2 x - 1}{\cos x} = ?$

A) cosx
B) 2cosx
C) sinx
D) 2·sinx
E) 1

3. $\frac{1+\sin x}{\cos x} - \frac{\cos x}{1-\sin x} = ?$

A) sinx
B) $\sin^2 x$
C) 0
D) cosx
E) $\cos^2 x$

4. $\tan x + \cot x = -2 \Rightarrow \tan^2 x + \cot^2 x = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

5. $\sin x - \cos x = \frac{3}{5} \Rightarrow \sin x \cdot \cos x = ?$

A) $\frac{1}{5}$
B) $\frac{6}{25}$
C) $\frac{7}{25}$
D) $\frac{8}{25}$
E) $\frac{9}{25}$

6. $\cos x - \sec x = \frac{4}{3} \Rightarrow \cos^2 x + \sec^2 x = ?$

A) $\frac{10}{3}$
B) $\frac{31}{9}$
C) $\frac{32}{9}$
D) $\frac{11}{3}$
E) $\frac{34}{9}$

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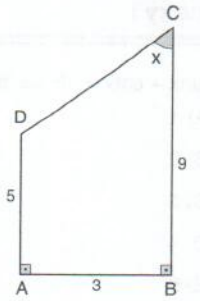
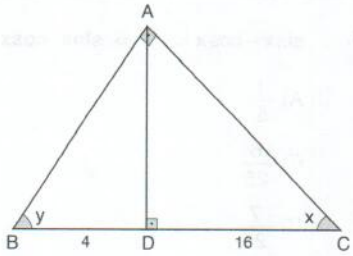
7. [DA] ⊥ [AB],
[AB] ⊥ [BC],
|AD| = 5 br(u),
|AB| = 3 br(u),
|BC| = 9 br(u).
sinx·cosx + tanx = ?



A) $\frac{113}{100}$ B) $\frac{123}{100}$ C) $\frac{133}{100}$ D) $\frac{143}{100}$ E) $\frac{153}{100}$

8. [BA] ⊥ [AC],
[AD] ⊥ [BC],
|BD| = 4 br(u),
|DC| = 16 br(u).
sinx + siny = ?



A) $\frac{\sqrt{5}}{5}$ B) $\frac{2\sqrt{5}}{5}$ C) $\frac{3\sqrt{5}}{5}$ D) $\frac{4\sqrt{5}}{5}$ E) 1

9. $x \in \left(0, \frac{\pi}{2}\right)$,
$\frac{3}{\sin x} = \frac{2}{\cos x} \Rightarrow$ **sinx = ?**

A) $\frac{\sqrt{13}}{13}$
B) $\frac{2\sqrt{13}}{13}$
C) $\frac{3\sqrt{13}}{13}$
D) $\frac{4\sqrt{13}}{13}$
E) $\frac{5\sqrt{13}}{13}$

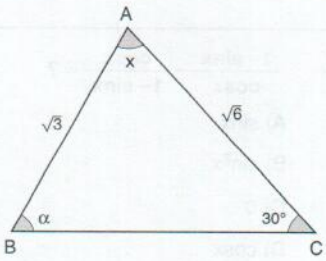
10. $\frac{3\cos x + \sin x}{2\sin x + 3\cos x} = \frac{4}{5} \Rightarrow$ **tanx = ?**

A) $\frac{3}{4}$
B) $\frac{4}{3}$
C) 1
D) $\frac{3}{5}$
E) $\frac{4}{5}$

11. $x \in \left(\pi, \frac{3\pi}{2}\right)$, $\tan x = 1 \Rightarrow$ **sinx·cos²x = ?**

A) $-\frac{\sqrt{2}}{2}$
B) $\frac{\sqrt{2}}{2}$
C) $-\frac{\sqrt{2}}{4}$
D) $\frac{\sqrt{2}}{4}$
E) $\frac{\sqrt{2}}{6}$

12. $\frac{\pi}{2} < \alpha < \pi$
$\Rightarrow$ **x = ?**



A) 15° B) 30° C) 45° D) 60° E) 75°

YANITLAR / ANSWERS

1. D	2. B	3. C	4. B	5. D	6. E	7. B	8. C
9. C	10. C	11. C	12. A				

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BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Cardano, Girolamo [1501 – 1576]

Italyan fizikçi ve matematikçi. Ikinci ve üçüncü dereceden denklemlerin çözümü için ilk öneri sunanlardandır. Cebir ve trigonometri alanlarında zamanın seçkin matematikçilerindendir.

Italian physician and mathematician, whose Ars magna contained the first published solutions of the general cubic equation and the general quadratic equation. Cardano was outstanding mathematician of the time in the fields of algebra and trigonometry.

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TRIGONOMETRY II

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TRİGONOMETRİ II
TRIGONOMETRY II

BÖLÜM 2
CHAPTER

TOPLAM VE FARK FORMÜLLERİ

1. $\sin(a + b) = \sin a \cdot \cos b + \cos a \cdot \sin b$
2. $\sin(a - b) = \sin a \cdot \cos b - \cos a \cdot \sin b$
3. $\cos(a + b) = \cos a \cdot \cos b - \sin a \cdot \sin b$
4. $\cos(a - b) = \cos a \cdot \cos b + \sin a \cdot \sin b$
5. $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$
6. $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$
7. $\cot(a+b) = \dfrac{\cot a \cdot \cot b - 1}{\cot b + \cot a}$
8. $\cot(a-b) = \dfrac{\cot a \cdot \cot b + 1}{\cot b - \cot a}$

NOT

$f(x) = a\sin x + b\cos x \Rightarrow$

$\max f(x) = \sqrt{a^2 + b^2}$

$\min f(x) = -\sqrt{a^2 + b^2}$

ÖRNEK 1

$\cos 105° = ?$

A) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$ B) $\dfrac{\sqrt{2} - \sqrt{6}}{4}$ C) $\dfrac{\sqrt{6} + 1}{2}$

D) $\dfrac{\sqrt{6} + \sqrt{2}}{2}$ E) $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

ÇÖZÜM

Yanıt **B**

$\cos 105° = \cos(60° + 45°)$

$= \cos 60° \cdot \cos 45° - \sin 60° \cdot \sin 45°$

$\cos 105° = \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

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NOTE

$f(x) = a\sin x + b\cos x \Rightarrow$

$\max f(x) = \sqrt{a^2 + b^2}$

$\min f(x) = -\sqrt{a^2 + b^2}$

SUM and DIFFERENCE FORMULAS

1. $\sin(a + b) = \sin a \cdot \cos b + \cos a \cdot \sin b$
2. $\sin(a - b) = \sin a \cdot \cos b - \cos a \cdot \sin b$
3. $\cos(a + b) = \cos a \cdot \cos b - \sin a \cdot \sin b$
4. $\cos(a - b) = \cos a \cdot \cos b + \sin a \cdot \sin b$
5. $\tan(a + b) = \dfrac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$
6. $\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$
7. $\cot(a + b) = \dfrac{\cot a \cdot \cot b - 1}{\cot b + \cot a}$
8. $\cot(a - b) = \dfrac{\cot a \cdot \cot b + 1}{\cot b - \cot a}$

EXAMPLE 1

$\cos 105° = ?$

A) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$ B) $\dfrac{\sqrt{2} - \sqrt{6}}{4}$ C) $\dfrac{\sqrt{6} + 1}{2}$

D) $\dfrac{\sqrt{6} + \sqrt{2}}{2}$ E) $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

SOLUTION

The answer is **B**

$\cos 105° = \cos(60° + 45°)$

$= \cos 60° \cdot \cos 45° - \sin 60° \cdot \sin 45°$

$\cos 105° = \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

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ÖRNEK 2

$a + b = \frac{\pi}{3} \Rightarrow (\cos a + \sin b)^2 + (\cos b + \sin a)^2 = ?$

A) $\sqrt{3}$ B) $1 + \sqrt{3}$ C) $-\sqrt{3}$

D) $2 + \sqrt{3}$ E) $\sqrt{3} - 2$

ÇÖZÜM

Yanıt **D**

$$(\cos a + \sin b)^2 + (\cos b + \sin a)^2 = \cos^2 a + 2\cos a \cdot \sin b + \sin^2 b + \cos^2 b + 2\cos b \cdot \sin a + \sin^2 a$$
$$= 1 + 1 + 2\sin(a + b)$$
$$= 2 + 2\sin(a + b)$$

$$a + b = \frac{\pi}{3} \Rightarrow 2 + 2\sin\frac{\pi}{3} = 2 + 2 \cdot \frac{\sqrt{3}}{2} = 2 + \sqrt{3}$$

ÖRNEK 3

$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = ?$

A) $\cos x$ B) $\sin x$ C) $\sin x \cdot \cos x$

D) $\operatorname{cosec} x$ E) $\sec x$

ÇÖZÜM

Yanıt **E**

$$\frac{\sin 2x}{\underset{(\cos x)}{\sin x}} - \frac{\cos 2x}{\underset{(\sin x)}{\cos x}} = \frac{\sin 2x \cdot \cos x - \cos 2x \cdot \sin x}{\sin x \cdot \cos x}$$

$$= \frac{\sin(2x - x)}{\sin x \cdot \cos x}$$

$$= \frac{\sin x}{\sin x \cdot \cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

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EXAMPLE 2

$a + b = \frac{\pi}{3} \Rightarrow (\cos a + \sin b)^2 + (\cos b + \sin a)^2 = ?$

A) $\sqrt{3}$ B) $1 + \sqrt{3}$ C) $-\sqrt{3}$

D) $2 + \sqrt{3}$ E) $\sqrt{3} - 2$

SOLUTION

The answer is D

$$(\cos a + \sin b)^2 + (\cos b + \sin a)^2 = \cos^2 a + 2\cos a \cdot \sin b + \sin^2 b + \cos^2 b + 2\cos b \cdot \sin a + \sin^2 a$$
$$= 1 + 1 + 2\sin(a + b)$$
$$= 2 + 2\sin(a + b)$$

$$a + b = \frac{\pi}{3} \Rightarrow 2 + 2\sin\frac{\pi}{3} = 2 + 2 \cdot \frac{\sqrt{3}}{2} = 2 + \sqrt{3}$$

EXAMPLE 3

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = ?$$

A) $\cos x$ B) $\sin x$ C) $\sin x \cdot \cos x$

D) $\operatorname{cosec} x$ E) $\sec x$

SOLUTION

The answer is E

$$\frac{\sin 2x}{\underset{(\cos x)}{\sin x}} - \frac{\cos 2x}{\underset{(\sin x)}{\cos x}} = \frac{\sin 2x \cdot \cos x - \cos 2x \cdot \sin x}{\sin x \cdot \cos x}$$

$$= \frac{\sin(2x - x)}{\sin x \cdot \cos x}$$

$$= \frac{\sin x}{\sin x \cdot \cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

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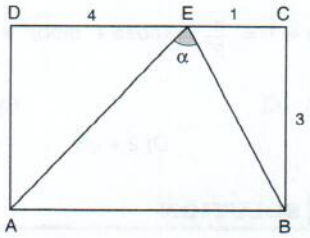
ÖRNEK 4

ABCD bir dikdörtgendir.
|CB| = 3 br,
|EC| = 1 br,
|DE| = 4 br
⇒ **tanα = ?**



A) 2 B) $\frac{5}{2}$ C) 3 D) $\frac{7}{2}$ E) 4

ÇÖZÜM

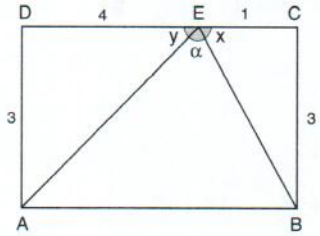
Yanıt **C**

$\tan\alpha = \tan(180°-(x+y))$

$= -\tan(x+y)$

$= -\frac{\tan x+\tan y}{1-\tan x\cdot\tan y}$

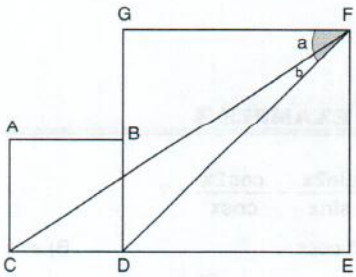
$= -\frac{3+\frac{3}{4}}{1-3\cdot\frac{3}{4}} = -\frac{15}{-5} = 3$



ÖRNEK 5

ACDB ve DEFG birer karedir.

$\tan a = \frac{2}{3}$ ⇒ **cotb = ?**



A) 5 B) 2 C) $\frac{1}{5}$ D) $\frac{1}{2}$ E) 1

ÇÖZÜM

Yanıt **A**

$\tan(a+b) = \tan 45° = 1$

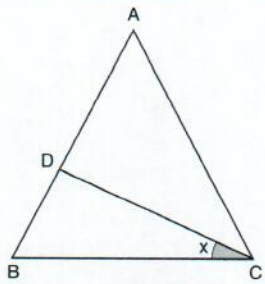
$\frac{\tan a+\tan b}{1-\tan a\cdot\tan b} = 1$

$\frac{\frac{2}{3}+\tan b}{1-\frac{2}{3}\tan b} = 1 \Rightarrow \frac{2}{3}+\tan b = 1-\frac{2}{3}\tan b$

$\frac{5}{3}\tan b = \frac{1}{3}$

$\cot b = 5$

Düşünün



ABC üçgeni, bir eşkenar üçgendir.
4·|BD| = |AB| ⇒ cotx = ?

Yanıt : $7/\sqrt{3}$

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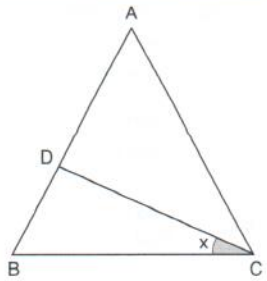
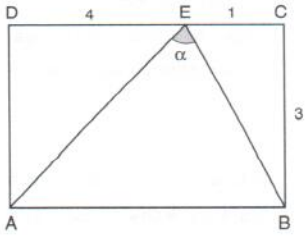
Think about it



ABC is an equilateral triangle.
$4 \cdot |IBDI| = |IABI| \Rightarrow \cot x = ?$

EXAMPLE 4

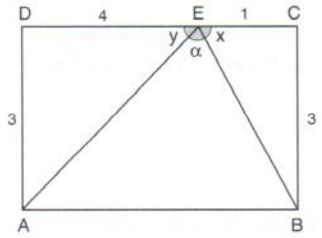
ABCD is a rectangle.
$|ICBI| = 3$ u,
$|IECI| = 1$ u,
$|IDEI| = 4$ u
$\Rightarrow \tan\alpha = ?$



A) 2 B) $\frac{5}{2}$ C) 3 D) $\frac{7}{2}$ E) 4

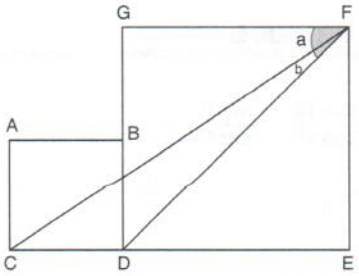
SOLUTION The answer is **C**

$$\tan\alpha = \tan(180^\circ - (x+y))$$
$$= -\tan(x+y)$$
$$= -\frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$
$$= -\frac{3 + \frac{3}{4}}{1 - 3 \cdot \frac{3}{4}} = -\frac{15}{-5} = 3$$



EXAMPLE 5

ACDB and DEFG are square.
$\tan a = \frac{2}{3} \Rightarrow \cot b = ?$



A) 5 B) 2 C) $\frac{1}{5}$ D) $\frac{1}{2}$ E) 1

SOLUTION The answer is **A**

$$\tan(a+b) = \tan 45^\circ = 1$$
$$\frac{\tan a + \tan b}{1 - \tan a \cdot \tan b} = 1$$
$$\frac{\frac{2}{3} + \tan b}{1 - \frac{2}{3}\tan b} = 1 \Rightarrow \frac{2}{3} + \tan b = 1 - \frac{2}{3}\tan b$$
$$\frac{5}{3}\tan b = \frac{1}{3}$$
$$\cot b = 5$$

Answer : $7/\sqrt{3}$

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YARIM AÇI FORMÜLLERİ

i. 1. toplam fark formulünde a = b alınırsa,

$\sin(a + a) = \sin a \cdot \cos a + \cos a \cdot \sin a$

$\sin 2a = 2\sin a \cdot \cos a$ elde edilir.

ii. 3. toplam fark formulünde a = b alınırsa,

$\cos(a + a) = \cos a \cdot \cos a - \sin a \cdot \sin a = \cos^2 a - \sin^2 a$ elde edilir.

$\cos^2 a = 1 - \sin^2 a$ olduğundan, $\cos 2a = 1 - \sin^2 a - \sin^2 a = 1 - 2\sin^2 a$ elde edilir veya

$\sin^2 a = 1 - \cos^2 a$ olduğundan, $\cos 2a = \cos^2 a - (1 - \cos^2 a) = 2\cos^2 a - 1$ elde edilir.

iii. 5. toplam fark formulünde a = b alınırsa,

$$\tan(a+a) = \frac{\tan a + \tan a}{1 - \tan a \cdot \tan a} = \frac{2\tan a}{1 - \tan^2 a}$$

iv. 7. toplam fark formulünde a = b alınırsa,

$$\cot(a+a) = \frac{\cot a \cdot \cot a - 1}{\cot a + \cot a} = \frac{\cot^2 a - 1}{2\cot a}$$

ÖZET

- ✓ $\sin 2a = 2\sin a \cdot \cos a$
- ✓ $\cos 2a = \cos^2 a - \sin^2 a$
  $= 2\cos^2 a - 1$
  $= 1 - 2\sin^2 a$
- ✓ $\tan 2a = \dfrac{2\tan a}{1 - \tan^2 a}$
- ✓ $\cot 2a = \dfrac{\cot^2 a - 1}{2\cot a}$

ÖRNEK 6

$\dfrac{\cos 15^\circ}{\cos 45^\circ} - \dfrac{\sin 15^\circ}{\sin 45^\circ} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

ÇÖZÜM

Yanıt C

$$\frac{\cos 15^\circ}{\cos 45^\circ} - \frac{\sin 15^\circ}{\sin 45^\circ} = \frac{\sin 45^\circ \cdot \cos 15^\circ - \cos 45^\circ \cdot \sin 15^\circ}{\sin 45^\circ \cdot \cos 45^\circ}$$

$$= \frac{\sin(45^\circ - 15^\circ)}{\frac{1}{2}\sin 90^\circ}$$

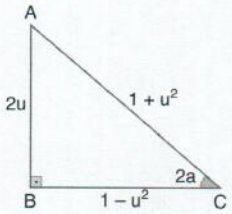
$$= \frac{\sin 30^\circ}{\frac{1}{2} \cdot 1}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

KANIT

$$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$$

$\tan a = u \Rightarrow \tan 2a = \dfrac{2u}{1-u^2}$



$$\Rightarrow \sin 2a = \frac{2\tan a}{1 + \tan^2 a}$$

$$\cos 2a = \frac{1 - \tan^2 a}{1 + \tan^2 a}$$

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SUMMARY

✓ $\sin 2a = 2\sin a \cdot \cos a$

✓ $\cos 2a = \cos^2 a - \sin^2 a$
$= 2\cos^2 a - 1$
$= 1 - 2\sin^2 a$

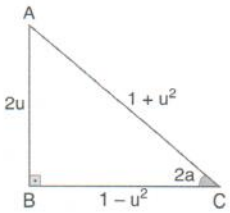
✓ $\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

✓ $\cot 2a = \dfrac{\cot^2 a - 1}{2\cot a}$

PROOF

$\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

$\tan a = u \Rightarrow \tan 2a = \dfrac{2u}{1-u^2}$



$\Rightarrow \sin 2a = \dfrac{2\tan a}{1+\tan^2 a}$

$\cos 2a = \dfrac{1-\tan^2 a}{1+\tan^2 a}$

HALF ANGLE FORMULAS

i. If we take a = b in the sum and difference formula – 1, we get
$\sin(a + a) = \sin a \cdot \cos a + \cos a \cdot \sin a$
$\sin 2a = 2\sin a \cdot \cos a$.

ii. If we take a = b in the sum and difference formula – 3, we get
$\cos(a + a) = \cos a \cdot \cos a - \sin a \cdot \sin a = \cos^2 a - \sin^2 a$.
Since $\cos^2 a = 1 - \sin^2 a$ we get $\cos 2a = 1 - \sin^2 a - \sin^2 a = 1 - 2\sin^2 a$ or
since $\sin^2 a = 1 - \cos^2 a$, we get $\cos 2a = \cos^2 a - (1 - \cos^2 a) = 2\cos^2 a - 1$.

iii. If we take a = b in the sum and difference formula – 5, we get

$$\tan(a+a) = \frac{\tan a + \tan a}{1 - \tan a \cdot \tan a} = \frac{2\tan a}{1-\tan^2 a}$$

iv. If we take a = b in the sum and difference formula – 7, we get

$$\cot(a+a) = \frac{\cot a \cdot \cot a - 1}{\cot a + \cot a} = \frac{\cot^2 a - 1}{2\cot a}$$

EXAMPLE 6

$\dfrac{\cos 15^\circ}{\cos 45^\circ} - \dfrac{\sin 15^\circ}{\sin 45^\circ} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

SOLUTION

The answer is C

$$\frac{\cos 15^\circ}{\cos 45^\circ} - \frac{\sin 15^\circ}{\sin 45^\circ} = \frac{\sin 45^\circ \cdot \cos 15^\circ - \cos 45^\circ \cdot \sin 15^\circ}{\sin 45^\circ \cdot \cos 45^\circ}$$

$$= \frac{\sin(45^\circ - 15^\circ)}{\frac{1}{2}\sin 90^\circ}$$

$$= \frac{\sin 30^\circ}{\frac{1}{2}\cdot 1}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

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ÖRNEK 7

$$\frac{\cot^2 22{,}5^\circ - 1}{2\cos 22{,}5^\circ} = ?$$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}$ C) 1 D) $\sqrt{2}$ E) 2

ÇÖZÜM

Yanıt C

$$\frac{\cot^2 22{,}5^\circ - 1}{2\cos 22{,}5^\circ} = \cot 45^\circ = 1$$

PEKİŞTİRME

$(\cos 45^\circ - \sin 45^\circ)\cdot(\cos 45^\circ + \sin 45^\circ) = ?$

YANIT : 0

ÖRNEK 8

tan 15° – cot 15° = ?

A) $-2\sqrt{3}$ B) $-2\sqrt{2}$ C) $-\sqrt{3}$ D) $2\sqrt{2}$ E) $2\sqrt{3}$

ÇÖZÜM

Yanıt A

$$\tan 15^\circ - \cot 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ} - \frac{\cos 15^\circ}{\sin 15^\circ}$$

$$= \frac{\sin^2 15^\circ - \cos^2 15^\circ}{\sin 15^\circ \cdot \cos 15^\circ}$$

$$= -\frac{\cos 30^\circ}{\frac{1}{2}\sin 30^\circ}$$

$$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{4}}$$

$$= -2\sqrt{3}$$

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REINFORCE

$(\cos 45° - \sin 45°)\cdot(\cos 45° + \sin 45°) = ?$

ANSWER : 0

EXAMPLE 7

$$\frac{\cot^2 22{,}5° - 1}{2\cos 22{,}5°} = ?$$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}$ C) 1 D) $\sqrt{2}$ E) 2

SOLUTION

The answer is C

$$\frac{\cot^2 22{,}5° - 1}{2\cos 22{,}5°} = \cot 45° = 1$$

EXAMPLE 8

$\tan 15° - \cot 15° = ?$

A) $-2\sqrt{3}$ B) $-2\sqrt{2}$ C) $-\sqrt{3}$ D) $2\sqrt{2}$ E) $2\sqrt{3}$

SOLUTION

The answer is A

$$\tan 15° - \cot 15° = \frac{\sin 15°}{\cos 15°} - \frac{\cos 15°}{\sin 15°}$$

$$= \frac{\sin^2 15° - \cos^2 15°}{\sin 15°\cdot\cos 15°}$$

$$= -\frac{\cos 30°}{\frac{1}{2}\sin 30°}$$

$$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{4}}$$

$$= -2\sqrt{3}$$

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ÖRNEK 9

$$\frac{\sin 4x+\sin 2x}{2\cos^2 2x-2\sin^2 x+1}=?$$

A) cot2x B) tan2x C) tan2x + 1 B) sec2x E) cosecx + 1

ÇÖZÜM

Yanıt **B**

$$\frac{\sin 4x+\sin 2x}{2\cos^2 2x-2\sin^2 x+1}=\frac{2\sin 2x\cdot\cos 2x+\sin 2x}{2\cos^2 2x+(1-2\sin^2 x)}$$

$$=\frac{\sin 2x\cdot(2\cos 2x+1)}{2\cos^2 2x+\cos 2x}$$

$$=\frac{\sin 2x\cdot(2\cos 2x+1)}{\cos 2x\cdot(2\cos 2x+1)}$$

$$=\frac{\sin 2x}{\cos 2x}=\tan 2x$$

ÖRNEK 10

$$\frac{2\tan 15^\circ}{1+\tan^2 15^\circ}=?$$

A) 1 B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{4}$ E) $\frac{\sqrt{3}}{4}$

ÇÖZÜM

Yanıt **B**

$$\frac{2\cdot\tan 15^\circ}{1+\tan^2 15^\circ}=\frac{2\cdot\frac{\sin 15^\circ}{\cos 15^\circ}}{1+\frac{\sin^2 15^\circ}{\cos^2 15^\circ}}$$

$$=\frac{2\cdot\frac{\sin 15^\circ}{\cos 15^\circ}}{\frac{\cos^2 15^\circ+\sin^2 15^\circ}{\cos^2 15^\circ}}$$

$$=\frac{2\sin 15^\circ}{\cos 15^\circ}\cdot\cos^2 15^\circ$$

$$=2\sin 15^\circ\cdot\cos 15^\circ$$

$$=\sin 30^\circ=\frac{1}{2}$$

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EXAMPLE 9

$$\frac{\sin 4x+\sin 2x}{2\cos^2 2x-2\sin^2 x+1}=?$$

A) cot2x B) tan2x C) tan2x + 1 B) sec2x E) cosecx + 1

SOLUTION

The answer is **B**

$$\frac{\sin 4x+\sin 2x}{2\cos^2 2x-2\sin^2 x+1}=\frac{2\sin 2x\cdot\cos 2x+\sin 2x}{2\cos^2 2x+(1-2\sin^2 x)}$$

$$=\frac{\sin 2x\cdot(2\cos 2x+1)}{2\cos^2 2x+\cos 2x}$$

$$=\frac{\sin 2x\cdot(2\cos 2x+1)}{\cos 2x\cdot(2\cos 2x+1)}$$

$$=\frac{\sin 2x}{\cos 2x}=\tan 2x$$

EXAMPLE 10

$$\frac{2\tan 15^\circ}{1+\tan^2 15^\circ}=?$$

A) 1 B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{4}$ E) $\frac{\sqrt{3}}{4}$

SOLUTION

The answer is **B**

$$\frac{2\cdot\tan 15^\circ}{1+\tan^2 15^\circ}=\frac{2\cdot\frac{\sin 15^\circ}{\cos 15^\circ}}{1+\frac{\sin^2 15^\circ}{\cos^2 15^\circ}}$$

$$=\frac{2\cdot\frac{\sin 15^\circ}{\cos 15^\circ}}{\frac{\cos^2 15^\circ+\sin^2 15^\circ}{\cos^2 15^\circ}}$$

$$=\frac{2\sin 15^\circ}{\cos 15^\circ}\cdot\cos^2 15^\circ$$

$$=2\sin 15^\circ\cdot\cos 15^\circ$$

$$=\sin 30^\circ=\frac{1}{2}$$

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ÖRNEK 11

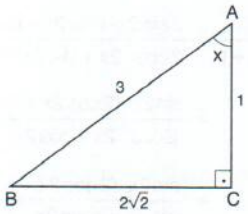
$\cos x = \frac{1}{3}$ ve $x \in (0, \frac{\pi}{2}) \Rightarrow \tan 2x = ?$

A) $\frac{3\sqrt{2}}{5}$ B) $\frac{4\sqrt{2}}{7}$ C) $\frac{6\sqrt{2}}{7}$ D) $\frac{-4\sqrt{2}}{7}$ E) $\frac{5}{2}$

ÇÖZÜM

Yanıt **D**

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2 \cdot 2\sqrt{2}}{1-\left(2\sqrt{2}\right)^2} = \frac{4\sqrt{2}}{1-8} = \frac{-4\sqrt{2}}{7}$$



ÖRNEK 12

$192\sqrt{3} \cdot \sin\frac{\pi}{48} \cdot \cos\frac{\pi}{48} \cdot \cos\frac{\pi}{24} \cdot \cos\frac{\pi}{12} \cdot \cos\frac{\pi}{6} \cdot \cos\frac{\pi}{3} = ?$

A) 1 B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) 2 E) 9

ÇÖZÜM

Yanıt **E**

$$192\sqrt{3} \underbrace{\sin\frac{\pi}{48} \cdot \cos\frac{\pi}{48}}_{\frac{\sin\frac{\pi}{24}}{2}} \cdot \cos\frac{\pi}{24} \cdot \cos\frac{\pi}{12} \cdot \cos\frac{\pi}{6} \cdot \cos\frac{\pi}{3}$$

$$= 96\sqrt{3} \underbrace{\sin\frac{\pi}{24} \cdot \cos\frac{\pi}{24}}_{\frac{\sin\frac{\pi}{12}}{2}} \cdot \cos\frac{\pi}{12} \cdot \cos\frac{\pi}{6} \cdot \cos\frac{\pi}{3}$$

$$= 48\sqrt{3} \sin\frac{\pi}{12}\cos\frac{\pi}{12}\cos\frac{\pi}{6}\cos\frac{\pi}{2} = 24\sqrt{3}\sin\frac{\pi}{6}\cos\frac{\pi}{6}\cos\frac{\pi}{3}$$

$$= 12\sqrt{3}\sin\frac{\pi}{3}\cos\frac{\pi}{3}$$

$$= 6\sqrt{3} \cdot \sin\frac{2\pi}{3} = 6\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 9$$

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EXAMPLE 11

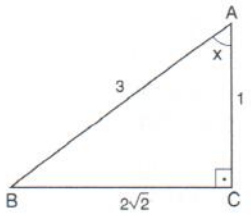
$\cos x = \frac{1}{3}$ and $x \in (0, \frac{\pi}{2}) \Rightarrow \tan 2x = ?$

A) $\frac{3\sqrt{2}}{5}$ B) $\frac{4\sqrt{2}}{7}$ C) $\frac{6\sqrt{2}}{7}$ D) $\frac{-4\sqrt{2}}{7}$ E) $\frac{5}{2}$

SOLUTION

The answer is D

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot 2\sqrt{2}}{1-\left(2\sqrt{2}\right)^2} = \frac{4\sqrt{2}}{1-8} = \frac{-4\sqrt{2}}{7}$$



EXAMPLE 12

$192\sqrt{3} \cdot \sin\frac{\pi}{48} \cdot \cos\frac{\pi}{48} \cdot \cos\frac{\pi}{24} \cdot \cos\frac{\pi}{12} \cdot \cos\frac{\pi}{6} \cdot \cos\frac{\pi}{3} = ?$

A) 1 B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) 2 E) 9

SOLUTION

The answer is E

$$192\sqrt{3}\underbrace{\sin\frac{\pi}{48}\cdot\cos\frac{\pi}{48}}_{\frac{\sin\frac{\pi}{24}}{2}}\cdot\cos\frac{\pi}{24}\cdot\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6}\cdot\cos\frac{\pi}{3}$$

$$= 96\sqrt{3}\underbrace{\sin\frac{\pi}{24}\cdot\cos\frac{\pi}{24}}_{\frac{\sin\frac{\pi}{12}}{2}}\cdot\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6}\cdot\cos\frac{\pi}{3}$$

$$= 48\sqrt{3}\sin\frac{\pi}{12}\cos\frac{\pi}{12}\cos\frac{\pi}{6}\cos\frac{\pi}{2} = 24\sqrt{3}\sin\frac{\pi}{6}\cos\frac{\pi}{6}\cos\frac{\pi}{3}$$

$$= 12\sqrt{3}\sin\frac{\pi}{3}\cos\frac{\pi}{3}$$

$$= 6\sqrt{3}\cdot\sin\frac{2\pi}{3} = 6\sqrt{3}\cdot\frac{\sqrt{3}}{2} = 9$$

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DÖNÜŞÜM FORMÜLLERİ

I. $\sin a+\sin b=2\cdot\sin\left(\frac{a+b}{2}\right)\cdot\cos\left(\frac{a-b}{2}\right)$

II. $\sin a-\sin b=2\cdot\sin\left(\frac{a-b}{2}\right)\cdot\cos\left(\frac{a+b}{2}\right)$

III. $\cos a+\cos b=2\cdot\cos\left(\frac{a-b}{2}\right)\cdot\cos\left(\frac{a+b}{2}\right)$

IV. $\cos a-\cos b=-2\cdot\sin\left(\frac{a+b}{2}\right)\cdot\sin\left(\frac{a-b}{2}\right)$

V. $\tan a+\tan b=\frac{\sin(a+b)}{\cos a\cdot\cos b}$

VI. $\tan a-\tan b=\frac{\sin(a-b)}{\cos a\cdot\cos b}$

VII. $\cot a+\cot b=\frac{\sin(a+b)}{\sin a\cdot\sin b}$

VIII. $\cot a-\cot b=\frac{-\sin(a-b)}{\sin a\cdot\sin b}$

KANIT

$\sin(x+y)=\sin x\cos y+\sin y\cos y$

$+\quad \sin(x-y)=\sin x\cos y-\sin y\cos y$

$\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$x+y=a$

$x-y=b$

$x=\frac{a+b}{2}$, $y=\frac{a-b}{2}$

$\sin a+\sin b=2\sin\frac{a+b}{2}\cdot\cos\frac{a-b}{2}$

ÖRNEK 13

$\frac{1}{\sin 15^\circ}+\frac{1}{\cos 15^\circ}=?$

A) $2\sqrt{3}$ B) $4\sqrt{2}$ C) 4 D) $2\sqrt{6}$ E) $4\sqrt{3}$

ÇÖZÜM

Yanıt **D**

$$\frac{1}{\sin 15^\circ}+\frac{1}{\cos 15^\circ}=\frac{\cos 15^\circ+\sin 15^\circ}{\sin 15^\circ\cdot\cos 15^\circ}$$

$$=\frac{\sin 75^\circ+\sin 15^\circ}{\frac{1}{2}\sin 30^\circ}$$

$$=\frac{2\sin 45^\circ\cdot\cos 30^\circ}{\frac{1}{4}}=8\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=2\sqrt{6}$$

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PROOF

$\sin(x+y) = \sin x\cos y + \sin y\cos y$

$+\quad \sin(x-y) = \sin x\cos y - \sin y\cos y$

$\sin(x+y) + \sin(x-y) = 2\sin x\cos y$

$x + y = a$

$x - y = b$

$x = \frac{a+b}{2}$, $y = \frac{a-b}{2}$

$\sin a + \sin b = 2\sin\frac{a+b}{2}\cdot\cos\frac{a-b}{2}$

CONVERSION FORMULAS

I. $\sin a + \sin b = 2\cdot\sin\left(\frac{a+b}{2}\right)\cdot\cos\left(\frac{a-b}{2}\right)$

II. $\sin a - \sin b = 2\cdot\sin\left(\frac{a-b}{2}\right)\cdot\cos\left(\frac{a+b}{2}\right)$

III. $\cos a + \cos b = 2\cdot\cos\left(\frac{a-b}{2}\right)\cdot\cos\left(\frac{a+b}{2}\right)$

IV. $\cos a - \cos b = -2\cdot\sin\left(\frac{a+b}{2}\right)\cdot\sin\left(\frac{a-b}{2}\right)$

V. $\tan a + \tan b = \frac{\sin(a+b)}{\cos a\cdot\cos b}$

VI. $\tan a - \tan b = \frac{\sin(a-b)}{\cos a\cdot\cos b}$

VII. $\cot a + \cot b = \frac{\sin(a+b)}{\sin a\cdot\sin b}$

VIII. $\cot a - \cot b = \frac{-\sin(a-b)}{\sin a\cdot\sin b}$

EXAMPLE 13

$\frac{1}{\sin 15^\circ} + \frac{1}{\cos 15^\circ} = ?$

A) $2\sqrt{3}$ B) $4\sqrt{2}$ C) 4 D) $2\sqrt{6}$ E) $4\sqrt{3}$

SOLUTION

The answer is D

$$\frac{1}{\sin 15^\circ} + \frac{1}{\cos 15^\circ} = \frac{\cos 15^\circ + \sin 15^\circ}{\sin 15^\circ\cdot\cos 15^\circ}$$

$$= \frac{\sin 75^\circ + \sin 15^\circ}{\frac{1}{2}\sin 30^\circ}$$

$$= \frac{2\sin 45^\circ\cdot\cos 30^\circ}{\frac{1}{4}} = 8\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = 2\sqrt{6}$$

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ÖRNEK 14

$$\frac{\sin 83^\circ - \sin 37^\circ}{\sin 68^\circ - \sin 22^\circ} = ?$$

A) $\frac{1}{2}$ B) $\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{1}{4}$ E) $\frac{\sqrt{3}}{4}$

ÇÖZÜM

Yanıt C

$$\frac{\sin 83^\circ - \sin 37^\circ}{\sin 68^\circ - \sin 22^\circ} = \frac{2.\sin\frac{83^\circ - 37^\circ}{2} \cdot \cos\frac{83^\circ + 37^\circ}{2}}{2\sin\frac{68^\circ - 22^\circ}{2} \cdot \cos\frac{68^\circ + 22^\circ}{2}}$$

$$= \frac{\sin 23^\circ \cdot \cos 60^\circ}{\sin 23^\circ \cdot \cos 45^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

ÖRNEK 15

$$\frac{\cos 79^\circ + \cos 41^\circ}{\cos 49^\circ + \cos 11^\circ} = ?$$

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $2\sqrt{3}$ D) 2 E) $\frac{\sqrt{3}}{3}$

ÇÖZÜM

Yanıt E

$$\frac{\cos 79^\circ + \cos 41^\circ}{\cos 49^\circ + \cos 11^\circ} = \frac{2\cos\frac{79^\circ + 41^\circ}{2} \cdot \cos\frac{79^\circ - 41^\circ}{2}}{2\cos\frac{49^\circ + 11^\circ}{2} \cdot \cos\frac{49^\circ - 11^\circ}{2}}$$

$$= \frac{\cos 60^\circ \cdot \cos 19^\circ}{\cos 30^\circ \cdot \cos 19^\circ}$$

$$= \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

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EXAMPLE 14

$$\frac{\sin 83^\circ - \sin 37^\circ}{\sin 68^\circ - \sin 22^\circ} = ?$$

A) $\frac{1}{2}$ B) $\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{1}{4}$ E) $\frac{\sqrt{3}}{4}$

SOLUTION

The answer is C

$$\frac{\sin 83^\circ - \sin 37^\circ}{\sin 68^\circ - \sin 22^\circ} = \frac{2.\sin\frac{83^\circ - 37^\circ}{2} \cdot \cos\frac{83^\circ + 37^\circ}{2}}{2\sin\frac{68^\circ - 22^\circ}{2} \cdot \cos\frac{68^\circ + 22^\circ}{2}}$$

$$= \frac{\sin 23^\circ \cdot \cos 60^\circ}{\sin 23^\circ \cdot \cos 45^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

EXAMPLE 15

$$\frac{\cos 79^\circ + \cos 41^\circ}{\cos 49^\circ + \cos 11^\circ} = ?$$

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $2\sqrt{3}$ D) 2 E) $\frac{\sqrt{3}}{3}$

SOLUTION

The answer is E

$$\frac{\cos 79^\circ + \cos 41^\circ}{\cos 49^\circ + \cos 11^\circ} = \frac{2\cos\frac{79^\circ + 41^\circ}{2} \cdot \cos\frac{79^\circ - 41^\circ}{2}}{2\cos\frac{49^\circ + 11^\circ}{2} \cdot \cos\frac{49^\circ - 11^\circ}{2}}$$

$$= \frac{\cos 60^\circ \cdot \cos 19^\circ}{\cos 30^\circ \cdot \cos 19^\circ}$$

$$= \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

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ÖRNEK 16

$$\frac{\sin x - \sin 3x + \sin 5x}{\cos x - \cos 3x + \cos 5x} = ?$$

A) sin2x B) cos2x C) cot4x D) cot3x E) tan3x

NOT

$\frac{x+y}{2} = z \Rightarrow$

$$\frac{\sin x + \sin z + \sin y}{\cos x + \cos z + \cos y} = \tan z$$

ÇÖZÜM

Yanıt **E**

Çözüm 1

$$\frac{\sin x - \sin 3x + \sin 5x}{\cos x - \cos 3x + \cos 5x} = \frac{\sin 5x + \sin x - \sin 3x}{\cos 5x + \cos x - \cos 3x}$$

$$= \frac{2\sin 3x \cdot \cos 2x - \sin 3x}{2\cos 3x \cdot \cos 2x - \cos 3x}$$

$$= \frac{\sin 3x(2\cos 2x - 1)}{\cos 3x(2\cos 2x - 1)}$$

$$= \frac{\sin 3x}{\cos 3x} = \tan 3x$$

Çözüm 2

$$\frac{x+5x}{2} = 3x \Rightarrow \frac{\sin x - \sin 3x + \sin 5x}{\cos x - \cos 3x + \cos 5x} = \frac{\sin 3x}{\cos 3x} = \tan 3x$$

ÖRNEK 17

$$\frac{\sin x - \sin y}{\cos x + \cos y} = ?$$

A) $\sin\frac{x-y}{2}$ B) $\cos\frac{x-y}{2}$ C) $\tan\frac{x-y}{2}$

D) $\cot\frac{x-y}{2}$ E) $\tan\frac{x+y}{2}$

ÇÖZÜM

Yanıt **C**

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \frac{2\sin\frac{x-y}{2} \cdot \cos\frac{x+y}{2}}{2\cos\frac{x-y}{2} \cdot \cos\frac{x+y}{2}}$$

$$= \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} = \tan\frac{x-y}{2}$$

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NOTE

$\frac{x+y}{2}=z \Rightarrow$

$\frac{\sin x+\sin z+\sin y}{\cos x+\cos z+\cos y}=\tan z$

EXAMPLE 16

$$\frac{\sin x-\sin 3x+\sin 5x}{\cos x-\cos 3x+\cos 5x}=?$$

A) sin2x B) cos2x C) cot4x D) cot3x E) tan3x

SOLUTION

The answer is E

Solution 1

$$\frac{\sin x-\sin 3x+\sin 5x}{\cos x-\cos 3x+\cos 5x}=\frac{\sin 5x+\sin x-\sin 3x}{\cos 5x+\cos x-\cos 3x}$$

$$=\frac{2\sin 3x\cdot\cos 2x-\sin 3x}{2\cos 3x\cdot\cos 2x-\cos 3x}$$

$$=\frac{\sin 3x(2\cos 2x-1)}{\cos 3x(2\cos 2x-1)}$$

$$=\frac{\sin 3x}{\cos 3x}=\tan 3x$$

Solution 2

$$\frac{x+5x}{2}=3x \Rightarrow \frac{\sin x-\sin 3x+\sin 5x}{\cos x-\cos 3x+\cos 5x}=\frac{\sin 3x}{\cos 3x}=\tan 3x$$

EXAMPLE 17

$$\frac{\sin x-\sin y}{\cos x+\cos y}=?$$

A) $\sin\frac{x-y}{2}$ B) $\cos\frac{x-y}{2}$ C) $\tan\frac{x-y}{2}$

D) $\cot\frac{x-y}{2}$ E) $\tan\frac{x+y}{2}$

SOLUTION

The answer is C

$$\frac{\sin x-\sin y}{\cos x+\cos y}=\frac{2\sin\frac{x-y}{2}\cdot\cos\frac{x+y}{2}}{2\cos\frac{x-y}{2}\cdot\cos\frac{x+y}{2}}$$

$$=\frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}}=\tan\frac{x-y}{2}$$

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TERS DÖNÜŞÜM FORMÜLLERİ

I. $\sin a \cdot \sin b = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$

II. $\cos a \cdot \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

III. $\sin a \cdot \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

IV. $\cos a \cdot \sin b = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$

KANIT

$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

$+ \quad \cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$

$\cos(a+b) + \cos(a-b) = 2\cos a \cdot \cos b$

$\cos a \cdot \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$

ÖRNEK 18

$\frac{\cos 55^\circ \cdot \cos 35^\circ}{\sin 70^\circ} = ?$

A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

ÇÖZÜM

Yanıt **E**

$$\frac{\cos 55^\circ \cdot \cos 35^\circ}{\sin 70^\circ} = \frac{\frac{1}{2}(\cos 90^\circ + \cos 20^\circ)}{\cos 20^\circ}$$

$$= \frac{1}{2}$$

ÖRNEK 19

$2 \cdot \cos 50^\circ \cdot \cos 70^\circ + \cos 160^\circ = ?$

A) 1 B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) -1

ÇÖZÜM

Yanıt **C**

$$2 \cdot \cos 50^\circ \cdot \cos 70^\circ + \cos 160^\circ = 2 \cdot \frac{1}{2}[\cos(50^\circ + 70^\circ) + \cos(50^\circ - 70^\circ)] + \cos 160^\circ$$

$$= \cos 120^\circ + \cos 20^\circ - \cos 20^\circ$$

$$= \cos 120^\circ = -\frac{1}{2}$$

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PROOF

$\cos(a+b) = \cos a\cdot\cos b - \sin a\cdot\sin b$

$+\quad \cos(a-b) = \cos a\cdot\cos b + \sin a\cdot\sin b$

$\cos(a+b) + \cos(a-b) = 2\cos a\cdot\cos b$

$\cos a\cdot\cos b = \frac{1}{2}(\cos(a+b)+\cos(a-b))$

INVERSE CONVERSION FORMULAS

I. $\sin a\cdot\sin b = -\frac{1}{2}[\cos(a+b)-\cos(a-b)]$

II. $\cos a\cdot\cos b = \frac{1}{2}[\cos(a+b)+\cos(a-b)]$

III. $\sin a\cdot\cos b = \frac{1}{2}[\sin(a+b)+\sin(a-b)]$

IV. $\cos a\cdot\sin b = \frac{1}{2}[\sin(a+b)-\sin(a-b)]$

EXAMPLE 18

$\frac{\cos 55^\circ\cdot\cos 35^\circ}{\sin 70^\circ} = ?$

A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

SOLUTION

The answer is **E**

$$\frac{\cos 55^\circ\cdot\cos 35^\circ}{\sin 70^\circ} = \frac{\frac{1}{2}(\cos 90^\circ+\cos 20^\circ)}{\cos 20^\circ}$$

$$= \frac{1}{2}$$

EXAMPLE 19

$2\cdot\cos 50^\circ\cdot\cos 70^\circ + \cos 160^\circ = ?$

A) 1 B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) -1

SOLUTION

The answer is **C**

$$2\cdot\cos 50^\circ\cdot\cos 70^\circ+\cos 160^\circ = 2\cdot\frac{1}{2}[\cos(50^\circ+70^\circ)+\cos(50^\circ-70^\circ)]+\cos 160^\circ$$

$$= \cos 120^\circ+\cos 20^\circ-\cos 20^\circ$$

$$= \cos 120^\circ = -\frac{1}{2}$$

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TERS TRİGONOMETRİK FONKSİYONLAR

1. $y = \arcsin x = \sin^{-1}x \Leftrightarrow x = \sin y$
   $x \in [-1,1]$ ve $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. $y = \arccos x = \cos^{-1}x \Rightarrow x = \cos y$
   $x \in [-1,1]$ ve $y \in [0, \pi]$

3. $y = \arctan x = \tan^{-1}x \Leftrightarrow x = \tan y$
   $x \in R$ ve $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

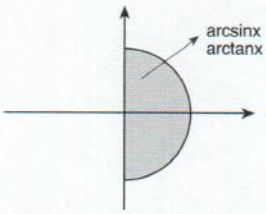
4. $y = \operatorname{arccot} x = \cot^{-1}x \Rightarrow x = \cot y$
   $x \in R$ ve $y \in [0, \pi]$

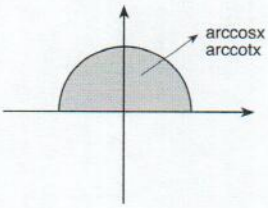
NOT

✓ I. $\arcsin(\sin x) = x$
II. $\arccos(\cos x) = x$
III. $\arctan(\tan x) = x$
IV. $\operatorname{arccot}(\cot x) = x$

✓ I. $\sin(\arcsin x) = x$
II. $\cos(\arccos x) = x$
III. $\tan(\arctan x) = x$
IV. $\cot(\operatorname{arccot} x) = x$

NOT





ÖRNEK 20

$x = \arccos\left(\frac{1}{2}\right) \Rightarrow x = ?$

A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{5\pi}{6}$ D) $\frac{4\pi}{3}$ E) $\frac{5\pi}{3}$

ÇÖZÜM

Yanıt **B**

$x = \arccos\left(\frac{1}{2}\right) \Rightarrow \cos x = \frac{1}{2}$

$x \in [0,\pi] \Rightarrow x = \frac{\pi}{3}$

ÖRNEK 21

$y = \arcsin\left(-\frac{1}{2}\right) \Rightarrow y = ?$

A) $\frac{\pi}{6}$ B) $\frac{7\pi}{6}$ C) $\frac{5\pi}{6}$ D) $\frac{4\pi}{3}$ E) $\frac{11\pi}{6}$

ÇÖZÜM

Yanıt **E**

$y = \arcsin\left(-\frac{1}{2}\right) \Rightarrow \sin y = -\frac{1}{2}$

$y \in \left[\frac{3\pi}{2}, 2\pi\right] \Rightarrow y = 2\pi - \frac{\pi}{6}$

$y = \frac{11\pi}{6}$

NOT

Fonksiyon	Tanım kümesi	Görüntü kümesi
arcsinx	[–1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
arccosx	[–1, 1]	[0, π]
arctanx	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
arccotx	R	[0, π]

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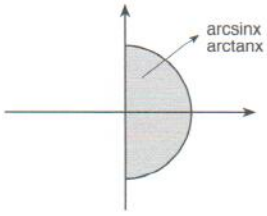
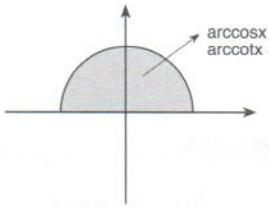
NOTE

✓ I. $\arcsin(\sin x) = x$
II. $\arccos(\cos x) = x$
III. $\arctan(\tan x) = x$
IV. $\operatorname{arccot}(\cot x) = x$

✓ I. $\sin(\arcsin x) = x$
II. $\cos(\arccos x) = x$
III. $\tan(\arctan x) = x$
IV. $\cot(\operatorname{arccot} x) = x$

NOTE





NOTE

Function	Domain	Range
arcsinx	[–1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
arccosx	[–1, 1]	[0, π]
arctanx	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
arccotx	R	[0, π]

INVERSE TRIGONOMETRIC FUNCTIONS

1. $y = \arcsin x = \sin^{-1} x \Leftrightarrow x = \sin y$
   $x \in [-1,1]$ and $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. $y = \arccos x = \cos^{-1} x \Rightarrow x = \cos y$
   $x \in [-1,1]$ and $y \in [0, \pi]$

3. $y = \arctan x = \tan^{-1} x \Leftrightarrow x = \tan y$
   $x \in R$ and $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. $y = \operatorname{arccot} x = \cot^{-1} x \Rightarrow x = \cot y$
   $x \in R$ and $y \in [0, \pi]$

EXAMPLE 20

$x = \arccos\left(\frac{1}{2}\right) \Rightarrow x = ?$

A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{5\pi}{6}$ D) $\frac{4\pi}{3}$ E) $\frac{5\pi}{3}$

SOLUTION

The answer is B

$x = \arccos\left(\frac{1}{2}\right) \Rightarrow \cos x = \frac{1}{2}$

$x \in [0,\pi] \Rightarrow x = \frac{\pi}{3}$

EXAMPLE 21

$y = \arcsin\left(-\frac{1}{2}\right) \Rightarrow y = ?$

A) $\frac{\pi}{6}$ B) $\frac{7\pi}{6}$ C) $\frac{5\pi}{6}$ D) $\frac{4\pi}{3}$ E) $\frac{11\pi}{6}$

SOLUTION

The answer is E

$y = \arcsin\left(-\frac{1}{2}\right) \Rightarrow \sin y = -\frac{1}{2}$

$y \in \left[\frac{3\pi}{2}, 2\pi\right] \Rightarrow y = 2\pi - \frac{\pi}{6}$

$y = \frac{11\pi}{6}$

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ÖRNEK 22

$A = \arccos\frac{\sqrt{2}}{\sqrt{3}} - \arccos\frac{\sqrt{6}+1}{2\sqrt{3}} \Rightarrow A = ?$

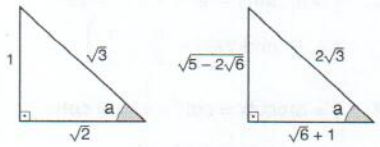
A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$ E) $\frac{5\pi}{6}$

ÇÖZÜM

Yanıt C

$a = \arccos\frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \cos a = \frac{\sqrt{2}}{\sqrt{3}}$

$b = \arccos\frac{\sqrt{6}+1}{2\sqrt{3}} \Rightarrow \cos b = \frac{\sqrt{6}+1}{2\sqrt{3}}$



$A = a - b \Rightarrow \cos A = \cos(a-b)$

$$\cos A = \cos a \cdot \cos b + \sin a \cdot \sin b = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{6}+1}{2\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{5-2\sqrt{6}}}{2\sqrt{3}}$$

$$= \frac{\sqrt{12}+\sqrt{2}}{6} + \frac{\sqrt{3}-\sqrt{2}}{6}$$

$$= \frac{2\sqrt{3}+\sqrt{3}}{6} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$\cos A = \frac{\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{6}$

ÖRNEK 23

$B = \arctan(3+2\sqrt{2}) - \arctan\frac{\sqrt{2}}{2} \Rightarrow B = ?$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{4}$ E) $\frac{7\pi}{4}$

ÇÖZÜM

Yanıt D

$a = \arctan(3+2\sqrt{2}) \Rightarrow \tan a = 3+2\sqrt{2}$

$b = \arctan\left(\frac{\sqrt{2}}{2}\right) \Rightarrow \tan b = \frac{\sqrt{2}}{2}$

$B = a - b$

$$\tan B = \frac{\tan a - \tan b}{1+\tan a \cdot \tan b} = \frac{3+2\sqrt{2}-\frac{\sqrt{2}}{2}}{1+(3+2\sqrt{2})\cdot\frac{\sqrt{2}}{2}} = \frac{\frac{6+4\sqrt{2}-\sqrt{2}}{2}}{\frac{2+3\sqrt{2}+4}{2}}$$

$$= \frac{6+3\sqrt{2}}{6+3\sqrt{2}} = 1$$

$\tan B = 1 \Rightarrow B = \frac{\pi}{4}$

PEKİŞTİRME

$3\arctan\left(\frac{x-1}{3}\right) = -\frac{\pi}{2} \Rightarrow x = ?$

YANIT : $1-\sqrt{3}$

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EXAMPLE 22

$A = \arccos\frac{\sqrt{2}}{\sqrt{3}} - \arccos\frac{\sqrt{6}+1}{2\sqrt{3}} \Rightarrow A = ?$

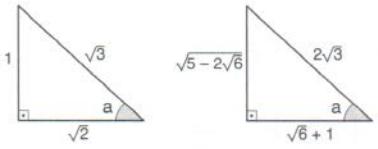
A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$ E) $\frac{5\pi}{6}$

SOLUTION

The answer is C

$a = \arccos\frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \cos a = \frac{\sqrt{2}}{\sqrt{3}}$

$b = \arccos\frac{\sqrt{6}+1}{2\sqrt{3}} \Rightarrow \cos b = \frac{\sqrt{6}+1}{2\sqrt{3}}$



$A = a - b \Rightarrow \cos A = \cos(a-b)$

$\cos A = \cos a \cdot \cos b + \sin a \cdot \sin b = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{6}+1}{2\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{5-2\sqrt{6}}}{2\sqrt{3}}$

$= \frac{\sqrt{12}+\sqrt{2}}{6} + \frac{\sqrt{3}-\sqrt{2}}{6}$

$= \frac{2\sqrt{3}+\sqrt{3}}{6} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\cos A = \frac{\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{6}$

REINFORCE

$3\arctan\left(\frac{x-1}{3}\right) = -\frac{\pi}{2} \Rightarrow x = ?$

ANSWER : $1 - \sqrt{3}$

EXAMPLE 23

$B = \arctan(3 + 2\sqrt{2}) - \arctan\frac{\sqrt{2}}{2} \Rightarrow B = ?$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{4}$ E) $\frac{7\pi}{4}$

SOLUTION

The answer is D

$a = \arctan(3+2\sqrt{2}) \Rightarrow \tan a = 3+2\sqrt{2}$

$b = \arctan\left(\frac{\sqrt{2}}{2}\right) \Rightarrow \tan b = \frac{\sqrt{2}}{2}$

$B = a - b$

$\tan B = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b} = \frac{3+2\sqrt{2} - \frac{\sqrt{2}}{2}}{1 + (3+2\sqrt{2}) \cdot \frac{\sqrt{2}}{2}} = \frac{\frac{6+4\sqrt{2}-\sqrt{2}}{2}}{\frac{2+3\sqrt{2}+4}{2}}$

$= \frac{6+3\sqrt{2}}{6+3\sqrt{2}} = 1$

$\tan B = 1 \Rightarrow B = \frac{\pi}{4}$

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ÖRNEK 24

$x = \text{arccot}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow x = ?$

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{4\pi}{3}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{6}$

ÇÖZÜM

Yanıt **D**

$x = \text{arccot}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \cot x = \frac{\sqrt{3}}{3}$

$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow x = \frac{\pi}{3}$

ÖRNEK 25

$\sin\left[\frac{1}{2}\text{arccot}\left(-\frac{3}{4}\right)\right] = ?$

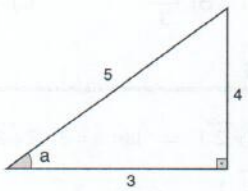
A) $\frac{1}{\sqrt{6}}$ B) $\frac{\sqrt{6}}{3}$ C) $\frac{\sqrt{3}}{4}$ D) $\frac{\sqrt{3}}{5}$ E) $\frac{2\sqrt{5}}{5}$

ÇÖZÜM

Yanıt **E**

$A = \sin\left[\frac{1}{2}\text{arccot}\left(-\frac{3}{4}\right)\right]$

$\text{arccot}\left(-\frac{3}{4}\right) = a$



$\cot a = -\frac{3}{4}$

$\cos a = -\frac{3}{5}$

$A = \sin\frac{a}{2} = \frac{\sqrt{1-\cos a}}{\sqrt{2}}$

$= \frac{\sqrt{1+\frac{3}{5}}}{\sqrt{2}} = \frac{\sqrt{\frac{8}{5}}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{\sqrt{5}\cdot\sqrt{2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

HATIRLATMA

$\cos 2a = 1 - 2\sin^2 a$

$\cos a = 1 - 2\sin^2\frac{a}{2}$

$\Rightarrow \sin\frac{a}{2} = \frac{\sqrt{1-\cos a}}{\sqrt{2}}$

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EXAMPLE 24

$x = \text{arccot}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow x = ?$

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{4\pi}{3}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{6}$

SOLUTION

The answer is D

$x = \text{arccot}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \cot x = \frac{\sqrt{3}}{3}$

$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow x = \frac{\pi}{3}$

REMINDER

$\cos 2a = 1 - 2\sin^2 a$

$\cos a = 1 - 2\sin^2 \frac{a}{2}$

$\Rightarrow \sin\frac{a}{2} = \frac{\sqrt{1-\cos a}}{\sqrt{2}}$

EXAMPLE 25

$\sin\left[\frac{1}{2}\text{arccot}\left(-\frac{3}{4}\right)\right] = ?$

A) $\frac{1}{\sqrt{6}}$ B) $\frac{\sqrt{6}}{3}$ C) $\frac{\sqrt{3}}{4}$ D) $\frac{\sqrt{3}}{5}$ E) $\frac{2\sqrt{5}}{5}$

SOLUTION

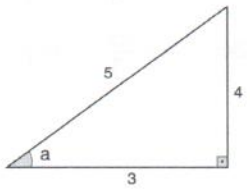
The answer is E

$A = \sin\left[\frac{1}{2}\text{arccot}\left(-\frac{3}{4}\right)\right]$

$\text{arccot}\left(-\frac{3}{4}\right) = a$

$\cot a = -\frac{3}{4}$

$\cos a = -\frac{3}{5}$



$A = \sin\frac{a}{2} = \frac{\sqrt{1-\cos a}}{\sqrt{2}}$

$= \frac{\sqrt{1+\frac{3}{5}}}{\sqrt{2}} = \frac{\sqrt{\frac{8}{5}}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{\sqrt{5}\cdot\sqrt{2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

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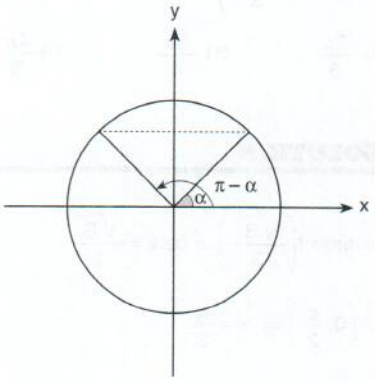
TRİGONOMETRİK DENKLEMLER

1. sinx = sinα Denkleminin Çözümü

$$\left.\begin{array}{l}\sin x = \sin\alpha \\ \sin x = \sin(\pi-\alpha)\end{array}\right\} \Rightarrow x_1 = \alpha \ , \ x_2 = \pi - \alpha$$

$$\sin x = \sin\alpha \Rightarrow \begin{cases} x_1 = \alpha + 2k\pi & , \ k \in Z \\ x_2 = (\pi-\alpha) + 2k\pi & , \ k \in Z \end{cases}$$



ÖRNEK 26

$\sin x = \frac{1}{2} \Rightarrow$ ÇK = ?

A) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

C) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

D) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

E) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{4\pi}{3} + 2k\pi\right\}$

ÇÖZÜM

Yanıt C

$$\sin x = \frac{1}{2} \Rightarrow \sin x = \sin\frac{\pi}{6}$$

$$\left.\begin{array}{l}\sin x = \sin\frac{\pi}{6} \\ \sin x = \sin\frac{5\pi}{6}\end{array}\right\} \Rightarrow x_1 = \frac{\pi}{6} + 2k\pi \ , \ x_2 = \frac{5\pi}{6} + 2k\pi$$

$$ÇK = \left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$$

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NOTE

$k \in Z$

✓ $\sin x = 1$

$\Rightarrow x = \frac{\pi}{2} + 2k\pi$

✓ $\sin x = -1$

$\Rightarrow x = \frac{3\pi}{2} + 2k\pi$

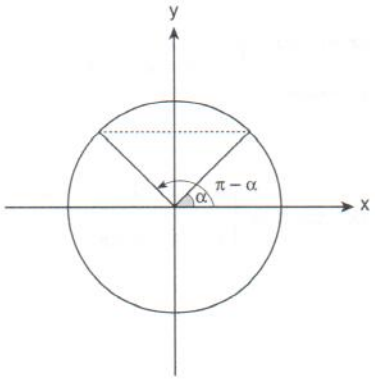
✓ $\sin x = 0 \Rightarrow x = k\pi$

TRIGONOMETRIC EQUATIONS

1. Solution of the Equation $\sin x = \sin\alpha$

$$\left.\begin{array}{l}\sin x = \sin\alpha \\ \sin x = \sin(\pi - \alpha)\end{array}\right\} \Rightarrow x_1 = \alpha \ , \ x_2 = \pi - \alpha$$

$$\sin x = \sin\alpha \Rightarrow \begin{cases} x_1 = \alpha + 2k\pi & , \ k \in Z \\ x_2 = (\pi - \alpha) + 2k\pi & , \ k \in Z \end{cases}$$



EXAMPLE 26

$\sin x = \frac{1}{2} \Rightarrow SS = ?$

A) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

C) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

D) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

E) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{4\pi}{3} + 2k\pi\right\}$

SOLUTION

The answer is **C**

$\sin x = \frac{1}{2} \Rightarrow \sin x = \sin\frac{\pi}{6}$

$$\left.\begin{array}{l}\sin x = \sin\frac{\pi}{6} \\ \sin x = \sin\frac{5\pi}{6}\end{array}\right\} \Rightarrow x_1 = \frac{\pi}{6} + 2k\pi \ , \ x_2 = \frac{5\pi}{6} + 2k\pi$$

$SS = \left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

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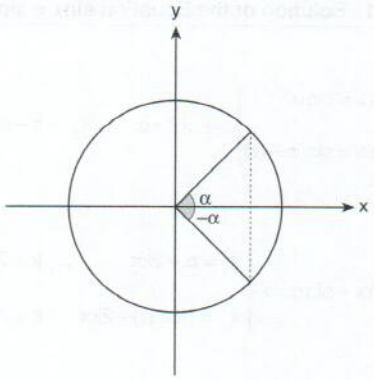
2. cosx = cosα Denkleminin Çözümü

$$\left.\begin{array}{l}\cos x = \cos\alpha \\ \cos x = \cos(-\alpha)\end{array}\right\} \Rightarrow x_1 = \alpha \ , \ x_2 = -\alpha$$

$$\cos x = \cos\alpha \Rightarrow \begin{cases} x_1 = \alpha + 2k\pi & , \ k \in Z \\ x_2 = -\alpha + 2k\pi & , \ k \in Z \end{cases}$$



NOT

$k \in Z$

✓ $\sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2k\pi$

✓ $\sin x = -1 \Rightarrow x = \frac{3\pi}{2} + 2k\pi$

✓ $\sin x = 0 \Rightarrow x = k\pi$

ÖRNEK 27

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow$ ÇK = ?

A) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{4\pi}{3} + 2k\pi\right\}$

C) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{5\pi}{3} + 2k\pi\right\}$

D) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{11\pi}{6} + 2k\pi\right\}$

E) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

ÇÖZÜM

Yanıt **D**

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow \cos x = \cos\frac{\pi}{6}$

$$\left.\begin{array}{l}\cos x = \cos\frac{\pi}{6} \\ \cos x = \cos\frac{11\pi}{6}\end{array}\right\} \Rightarrow x_1 = \frac{\pi}{6} + 2k\pi \ , \ x_2 = \frac{11\pi}{6} + 2k\pi$$

$ÇK = \left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{11\pi}{6} + 2k\pi\right\}$

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NOTE

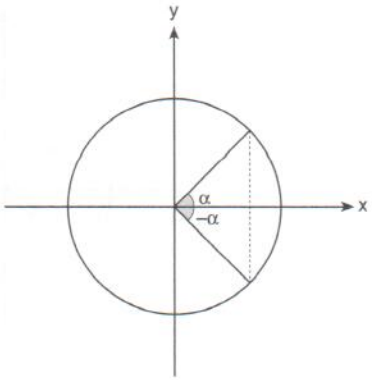
$k \in Z$

- ✓ $\cos x = 1 \Rightarrow x = 2k\pi$
- ✓ $\cos x = -1 \Rightarrow x = \pi + 2k\pi$
- ✓ $\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$

2. Solution of the Equation $\cos x = \cos\alpha$

$$\left.\begin{array}{l}\cos x = \cos\alpha \\ \cos x = \cos(-\alpha)\end{array}\right\} \Rightarrow x_1 = \alpha \ , \ x_2 = -\alpha$$

$$\cos x = \cos\alpha \Rightarrow \left\{\begin{array}{l} x_1 = \alpha + 2k\pi \ , \ k \in Z \\ x_2 = -\alpha + 2k\pi \ , \ k \in Z \end{array}\right.$$



EXAMPLE 27

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow SS = ?$

A) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{4\pi}{3} + 2k\pi\right\}$

C) $\left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{5\pi}{3} + 2k\pi\right\}$

D) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{11\pi}{6} + 2k\pi\right\}$

E) $\left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

SOLUTION

The answer is **D**

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow \cos x = \cos\frac{\pi}{6}$

$$\left.\begin{array}{l}\cos x = \cos\frac{\pi}{6} \\ \cos x = \cos\frac{11\pi}{6}\end{array}\right\} \Rightarrow x_1 = \frac{\pi}{6} + 2k\pi \ , \ x_2 = \frac{11\pi}{6} + 2k\pi$$

$SS = \left\{x: x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{11\pi}{6} + 2k\pi\right\}$

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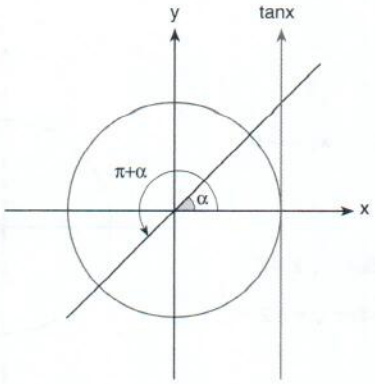
3. tanx = tanα Denkleminin Çözümü



tanx = tanα ⇒ x = α + kπ

NOT

k ∈ Z

- ✓ cosx = 1 ⇒ x = 2kπ
- ✓ cosx = −1 ⇒ x = π + 2kπ
- ✓ cosx = 0 ⇒ $x = \frac{\pi}{2} + k\pi$

ÖRNEK 28

tan(x − 30°) = 1 ⇒ **ÇK = ?**

A) $\left\{x: x = \frac{7\pi}{6} + k\pi \ , \ k \in Z\right\}$

B) $\left\{x: x = \frac{5\pi}{12} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

C) $\left\{x: x = \frac{5\pi}{12} + k\pi \ , \ k \in Z\right\}$

D) $\left\{x: x = \frac{5\pi}{24} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

E) $\left\{x: x = \frac{5\pi}{24} + k\pi \ , \ k \in Z\right\}$

ÇÖZÜM

Yanıt **C**

$\tan(x - 30°) = 1$

$\tan(x - 30°) = \tan 45°$

$x - 30° = 45° + k\pi$

$x - \frac{\pi}{6} = \frac{\pi}{4} + k\pi$

$x = \frac{\pi}{6} + \frac{\pi}{4} + k\pi$

$x = \frac{5\pi}{12} + k\pi$

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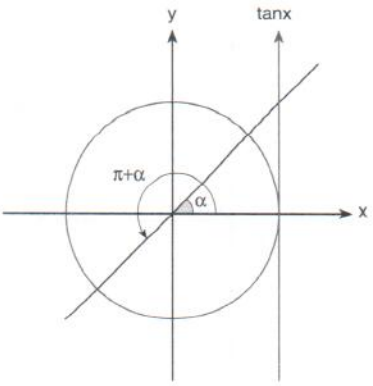
3. Solution of the Equation tanx = tanα



tanx = tanα ⇒ x = α + kπ

EXAMPLE 28

tan(x − 30°) = 1 ⇒ **SS = ?**

A) $\left\{x: x=\frac{7\pi}{6}+k\pi\ ,\ k\in Z\right\}$

B) $\left\{x: x=\frac{5\pi}{12}+\frac{k\pi}{2}\ ,\ k\in Z\right\}$

C) $\left\{x: x=\frac{5\pi}{12}+k\pi\ ,\ k\in Z\right\}$

D) $\left\{x: x=\frac{5\pi}{24}+\frac{k\pi}{2}\ ,\ k\in Z\right\}$

E) $\left\{x: x=\frac{5\pi}{24}+k\pi\ ,\ k\in Z\right\}$

SOLUTION

The answer is **C**

$\tan(x-30°)=1$

$\tan(x-30°)=\tan 45°$

$x-30°=45°+k\pi$

$x-\frac{\pi}{6}=\frac{\pi}{4}+k\pi$

$x=\frac{\pi}{6}+\frac{\pi}{4}+k\pi$

$x=\frac{5\pi}{12}+k\pi$

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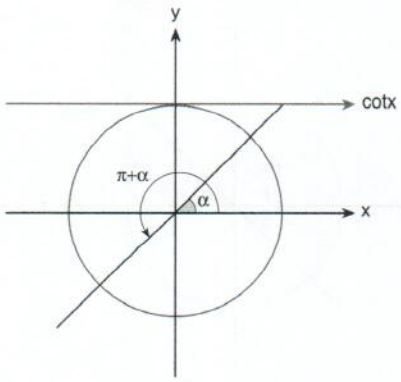
4. cotx = cotα Denkleminin Çözümü



cotx = cotα ⇒ x = α + kπ

ÖRNEK 29

cot(2x – 20°) = cotx ⇒ **ÇK = ?**

A) $\left\{x: x = \frac{\pi}{36} + k\pi\ ,\ k \in Z\right\}$

B) $\left\{x: x = \frac{\pi}{9} + k\pi\ ,\ k \in Z\right\}$

C) $\left\{x: x = \frac{\pi}{18} + k\pi\ ,\ k \in Z\right\}$

D) $\left\{x: x = \frac{\pi}{9} + \frac{k\pi}{2}\ ,\ k \in Z\right\}$

E) $\left\{x: x = \frac{\pi}{18} + \frac{k\pi}{4}\ ,\ k \in Z\right\}$

ÇÖZÜM

Yanıt **B**

cot(2x – 20°) = cotx

2x – 20° = x + 180°·k

x = 20° + 180°·k

⇒ $ÇK = \left\{x: x = \frac{\pi}{9} + k\pi\ ,\ k \in Z\right\}$

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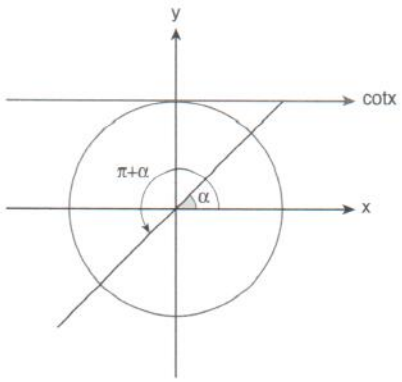
4. Solution of the Equation cotx = cotα



$\cot x = \cot\alpha \Rightarrow x = \alpha + k\pi$

EXAMPLE 29

$\cot(2x - 20°) = \cot x \Rightarrow$ **SS = ?**

A) $\left\{x: x = \frac{\pi}{36} + k\pi\ ,\ k \in Z\right\}$

B) $\left\{x: x = \frac{\pi}{9} + k\pi\ ,\ k \in Z\right\}$

C) $\left\{x: x = \frac{\pi}{18} + k\pi\ ,\ k \in Z\right\}$

D) $\left\{x: x = \frac{\pi}{9} + \frac{k\pi}{2}\ ,\ k \in Z\right\}$

E) $\left\{x: x = \frac{\pi}{18} + \frac{k\pi}{4}\ ,\ k \in Z\right\}$

SOLUTION

The answer is **B**

$\cot(2x - 20°) = \cot x$

$2x - 20° = x + 180°\cdot k$

$x = 20° + 180°\cdot k$

$\Rightarrow SS = \left\{x: x = \frac{\pi}{9} + k\pi\ ,\ k \in Z\right\}$

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1. $\sin 75^\circ = ?$

A) $\frac{\sqrt{2}+\sqrt{6}}{4}$

B) $\frac{\sqrt{2}+\sqrt{6}}{2}$

C) $\frac{\sqrt{6}-\sqrt{2}}{4}$

D) $\frac{\sqrt{6}-\sqrt{2}}{2}$

E) $\sqrt{2}+\sqrt{3}$

2. $\sin 18^\circ \cdot \cos 12^\circ + \cos 18^\circ \cdot \sin 12^\circ = ?$

A) 0

B) $\frac{1}{2}$

C) $\frac{\sqrt{2}}{2}$

D) $\frac{\sqrt{3}}{2}$

E) 1

3. $x = \frac{\pi}{8} \Rightarrow \sin x \cdot \cos x \cdot \cos 2x = ?$

A) 1

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) $\frac{1}{4}$

E) $\frac{1}{6}$

4. $\frac{\sin 80^\circ}{\cos 50^\circ \cdot \cos 40^\circ} = ?$

A) 1

B) 2

C) $2 \cdot \sin 40^\circ$

D) $2 \cdot \cos 40^\circ$

E) $2 \cdot \sin 50^\circ$

5. $\sin a = \frac{3}{5} \Rightarrow \tan 2a = ?$

A) $\frac{16}{7}$

B) $\frac{18}{7}$

C) $\frac{20}{7}$

D) $\frac{22}{7}$

E) $\frac{24}{7}$

6. $\frac{\sin 6x}{\sin 2x} + \frac{\cos 6x}{\cos 2x} = ?$

A) 4

B) $4 \cdot \sin 2x$

C) $4 \cdot \sin 4x$

D) $4 \cdot \cos 2x$

E) $4 \cdot \cos 4x$

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7. $\sin 20° = k \Rightarrow$ **sin50° = ?**

A) $1 - k^2$

B) $1 + k^2$

C) $1 - 2k^2$

D) $2 - k^2$

E) $2 + k^2$

8. **sin40° + tan25°·cos40° = ?**

A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) 1

D) $\frac{3}{2}$

E) 2

9. $0 < x < \frac{\pi}{4}$,

$\tan(2x) = \frac{3}{4} \Rightarrow$ **cotx = ?**

A) $\frac{1}{3}$

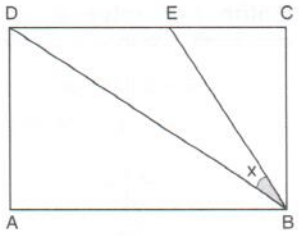
B) $\frac{1}{2}$

C) 1

D) 2

E) 3

10. ABCD bir dikdörtgendir.
ABCD is a rectangle.
2·|AB| = 5·|BC|,
|DE| = |EC|
$\Rightarrow$ **tanx = ?**



A) $\frac{7}{33}$ B) $\frac{8}{33}$ C) $\frac{10}{33}$

D) $\frac{12}{33}$ E) $\frac{17}{33}$

11. ABCD bir karedir.
ABCD is a square.
3·|DE| = |EC|,
$m(\widehat{AKB}) = x \Rightarrow$ **tanx = ?**

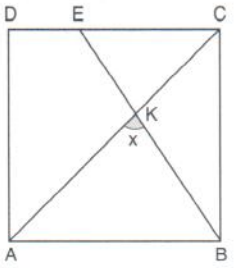
A) 3

B) 4

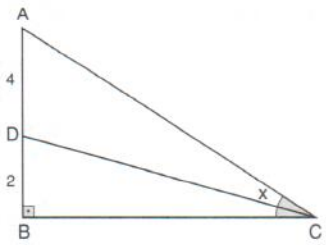
C) 5

D) 7

E) 9



12. $m(\widehat{ACD}) = m(\widehat{DCB})$
$\Rightarrow$ **tanx = ?**



A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{\sqrt{3}}$ C) $\frac{1}{\sqrt{5}}$

D) $\frac{1}{\sqrt{7}}$ E) $\frac{1}{\sqrt{10}}$

YANITLAR / ANSWERS

1. A	2. B	3. D	4. B	5. E	6. E	7. C	8. C
9. E	10. C	11. D	12. B				

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1. $\sin 75° = \sin(45°+30°)$

$= \sin 45°\cdot\cos 30° + \sin 30°\cdot\cos 45°$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

Yanıt / The Answer is A

2. $\sin a\cdot\cos b + \cos a\cdot\cos b = \sin(a+b)$

$\sin 18°\cdot\cos 12° + \cos 18°\cdot\sin 12° = \sin(18°+12°)$

$= \sin 30°$

$= \frac{1}{2}$

Yanıt / The Answer is B

3. $\sin x\cdot\cos x\cdot\cos 2x = \frac{\sin 2x}{2}\cdot\cos 2x$

$= \frac{1}{2}\cdot\frac{\sin 4x}{2}$

$= \frac{\sin 4x}{4}$

$= \frac{\sin\frac{4\pi}{8}}{4}$

$= \frac{\sin\frac{\pi}{2}}{4} = \frac{1}{4}$

Yanıt / The Answer is D

4. $\cos 50°\cdot\cos 40° = \frac{1}{2}\cdot[\cos 90° + \cos 10°]$

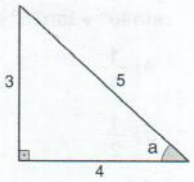
$\frac{\sin 80°}{\cos 50°\cdot\cos 40°} = \frac{\cos 10°}{\frac{1}{2}\cos 10°} = 2$

Yanıt / The Answer is B

5. $\tan 2a = \frac{2\tan a}{1-\tan^2 a} = \frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}}$

$= \frac{\frac{3}{2}}{\frac{7}{16}}$

$= \frac{3}{2}\cdot\frac{16}{7} = \frac{24}{7}$



Yanıt / The Answer is E

6. $\frac{\sin 6x}{\sin 2x} + \frac{\cos 6x}{\cos 2x} = \frac{\sin 6x}{\underset{(\cos 2x)}{\sin 2x}} + \frac{\cos 6x}{\underset{(\sin 2x)}{\cos 2x}}$

$= \frac{\sin 6x\cdot\cos 2x + \cos 6x\cdot\sin 2x}{\sin 2x\cdot\cos 2x}$

$= \frac{\sin 8x}{\frac{\sin 4x}{2}}$

$= \frac{2\sin 4x\cdot\cos 4x\cdot 2}{\sin 4x}$

$= 4\cos 4x$

Yanıt / The Answer is E

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7. $\sin 50° = \cos 40°$

$= 1 - 2\sin^2 20°$

$= 1 - 2k^2$

Yanıt / The Answer is C

8. $\sin 40° + \tan 25° \cdot \cos 40° = \sin 40° + \frac{\sin 25°}{\cos 25°} \cdot \cos 40°$

$= \frac{\sin 40° \cdot \cos 25° + \sin 25° \cdot \cos 40°}{\cos 25°}$

$= \frac{\sin 65°}{\sin 65°} = 1$

Yanıt / The Answer is C

9. $\tan 2x = \frac{3}{4}$

$\frac{2\tan x}{1 - \tan^2 x} = \frac{3}{4}$

$8\tan x = 3 - 3\tan^2 x$

$3\tan^2 x + 8\tan x - 3 = 0$

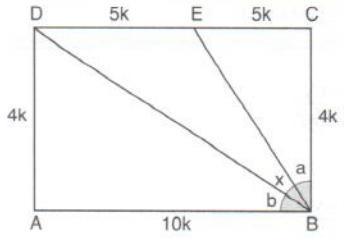
$(3\tan x - 1)(\tan x + 3) = 0$

$\tan x = \frac{1}{3}, \quad \tan x = -3$

$\cot x = \frac{1}{\tan x} = \frac{1}{\frac{1}{3}} = 3$

Yanıt / The Answer is E

10.



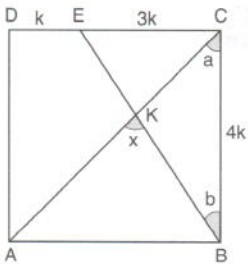
$\tan x = \tan(90° - (a+b)) = \cot(a+b)$

$= \frac{\cot a \cdot \cot b - 1}{\cot a + \cot b}$

$= \frac{\frac{4}{5} \cdot \frac{10}{4} - 1}{\frac{4}{5} + \frac{10}{4}} = \frac{1}{\frac{66}{20}} = \frac{10}{33}$

Yanıt / The Answer is C

11.



$\tan x = \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b} = \frac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = 7$

Yanıt / The Answer is D

12. $|AC|^2 = |BA|^2 + |BC|^2$

$(4k)^2 = 6^2 + (2k)^2$

$12k^2 = 36$

$k = \sqrt{3} \Rightarrow |BC| = 2\sqrt{3}$

$\tan x = \frac{|BD|}{|BC|} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

Yanıt / The Answer is B

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1. $\frac{\sin x+\sin y}{\cos x+\cos y}=?$

A) 1

B) $\sin\frac{x+y}{2}$

C) $\cos\frac{x+y}{2}$

D) $\tan\frac{x+y}{2}$

E) $\tan\frac{x-y}{2}$

2. $\frac{\sin 32^\circ+\sin 28^\circ}{\cos 32^\circ+\cos 28^\circ}=?$

A) $\frac{1}{2}$

B) $\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{3}}{3}$

D) $\sqrt{3}$

E) 1

3. $9x=\pi \Rightarrow \frac{\sin 9x+\sin 5x}{\cos 11x-\cos 17x}=?$

A) 1

B) $\frac{\sqrt{3}}{3}$

C) $-\frac{\sqrt{3}}{3}$

D) $-\frac{1}{2}$

E) -1

4. $\frac{\sin 2x+\sin 4x+\sin 6x}{1+\cos 2x+\cos 4x}=?$

A) tan4x

B) tan2x

C) 2·sin2x

D) cos2x

E) cot4x

5. $\frac{1+4\cos 70^\circ\cdot\cos 50^\circ}{\sin 20^\circ}=?$

A) $2\cdot\cot 20^\circ$

B) $2\cdot\tan 10^\circ$

C) 2

D) $2\cdot\cot 10^\circ$

E) $\frac{1}{\cos 10^\circ}$

6. $\cos 10^\circ\cdot\cos 50^\circ\cdot\cos 70^\circ=?$

A) $\frac{\sqrt{3}}{4}$

B) $\frac{\sqrt{3}}{8}$

C) $\frac{1}{4}$

D) $\frac{1}{8}$

E) $\frac{1}{16}$

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7. $\cos\left(\arctan\frac{1}{3}\right) = ?$

A) $\frac{\sqrt{10}}{10}$

B) $\frac{\sqrt{10}}{5}$

C) $\frac{3\sqrt{10}}{10}$

D) $\frac{4\sqrt{10}}{5}$

E) $\frac{\sqrt{10}}{2}$

8. $\tan\left(2\arctan\frac{1}{2}\right) = ?$

A) $\frac{5}{3}$

B) $\frac{4}{5}$

C) $\frac{4}{3}$

D) $\frac{3}{4}$

E) 1

9. $x > 0$.

$\arctan 8x = \operatorname{arccot} 2x \Rightarrow x = ?$

A) $\frac{1}{4}$

B) $\frac{1}{3}$

C) $\frac{1}{2}$

D) 1

E) $\frac{3}{4}$

10. $\sin\left(\arctan\frac{1}{3}\right) + \cos\left(\operatorname{arccot}\frac{1}{3}\right) = ?$

A) $\frac{\sqrt{10}}{10}$

B) $\frac{\sqrt{10}}{5}$

C) $\frac{3\sqrt{10}}{10}$

D) $\frac{2\sqrt{10}}{5}$

E) $\frac{\sqrt{10}}{2}$

11. $\sin\left(\frac{\pi}{2} + 2\arctan\frac{3}{4}\right) = ?$

A) $\frac{1}{5}$

B) $\frac{6}{25}$

C) $\frac{7}{25}$

D) $\frac{8}{25}$

E) $\frac{2}{5}$

12. $2\sin^2 x - 3\sin x + 1 = 0 \Rightarrow$ ÇK(SS) = ?

A) $\left\{\frac{\pi}{2}, \frac{\pi}{3}\right\}$

B) $\left\{\frac{\pi}{2} + 2k\pi, k \in Z\right\}$

C) $\left\{\frac{\pi}{6} + 2k\pi, k \in Z\right\}$

D) $\left\{\frac{5\pi}{6} + 2k\pi \vee \frac{\pi}{2} + 2k\pi, \ k \in Z\right\}$

E) $\left\{\frac{\pi}{6} + 2k\pi, \ \frac{\pi}{2} + 2k\pi, \ \frac{5\pi}{6} + 2k\pi, \ k \in Z\right\}$

YANITLAR / ANSWERS							
1. D	2. C	3. B	4. C	5. A	6. B	7. C	8. C
9. A	10. B	11. C	12. E				

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1. $$\frac{\sin x+\sin y}{\cos x+\cos y}=\frac{2\cdot\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}}{2\cdot\cos\frac{x-y}{2}\cdot\cos\frac{x+y}{2}}$$

$$=\frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}}=\tan\frac{x+y}{2}$$

Yanıt / The Answer is D

2. $$\frac{\sin32°+\sin28°}{\cos32°+\cos28°}=\frac{2\cdot\sin\frac{32°+28°}{2}\cdot\cos\frac{32°-28°}{2}}{2\cdot\cos\frac{32°-28°}{2}\cdot\cos\frac{32°+28°}{2}}$$

$$=\frac{\sin30°}{\cos30°}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

Yanıt / The Answer is C

3. $$\frac{\sin9x+\sin5x}{\cos11x-\cos17x}=\frac{2\cdot\sin\frac{9x+5x}{2}\cdot\cos\frac{9x-5x}{2}}{-2\sin\frac{11x+17x}{2}\cdot\sin\frac{11x-17x}{2}}$$

$$=\frac{\sin7x\cdot\cos2x}{\sin14x\cdot\sin3x}$$

$$=\frac{\sin7x\cdot\cos7x}{\sin14x\cdot\sin3x}$$

$$=\frac{\sin14x}{2\cdot\sin14x\sin3x}$$

$$=\frac{1}{2\sin\frac{\pi}{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

Yanıt / The Answer is B

4. $$\frac{\sin2x+\sin4x+\sin6x}{1+\cos2x+\cos4x}=\frac{\sin2x+\sin6x+\sin4x}{\cos0+\cos4x+\cos2x}$$

$$=\frac{2\sin4x\cos2x+\sin4x}{2\cos2x\cos2x+\cos2x}$$

$$=\frac{\sin4x(2\cos2x+1)}{\cos2x(2\cos2x+1)}$$

$$=\frac{2\sin2x\cos2x}{\cos2x}=2\sin2x$$

Yanıt / The Answer is C

5. $$\frac{1+4\cos70°\cdot\cos50°}{\sin20°}=\frac{1+4\cdot\frac{1}{2}[\cos120°+\cos20°]}{\sin20°}$$

$$=\frac{1+2\cos120°+2\cos20°}{\sin20°}$$

$$=\frac{2\cos20°}{\sin20°}$$

$$=2\cot20°$$

Yanıt / The Answer is A

6. $$\cos10°\cdot\cos70°\cdot\cos50°=\frac{1}{2}(\cos80°+\cos60°)\cdot\cos50°$$

$$=\frac{1}{2}\left[\cos80°\cdot\cos50°+\frac{1}{2}\cos50°\right]$$

$$=\frac{1}{2}\left[\frac{1}{2}(\cos130°+\cos30°)+\frac{1}{2}\cos50°\right]$$

$$=\frac{1}{2}\left[-\frac{1}{2}\cos50°+\frac{\cos30°}{2}+\frac{1}{2}\cos50°\right]$$

$$=\frac{1}{2}\cdot\frac{\cos30°}{2}=\frac{\sqrt{3}}{8}$$

Yanıt / The Answer is B

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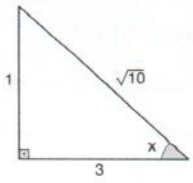
7. $x = \arctan\frac{1}{3}$

$\Rightarrow \tan x = \frac{1}{3}$

$\cos x = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$



Yanıt / The Answer is C

8. $\arctan\frac{1}{2} = x \Rightarrow \tan x = \frac{1}{2}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

Yanıt / The Answer is C

9. $a = \arctan 8x \Rightarrow \tan a = 8x$

$b = \operatorname{arccot} 2x \Rightarrow \cot b = 2x$

$\tan a = \tan b$

$a = b \Rightarrow \tan a = \frac{1}{2x}$

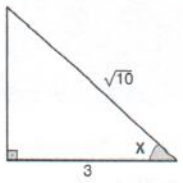
$8x = \frac{1}{2x}$

$16x^2 = 1$

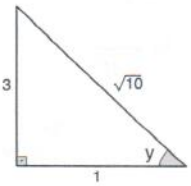
$x = \frac{1}{4}$

Yanıt / The Answer is A

10. $\arctan\frac{1}{3} = x \Rightarrow \tan x = \frac{1}{3}$



$\operatorname{arccot}\frac{1}{3} = y \Rightarrow \cot y = \frac{1}{3}$



$\sin x + \cos y = \frac{1}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{2}{\sqrt{10}} = \frac{\sqrt{10}}{5}$

Yanıt / The Answer is B

11. $2\arctan\frac{3}{4} = x \Rightarrow \arctan\frac{3}{4} = \frac{x}{2}$

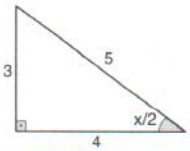
$\tan\frac{x}{2} = \frac{3}{4}$

$\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\cos x = 1 - 2\cdot\sin^2\frac{x}{2}$

$= 1 - 2\cdot\frac{9}{25}$

$= \frac{7}{25}$



Yanıt / The Answer is C

12. $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}, \quad \sin x = 1$

$x = \frac{\pi}{6} + 2k\pi, \ \frac{5\pi}{6} + 2k\pi, \ \frac{\pi}{2} + 2k\pi$

Yanıt / The Answer is E

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ZTC زنگنه مجموعه موسسات

YÖS Soruları / YÖS Questions

1. $\tan(x + 45°) = 2$

$\Rightarrow$ **sinx = ?**

A) $\frac{1}{\sqrt{10}}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$

D) $\frac{2}{\sqrt{10}}$ E) $\frac{3}{\sqrt{10}}$

[İSTANBUL ÜNİV. YÖS 2013]

2. $\sin(\cos^{-1}(\frac{\sqrt{3}}{2})) = ?$

A) 1 B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$

D) $\frac{\sqrt{3}}{2}$ E) 0

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

3. $\cos\left(\arctan\frac{1}{\sqrt{3}}\right) = ?$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) $\sqrt{3}$

D) 1 E) 0

[EGE ÜNİ. YÖS 2012]

4. $\sin x = 0{,}6 \Rightarrow$ **sin2x = ?**

A) $\frac{25}{24}$ B) $\frac{4}{3}$ C) $\frac{6}{5}$

D) $\frac{3}{4}$ E) $\frac{24}{25}$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

5. $\tan\left(\cos^{-1}\frac{1}{2}\right) = ?$

A) 1 B) 0 C) $\sqrt{3}$

D) -1 E) $-\sqrt{3}$

[ANKARA ÜNİV. YÖS 2011]

6. $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = ?$

A) $\frac{1}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{2}$ E) 1

[YÖS 2010]

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7. $AD \perp BD$

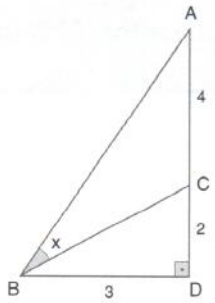
$|AC| = 4$ cm,

$|CD| = 2$ cm,

$|BD| = 3$ cm,

$m(\widehat{ABC}) = x$

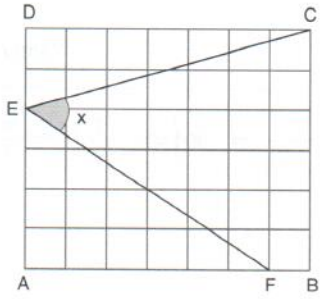
$\Rightarrow$ **tanx = ?**



A) $\frac{2}{3}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{3}{7}$ E) $\frac{4}{7}$

[YÖS 2009]

8.



tanx = ?

A) $\frac{20}{17}$ B) $\frac{20}{21}$ C) $\frac{17}{21}$ D) $\frac{21}{40}$ E) 1

[YÖS 2008]

9. $\dfrac{\cos\frac{\pi}{8}\cdot\sin\frac{\pi}{8}}{\left(2\cos^2\frac{\pi}{8}\right)-1} = ?$

A) $-\sqrt{3}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\sqrt{3}$

[YÖS 2008]

10. $\dfrac{\sin\left(\arctan\left(\sqrt{3}\right)\right)}{\cos 60^\circ} = ?$

A) $-\frac{\sqrt{2}}{2}$ B) $-\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\sqrt{3}$ E) 3

[YÖS 2008]

11. $\dfrac{\sin 30^\circ}{\sin 10^\circ} - \dfrac{\cos 30^\circ}{\cos 10^\circ} = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

[YÖS 2006]

12. $\tan 2x = \frac{3}{4}$

cotx = ?

A) 0 B) 1 C) 2 D) 3 E) 4

[YÖS 2005]

YANITLAR / ANSWERS

1. A	2. B	3. A	4. E	5. C	6. C	7. E	8. A
9. C	10. D	11. C	12. D				

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1.

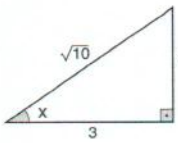
$$\tan(x+45°)=\frac{\tan x+\tan 45°}{1-\tan x\cdot\tan 45°}$$

$\tan x = t \Rightarrow$

$t+1=2-2t$

$3t=1$

$t=\frac{1}{3}$



$\Rightarrow \sin x=\frac{1}{\sqrt{10}}$

$\tan x=\frac{1}{3}$

Yanıt / The Answer is A

2. $\sin(\cos^{-1}(\frac{\sqrt{3}}{2}))=\sin(30°)=\frac{1}{2}$

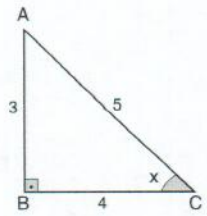
Yanıt / The Answer is B

3. $\arctan\frac{1}{\sqrt{3}}=\alpha \Rightarrow \alpha=30°$

$\cos\left(\arctan\frac{1}{\sqrt{3}}\right)=\cos 30°=\frac{\sqrt{3}}{2}$

Yanıt / The Answer is A

4.



$\sin x=\frac{6}{10}=\frac{3}{5}$

$\sin 2x=2\cdot\sin x\cdot\cos x$

$=2\cdot\frac{3}{5}\cdot\frac{4}{5}=\frac{24}{25}$

Yanıt / The Answer is E

5. $\cos^{-1}\frac{1}{2}=\frac{\pi}{3} \Rightarrow \tan\left(\cos^{-1}\frac{1}{2}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

Yanıt / The Answer is C

6. $\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$

$\cos\left(\sin\frac{-1}{2}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

Yanıt / The Answer is C

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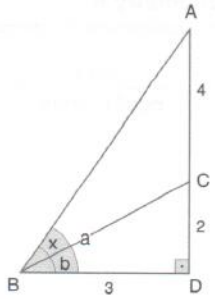
7. $\tan a = \frac{6}{3} = 2$

$\tan b = \frac{2}{3}$

$\tan(a-b) = \frac{\tan a - \tan b}{1+\tan a \cdot \tan b}$

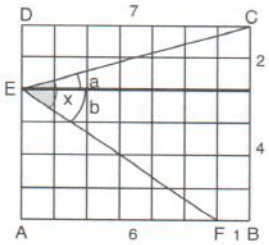
$= \frac{2-\frac{2}{3}}{1+2\cdot\frac{2}{3}} = \frac{\frac{4}{3}}{\frac{7}{3}}$

$= \frac{4}{7}$



Yanıt / The Answer is E

8.



$x = a+b \Rightarrow \tan x = \tan(a+b) = \frac{\tan a + \tan b}{1-\tan a \cdot \tan b}$

$\left.\begin{array}{l}\tan a = \frac{2}{7},\\ \tan b = \frac{4}{6} = \frac{2}{3}\end{array}\right\} \Rightarrow \frac{\frac{2}{7}+\frac{2}{3}}{1-\frac{2}{7}\cdot\frac{2}{3}} = \frac{\frac{20}{21}}{\frac{17}{21}} = \frac{20}{17}$

Yanıt / The Answer is A

9. $\frac{2\cos\frac{\pi}{8}\cdot\sin\frac{\pi}{8}}{2\cos^2\frac{\pi}{8}-1} = \frac{\frac{\sin\frac{\pi}{4}}{2}}{\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{4}}{\frac{\sqrt{2}}{2}} = \frac{1}{2}$

Yanıt / The Answer is C

10. $\arctan\sqrt{3} = \frac{\pi}{3} = 60° \Rightarrow$

$\frac{\sin(\arctan(\sqrt{3}))}{\cos 60°} = \frac{\sin 60°}{\cos 60°}$

$= \tan 60° = \sqrt{3}$

Yanıt / The Answer is D

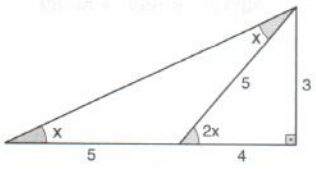
11. $\frac{\sin 30°}{\sin 10°} - \frac{\cos 30°}{\cos 10°} = \frac{\sin 30°\cdot\cos 10° - \cos 30°\cdot\sin 10°}{\sin 10°\cdot\cos 10°}$

$= \frac{\sin(30°-10°)}{\frac{\sin 20°}{2}} = 2$

Yanıt / The Answer is C

12. $\tan 2x = \frac{3}{4}$

$\Rightarrow \cot x = \frac{9}{3} = 3$



Yanıt / The Answer is D

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1. $\sin 15^\circ = ?$

Yanıt / The Answer is $\frac{\sqrt{6}-\sqrt{2}}{4}$

2. $\cos 20^\circ \cdot \cos 40^\circ - \sin 20^\circ \cdot \sin 40^\circ = ?$

Yanıt / The Answer is $\frac{1}{2}$

3. $\sin 10^\circ \cdot \sin 55^\circ + \sin 80^\circ \cdot \sin 35^\circ = ?$

Yanıt / The Answer is $\frac{\sqrt{2}}{2}$

4. $\frac{\sin 4x}{\cos 2x \cdot \cos x} = ?$

Yanıt / The Answer is $4\sin x$

5. $0 < x < \frac{\pi}{2}$,

$\tan x = \frac{3}{4} \Rightarrow \sin 2x = ?$

Yanıt / The Answer is $\frac{24}{25}$

6. $\cos 10^\circ = a \Rightarrow \cos 70^\circ = ?$

Yanıt / The Answer is $2a\sqrt{1-a^2}$

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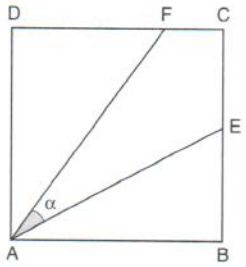
7. $\sin\frac{\pi}{12}\cdot\left(1-2\sin^2\frac{\pi}{24}\right)=?$

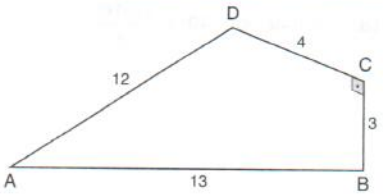
Yanıt / The Answer is $\frac{1}{4}$

8. ABCD bir karedir.
ABCD is a square.
|DF| = 4·|FC|,
|BE| = |EC|
$\Rightarrow \tan\alpha = ?$



Yanıt / The Answer is $\frac{6}{13}$

9. $\sin(\widehat{ADC}) = ?$



Yanıt / The Answer is $\frac{4}{5}$

10. $\frac{1}{\sin10^\circ}+\frac{\sqrt{3}}{\cos10^\circ}=?$

Yanıt / The Answer is $8\cdot\cos20^\circ$

11. $\frac{1}{\sin15^\circ}+\frac{1}{\cos75^\circ}=?$

Yanıt / The Answer is $2\text{cosec}15^\circ$

12. $16x=\pi \Rightarrow \frac{\sin9x-\sin5x}{\cos15x+\cos3x}=?$

Yanıt / The Answer is -1

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13. $9x = \pi \Rightarrow \dfrac{\sin 9x - \sin 3x}{\sin 12x} = ?$

Yanıt / The Answer is **1**

14. $x = \dfrac{\pi}{15} \Rightarrow \dfrac{\sin 3x + \sin 5x + \sin 7x}{\cos 3x + \cos 5x + \cos 7x} = ?$

Yanıt / The Answer is $\sqrt{3}$

15. $16x = \pi \Rightarrow \dfrac{\sin 6x + \sin 2x}{\cos 6x + \cos 2x} = ?$

Yanıt / The Answer is **1**

16. $\dfrac{\sin 8x + \sin 6x + \sin 4x}{\cos x + \cos 3x + \cos 5x} = ?$

Yanıt / The Answer is **2sin3x**

17. $\cos 18^\circ \cdot \cos 36^\circ \cdot \cos 72^\circ = ?$

Yanıt / The Answer is $\dfrac{1}{4}\cos 18^\circ$

18. $\sin 40^\circ \cdot \sin 140^\circ + \dfrac{\sin 10^\circ}{2} = ?$

Yanıt / The Answer is $\dfrac{1}{2}$

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19. $4\cdot\cos80^\circ-\frac{\sqrt{3}}{\cos50^\circ}=?$

Yanıt / The Answer is -2

20. $\arcsin\left(\frac{5}{13}\right)=\alpha \Rightarrow \tan\alpha=?$

Yanıt / The Answer is $\frac{5}{12}$

21. $\arctan\frac{1}{2}+\arctan\frac{1}{3}=?$

Yanıt / The Answer is $\frac{\pi}{4}$

22. $\cos\left(2\arcsin\frac{8}{17}\right)=?$

Yanıt / The Answer is $\frac{161}{289}$

23. $\sin x+\sin3x=2\cdot\sin x\cdot\cos x \Rightarrow$ ÇK(SS) = ?

Yanıt / The Answer is $\left\{0, \frac{\pi}{3}+2k\pi, \frac{5\pi}{3}+2k\pi\right\}$

24. $x\in(0, 2\pi)$,
$2\sin^2x-3\sin x-2=0 \Rightarrow$ ÇK(SS) = ?

Yanıt / The Answer is $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

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25. $\sin x + \cos x = \frac{4}{3} \Rightarrow$ **sin2x = ?**

Yanıt / The Answer is $\frac{7}{9}$

26. $\tan x = \frac{1}{2} \Rightarrow$ **cos2x = ?**

Yanıt / The Answer is $\frac{3}{5}$

27. $\cos 2x = \frac{7}{8} \Rightarrow$ **$\cos^4 x - \sin^4 x$ = ?**

Yanıt / The Answer is $\frac{7}{8}$

28. **arctan1 = ?**

Yanıt / The Answer is $\frac{\pi}{4}$

29. $\cos\left(\text{arcsin}\frac{\sqrt{7}}{4}\right)$ **= ?**

Yanıt / The Answer is $\frac{3}{4}$

30. $x \in [0, \pi]$,
$\cos^2 x - \sin^2 x = \sin x \Rightarrow$ **x = ?**

Yanıt / The Answer is $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

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1. $\sin27^\circ \cdot \cos33^\circ + \cos27^\circ \cdot \sin33^\circ = ?$

A) $\frac{1}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{4}$

E) 1

2. $\frac{\cos36^\circ}{\cos12^\circ} - \frac{\sin36^\circ}{\sin12^\circ} = ?$

A) –1

B) –2

C) –3

D) –4

E) –5

3. $\cos65^\circ \cdot \cos55^\circ - \cos25^\circ \cdot \cos35^\circ = ?$

A) $-\frac{1}{2}$

B) $-\frac{\sqrt{3}}{2}$

C) $\frac{1}{2}$

D) $\frac{\sqrt{3}}{2}$

E) 1

4. $A = 2\sin x - \cos x \Rightarrow \min(A) = ?$

A) –3

B) –1

C) $-\sqrt{5}$

D) $-2-\sqrt{2}$

E) $-1-\sqrt{3}$

5. $\left.\begin{matrix}\tan x = 2\\ \tan y = 3\end{matrix}\right\} \Rightarrow \tan(x+2y) = ?$

A) 1

B) 0

C) –1

D) $\frac{1}{2}$

E) $\frac{3}{2}$

6. $\sin75^\circ + \sqrt{3} \cdot \cos75^\circ = ?$

A) $\frac{\sqrt{3}}{2}$

B) $\sqrt{3}$

C) $\sqrt{2}$

D) $\frac{\sqrt{6}}{2}$

E) $\frac{\sqrt{6}}{3}$

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7. $a+b=\frac{5\pi}{6} \Rightarrow (\sin a+\cos b)^2+(\sin b+\cos a)^2=?$

A) 3

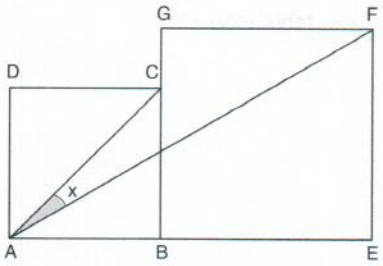
B) $2+\sqrt{3}$

C) $2-\sqrt{3}$

D) $1+\sqrt{3}$

E) 1

8.



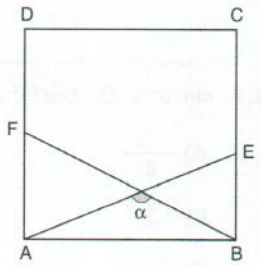
ABCD, BEFG birer karedir. / ABCD, BEFG are square.

$3\cdot|BC| = 2\cdot|BG|$, $m(\widehat{CAF}) = x \Rightarrow$ **tanx = ?**

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$

D) $\frac{1}{4}$ E) $\frac{1}{5}$

9. ABCD bir karedir.
ABCD is a square.
$|AF| = |FD|$,
$2\cdot|EB| = |EC|$
$\Rightarrow$ **tanα = ?**



A) 1 B) 0 C) –1

D) $\frac{1}{2}$ E) $-\frac{1}{2}$

10. $\cos x=\frac{2}{3}$, $\cos y=\frac{1}{3} \Rightarrow \sin(x+y)\cdot\sin(x-y)=?$

A) 1

B) $\frac{7}{9}$

C) $-\frac{1}{3}$

D) $-\frac{2}{3}$

E) –1

11. $\sin(x-45)=2\cdot\cos(45-x) \Rightarrow$ **tanx = ?**

A) –2

B) –3

C) –4

D) –5

E) –6

12. ABCDEFG bir küptür. / ABCDEFG is a cube.

$m(\widehat{ACE}) = \theta \Rightarrow$ **cosθ = ?**

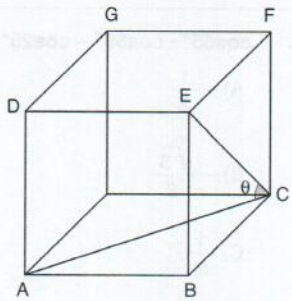
A) $\frac{1}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{2}}{3}$

E) $\frac{1}{3}$



YANITLAR / ANSWERS							
1. C	2. B	3. A	4. C	5. D	6. C	7. A	8. D
9. C	10. C	11. C	12. A				

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1. $\frac{\sin 2x}{1-\cos 2x} - \cot x = ?$

A) $\sec x$

B) $2 \cdot \sin 2x$

C) $2 \cdot \cot 2x$

D) 0

E) $\frac{1}{2}\cos 2x$

2. $0 < x < \frac{\pi}{2}$,

$\cos x = \frac{1}{\sqrt{10}} \Rightarrow \sin 2x + \tan x = ?$

A) $\frac{18}{5}$

B) $\frac{12}{5}$

C) $-\frac{9}{5}$

D) $-\frac{12}{5}$

E) $-\frac{21}{5}$

3. $\frac{8\cos^2 x - 4}{\tan x + \cot x} = ?$

A) 1

B) $2 \cdot \sin 2x$

C) $4 \cdot \sin 2x$

D) $\sin 4x$

E) $2 \cdot \sin 4x$

4. $\sin\frac{5\pi}{12}\left(\cos^2\frac{5\pi}{24} - \sin^2\frac{5\pi}{24}\right) = ?$

A) $-\frac{1}{4}$

B) $-\frac{1}{2}$

C) $\frac{1}{2}$

D) $\frac{1}{4}$

E) $\frac{1}{6}$

5. $\tan x = 3 \Rightarrow \sin 2x = ?$

A) $\frac{3}{5}$

B) $\frac{4}{5}$

C) $\frac{5}{13}$

D) $\frac{12}{13}$

E) $\frac{8}{17}$

6. $\sin 65° = k \Rightarrow \sin 40° = ?$

A) $2k^2 - 1$

B) $1 - 2k^2$

C) $2k$

D) $1 - k^2$

E) $2k - 1$

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7. $\cos\frac{\pi}{12} = ?$

A) $\frac{\sqrt{\sqrt{3}+2}}{2}$

B) $\frac{\sqrt{\sqrt{3}-2}}{2}$

C) $\frac{\sqrt{\sqrt{3}+2}}{4}$

D) $\frac{\sqrt{\sqrt{3}-2}}{4}$

E) $\frac{\sqrt{3}+2}{2}$

8. $\tan x = \frac{2}{3} \Rightarrow 2\cdot\sin 2x + 3\cdot\cos 2x = ?$

A) 6

B) 5

C) 4

D) 3

E) 2

9. $\cos 21° = m \Rightarrow \sin 48° = ?$

A) m^2

B) $2m^2$

C) $3m^2 + 1$

D) $2m^2 - 1$

E) $1 - m^2$

10. $x = \frac{\pi}{12} \Rightarrow \cos^2\left(x - \frac{\pi}{4}\right) - \sin^2\left(x - \frac{\pi}{4}\right) = ?$

A) 1

B) $\frac{\sqrt{2}}{2}$

C) $\frac{1}{2}$

D) $\frac{\sqrt{3}}{2}$

E) 2

11. $\cos 10° \cdot \cos 20° \cdot \cos 40° = ?$

A) $\tan 80°$

B) $\frac{\tan 80°}{2}$

C) $\frac{\tan 80°}{4}$

D) $\frac{\tan 80°}{8}$

E) $\frac{\tan 80°}{16}$

12. $\sin 4° = k \Rightarrow \cos^2 47° = ?$

A) $\frac{1-k}{2}$

B) $\frac{k-1}{2}$

C) $\frac{k^2-1}{2}$

D) $\frac{k^2+1}{2}$

E) $\frac{1-2k^2}{2}$

YANITLAR / ANSWERS							
1. D	2. A	3. D	4. D	5. A	6. A	7. A	8. D
9. D	10. C	11. D	12. A				

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1. $\cos\left(\arccos\frac{1}{2}+\pi\right)=?$

A) $-\frac{1}{2}$

B) $-\frac{1}{4}$

C) $-\frac{\sqrt{2}}{2}$

D) $-\frac{\sqrt{3}}{2}$

E) 0

2. $\arcsin\frac{1}{2}+\arccos\frac{\sqrt{2}}{2}=?$

A) 30°

B) 45°

C) 60°

D) 75°

E) 90°

3. $\sin\left(\arccos\frac{5}{13}\right)=?$

A) $\frac{8}{13}$

B) $\frac{9}{13}$

C) $\frac{10}{13}$

D) $\frac{11}{13}$

E) $\frac{12}{13}$

4. $\tan\left(\arctan\frac{1}{3}+\arctan\frac{3}{2}\right)=?$

A) $\frac{8}{3}$

B) 3

C) $\frac{10}{3}$

D) $\frac{11}{3}$

E) 4

5. $\tan\left(\frac{1}{2}\arcsin\frac{4}{5}\right)=?$

A) $\frac{1}{6}$

B) $\frac{1}{5}$

C) $\frac{1}{4}$

D) $\frac{1}{3}$

E) $\frac{1}{2}$

6. $\cos(\arccos x)=?$

A) -1

B) 0

C) 1

D) $\frac{1}{x}$

E) x

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7. $x \in \left(0, \frac{\pi}{2}\right)$,

$\cos(3x - 20°) = \sin(x - 14°) \Rightarrow$ **x = ?**

A) 16°

B) 31°

C) 32°

D) 40°

E) 48°

8. $\cos^2 x = 1 + \sin^2 x \Rightarrow$ **ÇK(SS) = ?**

A) $\{x: x = 0 + k\pi\}$

B) $\left\{x: x = \frac{\pi}{6} + 2k\pi\right\}$

C) $\left\{x: x = \frac{\pi}{4} + k\pi\right\}$

D) $\left\{x: x = \frac{\pi}{3} + 2k\pi\right\}$

E) $\left\{x: x = \frac{\pi}{2} + k\pi\right\}$

9. $\tan\left(3x - \frac{\pi}{3}\right) = \cot\left(\frac{\pi}{4} - x\right) \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x: x = \frac{7\pi}{24} + \frac{k\pi}{2}\right\}$

B) $\left\{x: x = \frac{7\pi}{48} + \frac{k\pi}{4}\right\}$

C) $\left\{x: x = \frac{5\pi}{12} + \frac{k\pi}{6}\right\}$

D) $\left\{x: x = \frac{5\pi}{24} + \frac{k\pi}{2}\right\}$

E) $\left\{x: x = \frac{\pi}{12} + \frac{k\pi}{3}\right\}$

10. $x \in (0, 2\pi)$,

$2\cos^2 x + \sin x - 1 = 0 \Rightarrow \sum x =$ **?**

A) $\frac{9\pi}{2}$

B) 4π

C) $\frac{7\pi}{2}$

D) $\frac{3\pi}{2}$

E) $\frac{5\pi}{6}$

11. $A = 5\cos x + 12\sin x \Rightarrow$ **max(A) = ?**

A) 5

B) 7

C) 9

D) 11

E) 13

12. $A = 6\sin x - 8\cos x \Rightarrow$ **min(A) = ?**

A) 0

B) –3

C) –4

D) –6

E) –10

YANITLAR / ANSWERS							
1. A	2. D	3. E	4. D	5. E	6. E	7. B	8. A
9. A	10. C	11. E	12. E				

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1. $\frac{\cos 9x - \cos 15x}{\cos 6x \cdot \sin 3x} = ?$

A) 4sin6x
B) –4sin6x
C) –2sin6x
D) 2cos6x
E) –4cos6x

2. $\frac{\sin 5x + \sin 3x}{\cos 2x \cdot \cos x} = ?$

A) 4·sin2x
B) 2·sin2x
C) sin2x
D) cos2x
E) 2·cosx

3. $\frac{\sin 10x - \sin 6x - \sin 2x}{\sin 9x - \sin 7x - \sin x} = ?$

A) 2·sinx
B) 2·tan4x
C) cot3x
D) 2·cosx
E) sin2x

4. $17x = \pi \Rightarrow \frac{\cos 12x + \cos 8x}{\cos 5x + \cos 9x} = ?$

A) 1
B) $\frac{\sqrt{3}}{2}$
C) $-\frac{1}{2}$
D) $-\frac{\sqrt{3}}{2}$
E) –1

5. $x = \frac{\pi}{6} \Rightarrow \frac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = ?$

A) $\frac{1}{2}$
B) $\frac{\sqrt{3}}{2}$
C) $\frac{\sqrt{3}}{3}$
D) $\sqrt{3}$
E) $\frac{2}{\sqrt{3}}$

6. $9x = \frac{\pi}{2} \Rightarrow \frac{\sin 7x + \sin x}{4 \cdot \cos 5x \cdot \sin 6x} = ?$

A) 4
B) 2
C) $\frac{1}{2}$
D) $\frac{1}{4}$
E) –2

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7. $a = \frac{\pi}{8} \Rightarrow \frac{\cos 5a + \cos 4a}{\cos 3a + \sin a} = ?$

A) 2

B) 1

C) $-\frac{1}{2}$

D) -1

E) -2

8. $\frac{\sin x + \sin 4x + \sin 9x}{\cos x + \cos 4x + \cos 9x} = ?$

A) 1

B) cot4x

C) tan4x

D) tanx

E) 0

9. $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = ?$

A) tanx

B) cotx

C) secx

D) 2·tanx

E) tan2x

10. $\sin 18° \cdot \sin 108° = ?$

A) $-\cos 54°$

B) $\frac{\cos 54°}{2}$

C) $\cos 54°$

D) $-\frac{\cos 54°}{2}$

E) $2 \cdot \cos 54°$

11. $2 \cdot \sin 235° \cdot \sin 145° = ?$

A) cos20°

B) –cos20°

C) 1

D) –2cos20°

E) 2cos20°

12. $\sin 10° \cdot \cos 20° \cdot \cos 40° = ?$

A) $\frac{1}{8}$

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) 1

E) 2

YANITLAR / ANSWERS							
1. A	2. A	3. D	4. E	5. D	6. C	7. C	8. C
9. A	10. B	11. B	12. A				

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ÜNİTE 3

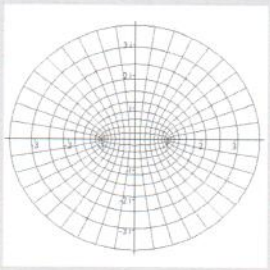
UNIT 3

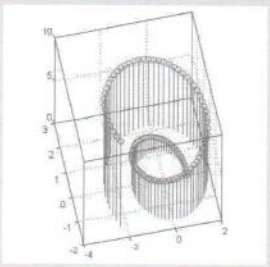
Birinci bölümde, reel sayılardan daha geniş bir küme olan karmaşık sayılar kümesine; ikinci bölümde de logaritma fonksiyonu, logaritmik denklem ve eşitsizlikler anlatılmıştır.

Bu ünite; 88 çözümlü örnek, 36 alıştırma ve 60 test sorusundan oluşmaktadır.

In the first chapter, complex number set is explained in detailed that enlarged from the real number set. In the second chapter, logarithmic function, equation and inequalities are explained in detailed.

This unit includes; 88 examples, 36 exercises, and 60 test questions.





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BÖLÜM 1
CHAPTER 1

TARİHSEL NOT / HISTORICAL NOTE

De Moivre, Abraham [1667 – 1754]

Fransa'da doğmuş, sonra İngiltere'ye yerleşmiştir. Karmaşık sayılarda trigonometrinin uygulaması olarak bilinen De Moivre Teoremi ile hatırlanır. Stirling formulü ve normal dağılım üzerine çalışmıştır.

He was born in France, who later settled in England. In De Moivre's Theorem, he is remembered for his use of complex numbers in trigonometry. But he was also the author of two notable early works on probability. Later work contains the result known as Stirling's formula and probably the first use of the normal frequency curve.

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COMPLEX NUMBERS

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KARMAŞIK SAYILAR
COMPLEX NUMBERS

BÖLÜM 1
CHAPTER

İkinci dereceden bir denklemde $\Delta < 0$ iken çözüm kümesi boş küme, yani; bu denklemi sağlayan reel kök yoktur. Bu nedenle, $\Delta < 0$ durumunda denklemin çözüm kümesini bulmak için reel sayılardan daha geniş bir kümeye gerek vardır.

NOT

$N^+ \subset N \subset Z \subset Q \subset R \subset C$

Tanım: Sanal sayı birimi $i = \sqrt{-1}$ ile tanımlanır.

Tanım: $a, b \in R$ olmak üzere, $z = a + bi$ şeklinde yazılan sayılara *karmaşık sayılar*, bu sayıların oluşturduğu kümeye de *karmaşık sayılar kümesi* denir ve C ile gösterilir.

Tanım: $z = a + bi$ karmaşık sayısında, a sayısına karmaşık sayının *reel kısmı* denir ve $Re(z) = a$ ile gösterilir.

Etkinlik

a. $z = 2 - 3i \Rightarrow Re(z) = 2$
b. $w = -3 + 4i \Rightarrow Re(w) = -3$
c. $t = 4i \Rightarrow Re(t) = 0$
d. $p = 3 \Rightarrow Re(p) = 3$

Tanım: $z = a + bi$ karmaşık sayısında, b sayısına karmaşık sayının *sanal (imajiner) kısmı* denir ve $Im(z)$ ile gösterilir.

Etkinlik

a. $z = 2 + 5i \Rightarrow Im(z) = 5$
b. $k = 2 - 4i \Rightarrow Im(k) = -4$
c. $p = 5 \Rightarrow Im(p) = 0$
d. $w = 3i \Rightarrow Im(w) = 3$

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NOTE

$N^+ \subset N \subset Z \subset Q \subset R \subset C$

We said that the solution set is empty set when $\Delta < 0$ in quadratic equation i.e, we have not got real root of this equation. In situation $\Delta < 0$, there are need to set more larger than real numbers for the solution set.

Definition: The imaginary unit i is defined by $i = \sqrt{-1}$.

Definition: A complex number is a number of the form a + bi where a and b are real numbers. *The set of complex numbers* is consist of complex numbers and this set is denoted by C.

Definition: In complex number z = a + bi, the real number a is called the *real part* of the complex number and that is denoted by Re(z) = a.

Activity

a. $z = 2 - 3i \Rightarrow Re(z) = 2$

b. $w = -3 + 4i \Rightarrow Re(w) = -3$

c. $t = 4i \Rightarrow Re(t) = 0$

d. $p = 3 \Rightarrow Re(p) = 3$

Definition: In complex number z = a + bi, the real number b is called the *imaginary part* of the complex number and that is denoted by Im(z) = b.

Activity

a. $z = 2 + 5i \Rightarrow Im(z) = 5$

b. $k = 2 - 4i \Rightarrow Im(k) = -4$

c. $p = 5 \Rightarrow Im(p) = 0$

d. $w = 3i \Rightarrow Im(w) = 3$

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ÖRNEK 1

$$\frac{\sqrt{-36}+\sqrt{-16}-\sqrt{-25}}{\sqrt{-4}+\sqrt{-9}} = ?$$

A) 1 B) –i C) 0 D) –1 E) i

ÇÖZÜM

Yanıt A

$$\left.\begin{array}{l}\sqrt{-36}=6\sqrt{-1}=6i\\ \sqrt{-16}=4\sqrt{-1}=2i\\ \sqrt{-25}=5\sqrt{-1}=5i\\ \sqrt{-4}=2\sqrt{-1}=2i\\ \sqrt{-9}=3\sqrt{-1}=3i\end{array}\right\} \Rightarrow \frac{\sqrt{-36}+\sqrt{-16}-\sqrt{-25}}{\sqrt{-4}+\sqrt{-9}} = \frac{6i+4i-5i}{2i+3i} = \frac{5i}{5i} = 1$$

PEKİŞTİRME

$$\frac{\sqrt{-16}+\sqrt{-25}}{\sqrt{-81}} = ?$$

YANIT : 1

ÖRNEK 2

$2x^2 + x + 1 = 0 \Rightarrow$ ÇK = ?

A) $-1 \pm \sqrt{7}\,i$ B) $\frac{-1 \pm \sqrt{7}\,i}{2}$ C) $\frac{-1 \pm \sqrt{7}\,i}{3}$

D) $\frac{-1 \pm \sqrt{7}\,i}{4}$ E) $\frac{-1 \pm \sqrt{7}\,i}{5}$

ÇÖZÜM

Yanıt D

$\Delta = 1^2 - 4 \cdot 2 \cdot 1 = -7 < 0$

$x_{1,2} = \frac{-1 \pm \sqrt{-7}}{4} = \frac{-1 \pm \sqrt{7}\,i}{4}$

PEKİŞTİRME

$x^2 + x + 4 = 0 \Rightarrow$ ÇK = ?

YANIT : $(-1 \mp \sqrt{15}\,i)\,/\,2$

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REINFORCE

$\frac{\sqrt{-16}+\sqrt{-25}}{\sqrt{-81}} = ?$

EXAMPLE 1

$\frac{\sqrt{-36}+\sqrt{-16}-\sqrt{-25}}{\sqrt{-4}+\sqrt{-9}} = ?$

A) 1 B) $-i$ C) 0 D) -1 E) i

ANSWER : 1

SOLUTION

The answer is A

$$\left.\begin{array}{l}\sqrt{-36}=6\sqrt{-1}=6i\\ \sqrt{-16}=4\sqrt{-1}=2i\\ \sqrt{-25}=5\sqrt{-1}=5i\\ \sqrt{-4}=2\sqrt{-1}=2i\\ \sqrt{-9}=3\sqrt{-1}=3i\end{array}\right\} \Rightarrow \frac{\sqrt{-36}+\sqrt{-16}-\sqrt{-25}}{\sqrt{-4}+\sqrt{-9}}=\frac{6i+4i-5i}{2i+3i}=\frac{5i}{5i}=1$$

REINFORCE

$x^2 + x + 4 = 0 \Rightarrow SS = ?$

EXAMPLE 2

$2x^2 + x + 1 = 0 \Rightarrow$ **SS = ?**

A) $-1 \pm \sqrt{7}\,i$ B) $\frac{-1 \pm \sqrt{7}\,i}{2}$ C) $\frac{-1 \pm \sqrt{7}\,i}{3}$

D) $\frac{-1 \pm \sqrt{7}\,i}{4}$ E) $\frac{-1 \pm \sqrt{7}\,i}{5}$

ANSWER : $(-1 \mp \sqrt{15}i)/2$

SOLUTION

The answer is D

$\Delta = 1^2 - 4 \cdot 2 \cdot 1 = -7 < 0$

$x_{1,2} = \frac{-1 \pm \sqrt{-7}}{4} = \frac{-1 \pm \sqrt{7}\,i}{4}$

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i' nin TAMSAYI KUVVETLERİ

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$i^5 = i$
$i^6 = -1$

$$i^n = \begin{cases} 1, & n \equiv 0 \pmod 4 \\ i, & n \equiv 1 \pmod 4 \\ -1, & n \equiv 2 \pmod 4 \\ -i, & n \equiv 3 \pmod 4 \end{cases}$$

NOT

$i^m \cdot i^n = i^{m+n}$

ÖRNEK 3

$i^{49} = ?$

A) i B) 2i C) 3i D) 4i E) 5i

ÇÖZÜM

Yanıt **A**

$49 \equiv 1 \pmod 4$
$i^{49} = i$

▼ I. $(i+1)^2 = i^2 + 2i + 1$
$= -1 + 2i + 1 = 2i$

II. $(i-1)^2 = i^2 - 2i + 1$
$= -1 + 2i + 1 = -2i$

ÖRNEK 4

$z = (1+i)^{27} = ?$

A) $2^{11}(-1+i)$ B) $2^{12}(-1+i)$ C) $2^{13}(-1+i)$
D) $2^{14}(-1+i)$ E) $2^{15}(-1+i)$

ÇÖZÜM

Yanıt **C**

$z = (1+i)^{27} = ((1+i)^2)^{13} \cdot (1+i)$
$z = (2i)^{13} \cdot (1+i) = 2^{13} \cdot i^{13} \cdot (1+i)$
$z = 2^{13} \cdot i(1+i) = 2^{13} \cdot (i-1)$
$= 2^{13}(-1+i)$

PEKİŞTİRME

$(i-1)^{12} = ?$

YANIT : –64

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NOTE

$i^m \cdot i^n = i^{m+n}$

INTEGER POWERS of i

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$$i^n = \begin{cases} 1, & n \equiv 0 \pmod 4 \\ i, & n \equiv 1 \pmod 4 \\ -1, & n \equiv 2 \pmod 4 \\ -i, & n \equiv 3 \pmod 4 \end{cases}$$

EXAMPLE 3

$i^{49} = ?$

A) i B) 2i C) 3i D) 4i E) 5i

SOLUTION

The answer is A

$49 \equiv 1 \pmod 4$

$i^{49} = i$

▼ I. $(i+1)^2 = i^2 + 2i + 1$
$= -1 + 2i + 1 = 2i$

II. $(i-1)^2 = i^2 - 2i + 1$
$= -1 + 2i + 1 = -2i$

REINFORCE

$(i-1)^{12} = ?$

ANSWER : –64

EXAMPLE 4

$z = (1+i)^{27} = ?$

A) $2^{11}(-1+i)$ B) $2^{12}(-1+i)$ C) $2^{13}(-1+i)$

D) $2^{14}(-1+i)$ E) $2^{15}(-1+i)$

SOLUTION

The answer is C

$z = (1+i)^{27} = ((1+i)^2)^{13} \cdot (1+i)$

$z = (2i)^{13} \cdot (1+i) = 2^{13} \cdot i^{13} \cdot (1+i)$

$z = 2^{13} \cdot i(1+i) = 2^{13} \cdot (i-1)$

$= 2^{13}(-1+i)$

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İKİ KARMAŞIK SAYININ EŞİTLİĞİ

$z_1 = a + bi$

$z_2 = c + di,$

$z_1 = z_2 \Rightarrow a = c,\ b = d$

ÖRNEK 5

$z_1 = (2a - 3) + (b - 2)i$

$z_2 = (a - 9) + (4 - b)i,$

$z_1 = z_2 \Rightarrow a \cdot b = ?$

A) –18 B) –16 C) 0 D) 16 E) 18

ÇÖZÜM

Yanıt **A**

$z_1 = z_2 \Rightarrow 2a - 3 = a - 9\ ,\ b - 2 = 4 - b$

$a = -6\ ,\ 2b = 6$

$a = -6\ ,\ b = 3$

$\Rightarrow a \cdot b = (-6) \cdot (3) = -18$

KARMAŞIK SAYILARIN EŞLENİĞİ

$z = a + bi$ karmaşık sayısının eşleniği $\bar{z} = a - bi$ dir.

Etkinlik

a. $z = 3 + 4i \Rightarrow \bar{z} = 3 - 4i$

b. $w = 3 - i \Rightarrow \bar{w} = 3 + i$

c. $k = 5 \Rightarrow \bar{k} = 5$

d. $p = 4i \Rightarrow \bar{p} = -4i$

Eşlenik Özellikleri

1. $\overline{(\bar{z})} = z$
2. $\overline{z_1 \pm z_2} = \bar{z_1} \pm \bar{z_2}$
3. $\overline{z_1 \cdot z_2} = \bar{z_1} \cdot \bar{z_2}$
4. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z_1}}{\bar{z_2}},\ (z_2 \neq 0)$
5. $z = a + bi \Rightarrow z \cdot \bar{z} = a^2 + b^2$

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EQUALITY of TWO COMPLEX NUMBERS

$z_1 = a + bi$

$z_2 = c + di,$

$z_1 = z_2 \Rightarrow a = c,\ b = d$

EXAMPLE 5

$z_1 = (2a - 3) + (b - 2)i$

$z_2 = (a - 9) + (4 - b)i,$

$z_1 = z_2 \Rightarrow$ **a·b = ?**

A) –18 B) –16 C) 0 D) 16 E) 18

SOLUTION

The answer is **A**

$z_1 = z_2 \Rightarrow 2a - 3 = a - 9\ ,\ b - 2 = 4 - b$

$a = -6\ ,\ 2b = 6$

$a = -6\ ,\ \ b = 3$

$\Rightarrow a \cdot b = (-6)\cdot(3) = -18$

CONJUGATE COMPLEX NUMBERS

The conjugate of $z = a + bi$ is denoted by $\bar{z}$ and $\bar{z} = a - bi$.

Etkinlik

a. $z = 3 + 4i \Rightarrow \bar{z} = 3 - 4i$

b. $w = 3 - i \Rightarrow \bar{w} = 3 + i$

c. $k = 5 \Rightarrow \bar{k} = 5$

d. $p = 4i \Rightarrow \bar{p} = -4i$

Properties of Conjugates

1. $\overline{(\bar{z})} = z$
2. $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$
3. $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$
4. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2},\ (z_2 \neq 0)$
5. $z = a + bi \Rightarrow z \cdot \bar{z} = a^2 + b^2$

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ÖRNEK 6

$3 + \bar{z} = (1 + 3i)z \Rightarrow z = ?$

A) $\frac{2}{3} - i$ B) $\frac{2}{3} + i$ C) $\frac{3}{2} - i$ D) $\frac{3}{2} + 2i$ E) $\frac{2}{3} + \frac{1}{2}i$

ÇÖZÜM

Yanıt **A**

$z = a + bi \Rightarrow \bar{z} = a - bi$

$3 + a - bi = (1 + 3i)(a + bi)$

$3 + a - bi = a - 3b + (3a + b)i$

$3 + a = a - 3b \quad , \quad -b = 3a + b$

$b = -1 \quad , \quad a = \frac{2}{3}$

$\Rightarrow z = \frac{2}{3} - i$

ÖRNEK 7

$z + 3i = \bar{z} \cdot 4i \Rightarrow z = ?$

A) $\frac{1}{3} - \frac{4}{3}i$ B) $\frac{1}{2} + \frac{3}{2}i$ C) $\frac{4}{5} + \frac{1}{5}i$

D) $\frac{8}{9} - \frac{1}{9}i$ E) $\frac{3}{7} - \frac{1}{7}i$

ÇÖZÜM

Yanıt **C**

$z = a + bi \Rightarrow \bar{z} = a - bi$

$a + (b + 3)i = (a - bi) \cdot 4i$

$a + (b + 3)i = 4b + 4ai$

$a = 4b \quad , \quad b + 3 = 4a \Rightarrow 4 / a - 4b = 0$

$b - 4a = -3$

$-15b = -3 \Rightarrow b = \frac{1}{5} \ , \ a = \frac{4}{5}$

$\Rightarrow z = \frac{4}{5} + \frac{1}{5}i$

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EXAMPLE 6

$3 + \bar{z} = (1 + 3i)z \Rightarrow z = ?$

A) $\frac{2}{3} - i$ B) $\frac{2}{3} + i$ C) $\frac{3}{2} - i$ D) $\frac{3}{2} + 2i$ E) $\frac{2}{3} + \frac{1}{2}i$

SOLUTION

The answer is A

$z = a + bi \Rightarrow \bar{z} = a - bi$

$3 + a - bi = (1 + 3i)(a + bi)$

$3 + a - bi = a - 3b + (3a + b)i$

$3 + a = a - 3b \quad , \quad -b = 3a + b$

$b = -1 \quad , \quad a = \frac{2}{3}$

$\Rightarrow z = \frac{2}{3} - i$

EXAMPLE 7

$z + 3i = \bar{z} \cdot 4i \Rightarrow z = ?$

A) $\frac{1}{3} - \frac{4}{3}i$ B) $\frac{1}{2} + \frac{3}{2}i$ C) $\frac{4}{5} + \frac{1}{5}i$

D) $\frac{8}{9} - \frac{1}{9}i$ E) $\frac{3}{7} - \frac{1}{7}i$

SOLUTION

The answer is C

$z = a + bi \Rightarrow \bar{z} = a - bi$

$a + (b + 3)i = (a - bi) \cdot 4i$

$a + (b + 3)i = 4b + 4ai$

$a = 4b \quad , \quad b + 3 = 4a \Rightarrow 4 / a - 4b = 0$

$b - 4a = -3$

$-15b = -3 \Rightarrow b = \frac{1}{5} \ , \ a = \frac{4}{5}$

$\Rightarrow z = \frac{4}{5} + \frac{1}{5}i$

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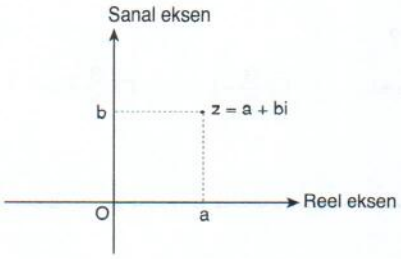
KARMAŞIK SAYILARIN ANALİTİK DÜZLEMDE GÖSTERİLMESİ



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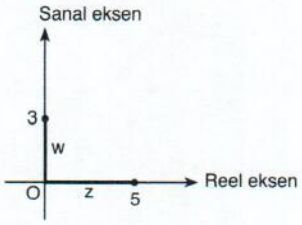
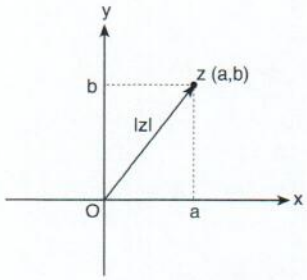


z = 5, w = 3i

BİR KARMAŞIK SAYININ MUTLAK DEĞERİ (MODÜLÜ)

Bir karmaşık sayının orijine olan uzaklığına o karmaşık sayının *mutlak değeri (modülü)* denir. |z| biçiminde gösterilir.

$$|z| = \sqrt{a^2 + b^2}$$



Modülün Özellikleri

1. $z \cdot \bar{z} = |z|^2$
2. $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
3. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ $(z_2 \neq 0)$
4. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
5. $|z^n| = |z|^n$
6. $\left|\frac{1}{z}\right| = \frac{1}{|z|}$

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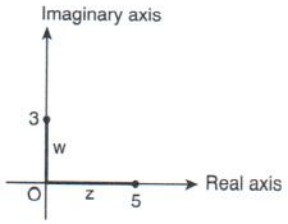
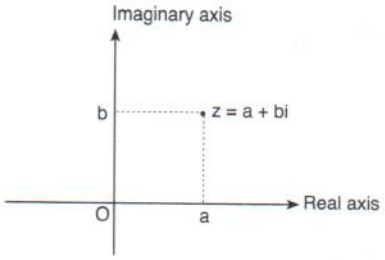
NOTE



$z = 5,\ w = 3i$

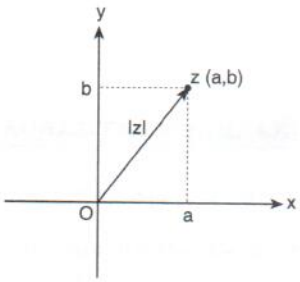
INTERPRETATION of COMPLEX NUMBERS on ANALYTIC PLANE



ABSOLUTE VALUE (MODULUS) of a COMPLEX NUMBER

The distance from a complex number z to the origin is called the *absolute value or modulus of z*. It is shown by $|z|$.

$$|z| = \sqrt{a^2 + b^2}$$



Properties of Modulus

1. $z \cdot \bar{z} = |z|^2$
2. $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
3. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\ (z_2 \neq 0)$
4. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
5. $|z^n| = |z|^n$
6. $\left|\frac{1}{z}\right| = \frac{1}{|z|}$

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ÖRNEK 8

$z = \dfrac{(3+\sqrt{3}i)^2 \cdot (2-2\sqrt{3}i)}{3-3\sqrt{3}i} \Rightarrow |z| = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

ÇÖZÜM

Yanıt **D**

$$|z| = \left|\frac{(3+\sqrt{3}i)^2 \cdot (2-2\sqrt{3}i)}{3-3\sqrt{3}i}\right|$$

$$= \frac{|3+\sqrt{3}i|^2 \cdot |2-2\sqrt{3}i|}{|3-3\sqrt{3}i|}$$

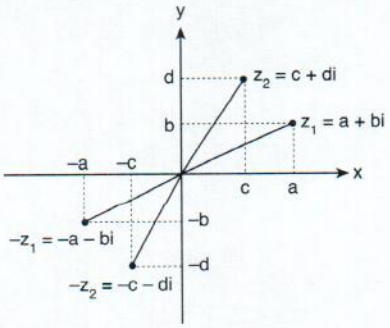
$$= \frac{(\sqrt{9+3})^2(\sqrt{4+12})}{\sqrt{9+27}} = \frac{12\cdot 4}{6} = 2\cdot 4 = 8$$

KARMAŞIK SAYILARDA DÖRT İŞLEM

$z_1 = a + bi$, $z_2 = c + di$

1. $z_1 + z_2 = (a + c) + (b + d)i$
2. $z_1 - z_2 = (a - c) + (b - d)i$
3. $z_1 \cdot z_2 = (a + bi)\cdot(c + di)$
4. $\dfrac{z_1}{z_2} = \dfrac{a+bi}{c+di}$

NOT



ÖRNEK 9

$z_1 = 4 - 3i$, $z_2 = 3 + i \Rightarrow z_1 \cdot z_2 = ?$

A) 15 + 5i B) 15 – 5i C) 10 + 5i D) 10 – 5i E) 10 + 15i

ÇÖZÜM

Yanıt **B**

$z_1 \cdot z_2 = (4 - 3i)\cdot(3 + i)$

$= 12 + 4i - 9i - 3i^2$

$= 12 - 5i + 3$

$= 15 - 5i$

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EXAMPLE 8

$$z = \frac{(3+\sqrt{3}i)^2 \cdot (2-2\sqrt{3}i)}{3-3\sqrt{3}i} \Rightarrow |z| = ?$$

A) 2 B) 4 C) 6 D) 8 E) 10

SOLUTION

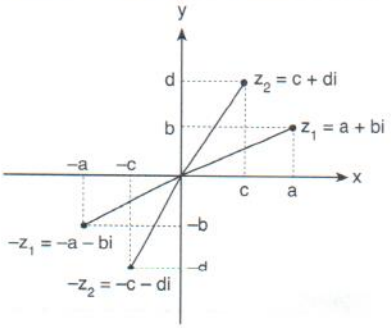
The answer is **D**

$$|z| = \left|\frac{(3+\sqrt{3}i)^2 \cdot (2-2\sqrt{3}i)}{3-3\sqrt{3}i}\right|$$

$$= \frac{|3+\sqrt{3}i|^2 \cdot |2-2\sqrt{3}i|}{|3-3\sqrt{3}i|}$$

$$= \frac{(\sqrt{9+3})^2(\sqrt{4+12})}{\sqrt{9+27}} = \frac{12 \cdot 4}{6} = 2 \cdot 4 = 8$$

NOTE



FOUR OPERATIONS in the COMPLEX NUMBERS

$z_1 = a + bi$, $z_2 = c + di$

1. $z_1 + z_2 = (a + c) + (b + d)i$
2. $z_1 - z_2 = (a - c) + (b - d)i$
3. $z_1 \cdot z_2 = (a + bi) \cdot (c + di)$
4. $\frac{z_1}{z_2} = \frac{a+bi}{c+di}$

EXAMPLE 9

$z_1 = 4 - 3i$, $z_2 = 3 + i \Rightarrow z_1 \cdot z_2 = ?$

A) 15 + 5i B) 15 – 5i C) 10 + 5i D) 10 – 5i E) 10 + 15i

SOLUTION

The answer is **B**

$$z_1 \cdot z_2 = (4 - 3i) \cdot (3 + i)$$
$$= 12 + 4i - 9i - 3i^2$$
$$= 12 - 5i + 3$$
$$= 15 - 5i$$

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ÖRNEK 10

$z_1 = 3 - 4i$,

$z_2 = 2 + i \Rightarrow \dfrac{z_1}{z_2} = ?$

A) $\dfrac{4+7i}{6}$ B) $\dfrac{5-9i}{5}$ C) $\dfrac{2-11i}{5}$

D) $\dfrac{1-5i}{4}$ E) $\dfrac{4-11i}{5}$

ÇÖZÜM

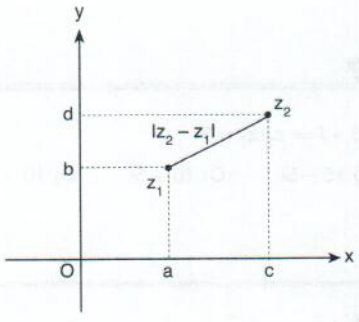
Yanıt C

$$\frac{z_1}{z_2} = \frac{3-4i}{2+i} \quad (2-i)$$

$$= \frac{6-3i-8i-4}{4-i^2}$$

$$= \frac{2-11i}{5}$$

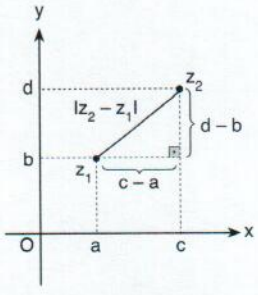
KARMAŞIK DÜZLEMDE İKİ NOKTA ARASINDAKİ UZAKLIK



$z_1 = a + bi$, $z_2 = c + di$

$$|z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$$

KANIT



$(d-b)^2 + (c-a)^2 = (z_2 - z_1)^2$ [Pisagor]

$$\Rightarrow |z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$$

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EXAMPLE 10

$z_1 = 3 - 4i$,

$z_2 = 2 + i \Rightarrow \frac{z_1}{z_2} = ?$

A) $\frac{4+7i}{6}$ B) $\frac{5-9i}{5}$ C) $\frac{2-11i}{5}$

D) $\frac{1-5i}{4}$ E) $\frac{4-11i}{5}$

SOLUTION

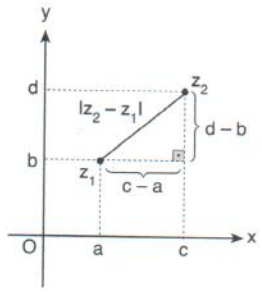
The answer is **C**

$$\frac{z_1}{z_2} = \frac{3-4i}{2+i} \quad (2-i)$$

$$= \frac{6-3i-8i-4}{4-i^2}$$

$$= \frac{2-11i}{5}$$

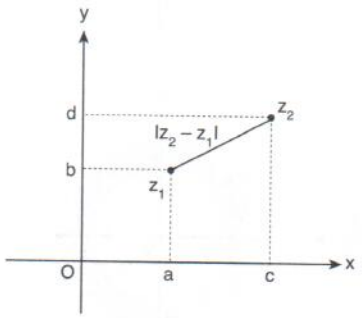
PROOF



$(d-b)^2 + (c-a)^2 = (z_2 - z_1)^2$ [Pythagorean]

$\Rightarrow |z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$

THE DISTANCE BETWEEN the TWO POINTS on the COMPLEX PLANE



$z_1 = a + bi$, $z_2 = c + di$

$|z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$

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ÖRNEK 11

$z_1 = x + 5i$,

$z_2 = 3 - 4i$,

$|z_1 - z_2| = 15 \Rightarrow \sum x = ?$

A) 3 B) 6 C) 9 D) 10 E) 12

ÇÖZÜM

Yanıt B

$z_1 - z_2 = x - 3 + 9i$

$|z_1 - z_2| = \sqrt{(x-3)^2 + 9^2} = 15$

$(x-3)^2 + 81 = 225$

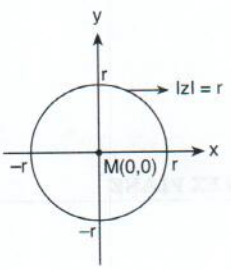
$(x-3)^2 = 144$

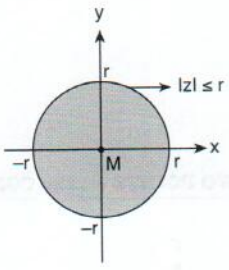
$x - 3 = 12 \quad x = 15$

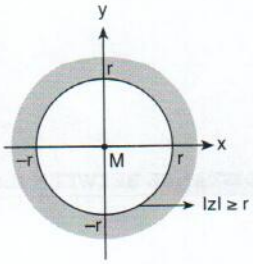
$x - 3 = -12 \quad x = -9$

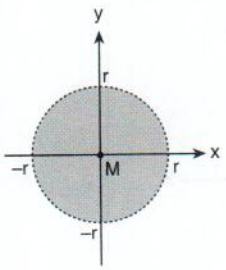
$\Rightarrow \sum x = 15 - 9 = 6$

Sonuçlar

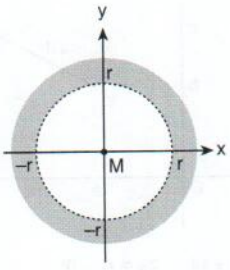




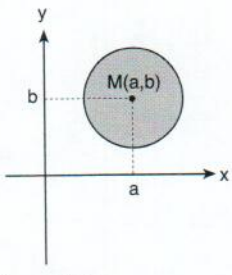




$|z| < r$



$|z| > r$



$|z - (a+bi)| \le r$

NOT

$z = x + yi$, $z_1 = a + bi$ olmak üzere,

- ✓ $|z - z_1| = r$, M(a,b) merkezli r yarıçaplı bir çember belirtir.
- ✓ $|z - z_1| < r$, çemberin içini belirtir.
- ✓ $|z - z_1| > r$, çemberin dışını belirtir.

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EXAMPLE 11

$z_1 = x + 5i$,

$z_2 = 3 - 4i$,

$|z_1 - z_2| = 15 \Rightarrow \sum x = ?$

A) 3 B) 6 C) 9 D) 10 E) 12

SOLUTION

The answer is B

$z_1 - z_2 = x - 3 + 9i$

$|z_1 - z_2| = \sqrt{(x-3)^2 + 9^2} = 15$

$(x-3)^2 + 81 = 225$

$(x-3)^2 = 144$

$x - 3 = 12 \quad x - 3 = -12$

$x = 15 \quad x = -9$

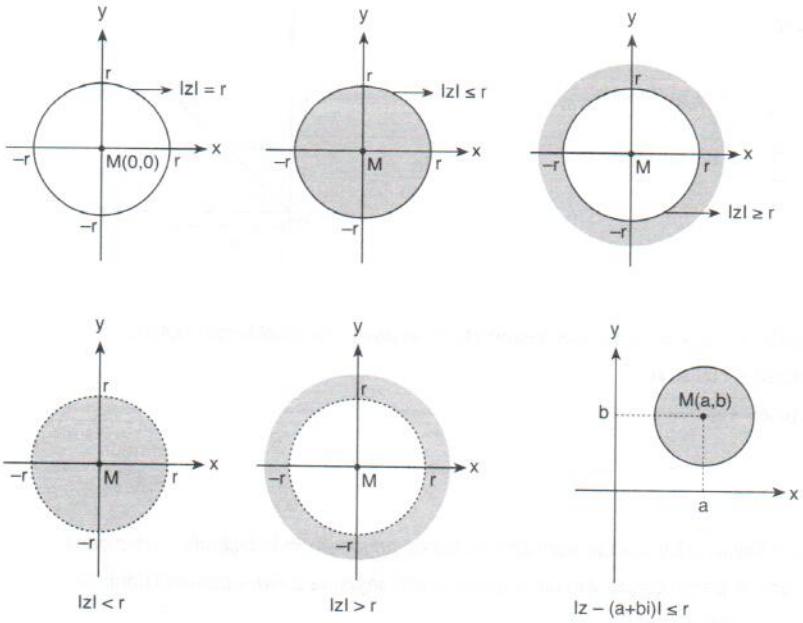
$\Rightarrow \sum x = 15 - 9 = 6$

NOTE

Let $z = x + yi$, $z_1 = a + bi$,

- ✓ $|z - z_1| = r$ represents a circle with the center M(a,b) and radius of r.
- ✓ $|z - z_1| < r$ represents interior of the circle.
- ✓ $|z - z_1| > r$ represents exterior of the circle.

Conclusions



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ÖRNEK 12

$|z| \le 4 \Rightarrow \min(|z - 5 - 12i|) = ?$

A) 17 B) 15 C) 14 D) 9 E) 7

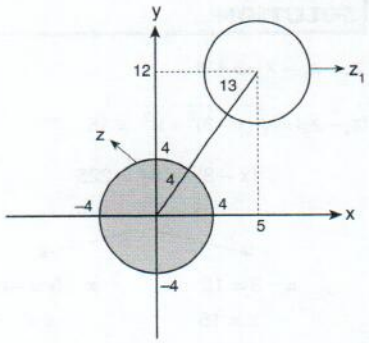
ÇÖZÜM

Yanıt **D**

$|z| \le 4$, $z_1 = 5 + 12i$

$|z_1| = 13$, $|z| = 4$

$\Rightarrow \min(|z - 5 - 12i|) = 13 - 4 = 9$

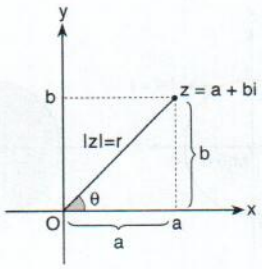


KARMAŞIK SAYILARIN KUTUPSAL FORMDA YAZILIŞI

$z = a + bi$,

$\cos\theta = \frac{a}{|z|} \Rightarrow a = |z| \cdot \cos\theta$

$\sin\theta = \frac{b}{|z|} \Rightarrow b = |z| \cdot \sin\theta$



Bu değerler $z = a + bi$ ifadesinde yerine yazılırsa aşağıdaki denklemler bulunur.

$z = |z| \cdot \cos\theta + i\,|z| \cdot \sin\theta$

$z = |z|\,(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\theta$

Karmaşık sayıların bu şekilde yazılışına *kutupsal (trigonometrik) biçimde yazılış* denir.
Ox eksenli ile pozitif yönde yapılan θ gerçel (reel) sayısına *z nin argümenti* denir ve arg(z) ile gösterilir.

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EXAMPLE 12

$|z| \le 4 \Rightarrow \min(|z - 5 - 12i|) = ?$

A) 17 B) 15 C) 14 D) 9 E) 7

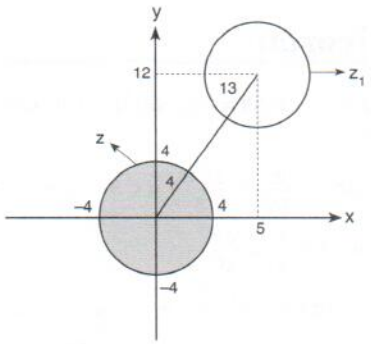
SOLUTION

The answer is D

$|z| \le 4, \quad z_1 = 5 + 12i$

$|z_1| = 13, \quad |z| = 4$

$\Rightarrow \min(|z - 5 - 12i|) = 13 - 4 = 9$

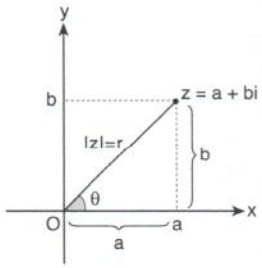


POLAR (Trigonometric) FORM of COMPLEX NUMBERS

$z = a + bi,$

$\cos\theta = \frac{a}{|z|} \Rightarrow a = |z| \cdot \cos\theta$

$\sin\theta = \frac{b}{|z|} \Rightarrow b = |z| \cdot \sin\theta$



Substituting these values in $z = a + bi$ we get

$z = |z| \cdot \cos\theta + i\,|z| \cdot \sin\theta$

$z = |z|\,(\cos\theta + i\sin\theta) = r\,\text{cis}\theta$

This form is called the *polar* or *trigonometric* form of a complex number. The angle θ is called the *argument of z*, shown by arg(z).

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ÖRNEK 13

$z = -3 - 3\sqrt{3}i \Rightarrow z = r\text{cis}\theta = ?$

A) $2\text{cis}\frac{4\pi}{3}$ B) $4\text{cis}\frac{4\pi}{3}$ C) $6\text{cis}\frac{4\pi}{3}$

D) $8\text{cis}\frac{4\pi}{3}$ E) $10\text{cis}\frac{4\pi}{3}$

ÇÖZÜM

Yanıt C

$z = -3 - 3\sqrt{3}i = (-3, -3\sqrt{3})$ [III. bölgede]

$\theta = \pi + \alpha$

$\tan\alpha = \frac{|b|}{|a|} = \frac{3\sqrt{3}}{3} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$r = |z| = \sqrt{a^2 + b^2} = \sqrt{9+27} = \sqrt{36} = 6$

$z = r(\cos\theta + i\ \sin\theta) = 6\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = 6\text{cis}\frac{4\pi}{3}$

ÖRNEK 14

$z = r\text{cis}\theta = ?$

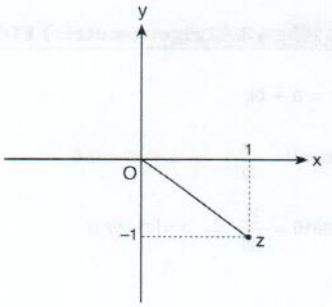
A) $2\text{cis}\frac{7\pi}{4}$

B) $2\text{cis}\frac{3\pi}{4}$

C) $\sqrt{2}\ \text{cis}\frac{7\pi}{4}$

D) $\sqrt{2}\ \text{cis}\frac{3\pi}{4}$

E) $\text{cis}\frac{7\pi}{4}$



ÇÖZÜM

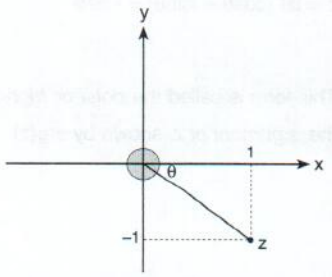
Yanıt C

$\tan\theta = \frac{\text{Im}(z)}{\text{Re}(z)} = \frac{1}{-1} = -1$

$\Rightarrow \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$|z| = r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$z = \sqrt{2}\ \text{cis}\frac{7\pi}{4}$



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EXAMPLE 13

$z = -3 - 3\sqrt{3}i \Rightarrow z = r\text{cis}\theta = ?$

A) $2\text{cis}\frac{4\pi}{3}$ B) $4\text{cis}\frac{4\pi}{3}$ C) $6\text{cis}\frac{4\pi}{3}$

D) $8\text{cis}\frac{4\pi}{3}$ E) $10\text{cis}\frac{4\pi}{3}$

SOLUTION

The answer is C

$z = -3 - 3\sqrt{3}i = (-3, -3\sqrt{3})$ [Quadrant 3]

$\theta = \pi + \alpha$

$\tan\alpha = \frac{|b|}{|a|} = \frac{3\sqrt{3}}{3} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$r = |z| = \sqrt{a^2 + b^2} = \sqrt{9+27} = \sqrt{36} = 6$

$z = r(\cos\theta + i\ \sin\theta) = 6\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = 6\text{cis}\frac{4\pi}{3}$

EXAMPLE 14

$z = r\text{cis}\theta = ?$

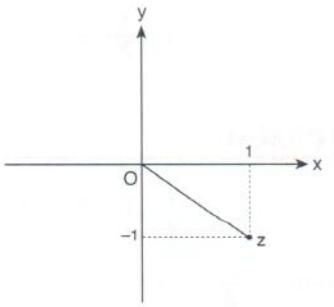
A) $2\text{cis}\frac{7\pi}{4}$

B) $2\text{cis}\frac{3\pi}{4}$

C) $\sqrt{2}\ \text{cis}\frac{7\pi}{4}$

D) $\sqrt{2}\ \text{cis}\frac{3\pi}{4}$

E) $\text{cis}\frac{7\pi}{4}$



SOLUTION

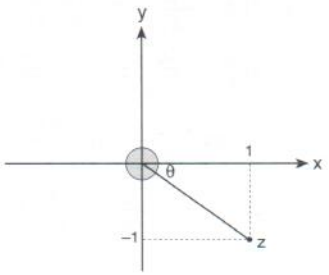
The answer is C

$\tan\theta = \frac{\text{Im}(z)}{\text{Re}(z)} = \frac{1}{-1} = -1$

$\Rightarrow \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$|z| = r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$z = \sqrt{2}\ \text{cis}\frac{7\pi}{4}$



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Kutupsal Gösterimin Özellikleri

$z_1 = r_1 \text{cis}\theta_1$, $z_2 = r_1 \text{cis}\theta_2$

1. $z_1 \cdot z_2 = r_1 \cdot r_2 \text{cis}(\theta_1 + \theta_2)$
2. $\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
3. $z_1^n = r_1^n \text{cis}(\theta \cdot n)$ [De Moivre]

ÖRNEK 15

$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i \Rightarrow z^{23} = ?$

A) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ B) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ C) $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

D) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ E) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

ÇÖZÜM

Yanıt **A**

$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ [IV.bölgede]

$\tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6}$

$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$r = |z| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$z = 1\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

$z^{23} = 1^{23}\left(\cos\frac{23 \cdot 11\pi}{6} + i\sin\frac{23 \cdot 11\pi}{6}\right)$

$z^{23} = 1\left(\cos\frac{253\pi}{6} + i\sin\frac{253\pi}{6}\right)$

$z^{23} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

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Properties of Polar Form

$z_1 = r_1 \text{cis}\theta_1$, $z_2 = r_1 \text{cis}\theta_2$

1. $z_1 \cdot z_2 = r_1 \cdot r_2 \text{cis}(\theta_1 + \theta_2)$
2. $\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
3. $z_1^n = r_1^n \text{cis}(\theta \cdot n)$ [De Moivre]

EXAMPLE 15

$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i \Rightarrow z^{23} = ?$

A) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ B) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ C) $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

D) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ E) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

SOLUTION

The answer is **A**

$$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \quad [\text{ Quadrant IV }]$$

$$\tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6}$$

$$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$r = |z| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$$

$$z = 1\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$

$$z^{23} = 1^{23}\left(\cos\frac{23 \cdot 11\pi}{6} + i\sin\frac{23 \cdot 11\pi}{6}\right)$$

$$z^{23} = 1\left(\cos\frac{253\pi}{6} + i\sin\frac{253\pi}{6}\right)$$

$$z^{23} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

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ÖRNEK 16

$z_1 = (1+i)^4$, $z_2 = (1+\sqrt{3}i)^6 \Rightarrow \frac{z_1}{z_2} = ?$

A) $-\frac{1}{32}$ B) $-\frac{1}{24}$ C) $-\frac{1}{16}$ D) $-\frac{1}{8}$ E) $-\frac{1}{4}$

ÇÖZÜM

Yanıt C

$$z_1 = \left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^4 , \quad z_2 = \left(2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^6$$

$$\frac{z_1}{z_2} = \frac{4(\cos\pi+i\sin\pi)}{64(\cos 2\pi+i\sin 2\pi)} = \frac{1}{16}(\cos(-\pi)+i\sin(-\pi))$$

$$= \frac{1}{16}(\cos\pi - i\sin\pi)) = \frac{1}{16}(-1-0) = -\frac{1}{16}$$

BİR KARMAŞIK SAYININ KÖKLERİ

z bir karmaşık sayı ise,

$z = r(\cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi))$.

$$w_k = \sqrt[n]{z} = r^{\frac{1}{n}}\left(\cos\frac{1}{n}(\theta+2k\pi)+i\sin\frac{1}{n}(\theta+2k\pi)\right)$$

$(k \in \{0, 1, 2, 3, \ldots\ldots, k-1\})$ ifadesine z nin n. *kuvvetten kökleri* denir.

✓ z karmaşık sayısının kare köklerinden biri w_0 ise ikinci kök $w_1 = -w_0$ dır.

✓ $z = a \pm bi$ karmaşık sayısının kare kökleri aşağıdaki formül ile bulunabilir.

$$w_{0,1} = \pm\left(\sqrt{\frac{|z|+a}{2}} \pm \sqrt{\frac{|z|-a}{2}}\, i\right)$$

ÖRNEK 17

$z = -2 + 2\sqrt{3}i$ karmaşık sayısının kare köklerinden biri aşağıdakilerden hangisidir?

A) $1 - \sqrt{3}i$ B) $-1 + \sqrt{3}i$ C) $2 + \sqrt{3}i$

D) $-1 - \sqrt{3}i$ E) $2 - \sqrt{3}i$

ÇÖZÜM

Yanıt D

$z = -2 + 2\sqrt{3}i = (-2, 2\sqrt{3})$ [II. bölgede]

$$\tan\alpha = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$|z| = \sqrt{4+12} = \sqrt{16} = 4$$

$$w_0 = 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = 1+\sqrt{3}\,i$$

$$w_1 = 2\left(\cos\left(\frac{\pi}{3}+\pi\right)+i\sin\left(\frac{\pi}{3}+\pi\right)\right) = 2\left(-\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right) = -1-\sqrt{3}\,i$$

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EXAMPLE 16

$z_1 = (1+i)^4$, $z_2 = (1+\sqrt{3}i)^6 \Rightarrow \frac{z_1}{z_2} = ?$

A) $-\frac{1}{32}$ B) $-\frac{1}{24}$ C) $-\frac{1}{16}$ D) $-\frac{1}{8}$ E) $-\frac{1}{4}$

SOLUTION

The answer is C

$$z_1 = \left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^4 , \quad z_2 = \left(2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^6$$

$$\frac{z_1}{z_2} = \frac{4(\cos\pi+i\sin\pi)}{64(\cos 2\pi+i\sin 2\pi)} = \frac{1}{16}(\cos(-\pi)+i\sin(-\pi))$$

$$= \frac{1}{16}(\cos\pi - i\sin\pi) = \frac{1}{16}(-1-0) = -\frac{1}{16}$$

ROOTS of a COMPLEX NUMBER

If z is a complex number,

$z = r(\cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi))$.

$$w_k = \sqrt[n]{z} = r^{\frac{1}{n}}\left(\cos\frac{1}{n}(\theta+2k\pi)+i\sin\frac{1}{n}(\theta+2k\pi)\right)$$

($k \in \{0, 1, 2, 3, \ldots\ldots, k-1\}$) the expression above is the *n^{th} roots* of z.

✓ If one of the square roots of z is w_0, then other root is $w_1 = -w_0$

✓ The square roots of $z = a + bi$ can be found also by the following formula.

$$w_{0,1} = \pm\left(\sqrt{\frac{|z|+a}{2}} \pm \sqrt{\frac{|z|-a}{2}}\, i\right)$$

EXAMPLE 17

If $z = -2 + 2\sqrt{3}i$, which of the following is one of the square roots of z ?

A) $1 - \sqrt{3}i$ B) $-1 + \sqrt{3}i$ C) $2 + \sqrt{3}i$

D) $-1 - \sqrt{3}i$ E) $2 - \sqrt{3}i$

SOLUTION

The answer is D

$z = -2 + 2\sqrt{3}i = (-2, 2\sqrt{3})$ [Quadrant 2]

$$\tan\alpha = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$|z| = \sqrt{4+12} = \sqrt{16} = 4$$

$$w_0 = 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = 1+\sqrt{3}\,i$$

$$w_1 = 2\left(\cos\left(\frac{\pi}{3}+\pi\right)+i\sin\left(\frac{\pi}{3}+\pi\right)\right) = 2\left(-\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right) = -1-\sqrt{3}\,i$$

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1. $z = i^{85} + i^{23} - i^{41} - i^{9} \Rightarrow z = ?$

A) $-i$

B) $-2i$

C) $-3i$

D) $-4i$

E) $-5i$

2. $i^{48} \cdot i^{16} \cdot i^{-1} = ?$

A) $-i$

B) i

C) $-2i$

D) $2i$

E) $-3i$

3. $(1 + i)^5 + (1 - i)^5 + i^{25} = ?$

A) $4 - i$

B) $i - 8$

C) $8 + i$

D) $8i - 1$

E) $1 + 8i$

4. $\dfrac{(1+i)^{36}}{(1-i)^{24}} = ?$

A) -64

B) 64

C) $64i$

D) $-64i$

E) 16

5. $i^2 = -1 \Rightarrow (1 + i^5)\cdot(1 + i^7)\cdot(1 + i^9)\cdot(1 + i^{18}) = ?$

A) -2

B) 0

C) $1 + i$

D) $1 - i$

E) $-4i$

6. $z = \dfrac{1 - 3i}{3 + i} \Rightarrow \text{Im}(z) = ?$

A) -1

B) -2

C) -3

D) -4

E) -5

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7. $z = (1 + 7i)\cdot(2 - 6i) \Rightarrow |z| = ?$

A) $10\sqrt{5}$

B) $15\sqrt{2}$

C) $20\sqrt{2}$

D) $16\sqrt{5}$

E) $20\sqrt{5}$

8. $i = \sqrt{-1}$,

$z\cdot(2 - i) = 7 - i \Rightarrow z = ?$

A) $3 - i$

B) $6 + i$

C) $3 + i$

D) $2 + i$

E) $5 - i$

9. $z\cdot(1 + i) = 5 + i \Rightarrow z^{-1} = ?$

A) $\frac{1}{7} + \frac{3}{7}i$

B) $\frac{3}{11} - \frac{2}{11}i$

C) $\frac{3}{13} + \frac{2}{13}i$

D) $-\frac{2}{13} + \frac{3}{13}i$

E) $\frac{2}{3} - \frac{\sqrt{2}}{3}i$

10. $z = \frac{(3+4i)\cdot(-1+2i)}{(-1-i)\cdot(3-i)} \Rightarrow |z| = ?$

A) 5

B) 4

C) 3

D) $\frac{5}{2}$

E) 2

11. $|z| = 2 \Rightarrow z^5 = ?$

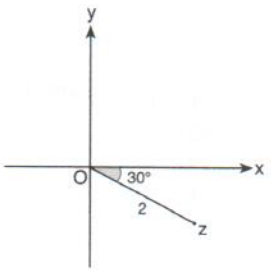
A) $32(\sqrt{3} - i)$

B) $16(\sqrt{3} - i)$

C) $16(-\sqrt{3} + i)$

D) $16(-\sqrt{3} - i)$

E) $8(1 + i)$



12. $z = 4\cdot\text{cis}\frac{11\pi}{6} = ?$

A) $2 - 2\sqrt{3}i$

B) $2 + 2\sqrt{3}i$

C) $-2\sqrt{3} + 2i$

D) $\sqrt{3} - i$

E) $2\sqrt{3} - 2i$

YANITLAR / ANSWERS

1. B	2. A	3. B	4. A	5. B	6. A	7. E	8. C
9. C	10. D	11. D	12. E				

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1. $85 \mid 4$, $84 \mid 21$, remainder 1 — $23 \mid 4$, $20 \mid 5$, remainder 3 — $41 \mid 4$, $40 \mid 10$, remainder 1 — $9 \mid 4$, $8 \mid 2$, remainder 1

$z = i^{85} + i^{23} - i^{41} - i^{9}$

$= i + i^3 - i - i$

$= -2i$

Yanıt / The Answer is **B**

2. $i^{48} \cdot i^{16} \cdot i^{-1} = i^{48+16-1} = i^{63}$

$63 \mid 4$, $60 \mid 15$, remainder 3 $\Rightarrow i^{63} = i^3 = -i$

Yanıt / The Answer is **A**

3. $(1+i)^5 + (1-i)^5 + i^{25}$

$= ((1+i)^2)^2 (1+i) + ((1-i)^2)^2 (1-i) + i$

$= (2i)^2(1+i) + (-2i)^2(1-i) + i$

$= -4(1+i) + (-4)(1-i) + i$

$= -4 - 4i - 4 + 4i + i = -8 + i$

Yanıt / The Answer is **B**

4. $\dfrac{(1+i)^{36}}{(1-i)^{24}} = \dfrac{((i+2)^2)^{18}}{((i-1)^2)^{12}}$

$= \dfrac{(2i)^{18}}{(-2i)^{12}}$

$= \dfrac{2^{18} \cdot i^{18}}{2^{12} \cdot i^{12}}$

$= 2^6 i^6$

$= -64$

Yanıt / The Answer is **A**

5. $18 \mid 4$, $16 \mid 4$, remainder 2 $\Rightarrow 1 + i^{18} = 1 + i^2 = 1 - 1 = 0$

$(1+i^5)\cdot(1+i^7)\cdot(1+i^9)\cdot\underbrace{(1+i^{18})}_{0} = 0$

Yanıt / The Answer is **B**

6. $z = \dfrac{1-3i}{3+i} \underset{(3-i)}{=} \dfrac{3-3-9i-i}{10}$

$= -i \Rightarrow \text{Im}(z) = -1$

Yanıt / The Answer is **A**

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7. $|z| = \sqrt{1^2+7^2}\cdot\sqrt{2^2+(-6)^2}$

$= \sqrt{50}\cdot\sqrt{40}$

$= 5\sqrt{2}\cdot 2\sqrt{10}$

$= 20\sqrt{5}$

Yanıt / The Answer is E

8. $z = a + bi$

$z(2-i) = 7-i$

$(a+bi)(2-i) = 7-i$

$2a + b + (2b-a)i = 7-i$

$2a + b = 7$

$2 / 2b - a = -1$

$5b = 5$

$b = 1 \Rightarrow a = 3$

$z = 3 + i$

Yanıt / The Answer is C

9. $z = a + bi$

$z(1+i) = 5+i$

$(a+bi)(1+i) = 5+i$

$a - b + (a+b)i = 5+i$

$a - b = 5$

$a + b = 1$

$2a = 6$

$a = 3 \Rightarrow b = -2 \Rightarrow z = 3-2i$

$z^{-1} = \frac{1}{3-2i} = \frac{3+2i}{13} = \frac{3}{13} + \frac{2}{13}i$

$(3+2i)$

Yanıt / The Answer is C

10. $|z| = \frac{\sqrt{3^2+4^2}\cdot\sqrt{(-1)^2+2^2}}{\sqrt{(-1)^2+(-1)^2}\cdot\sqrt{3^2+(-1)^2}}$

$= \frac{5\sqrt{5}}{\sqrt{2}\cdot\sqrt{10}}$

$= \frac{5}{2}$

Yanıt / The Answer is D

11. $z = 2\text{cis}(360° - 30°) = 2\text{cis}330°$

$z^5 = 2^5\text{cis}(330°\cdot 5)$

$= 32\cdot\text{cis}1650°$

$= 32\cdot\text{cis}210°$

$= 32(\cos 210° + i\sin 210°)$

$= 32\left(-\frac{\sqrt{3}}{2} - i\cdot\frac{1}{2}\right)$

$= 16(-\sqrt{3} - i)$

$1650° \mid 360°$

$-1440° \mid 4$

$210°$

Yanıt / The Answer is D

12. $z = 4(\cos 330° + i\sin 330°)$

$= 4\left(\frac{\sqrt{3}}{2} - i\cdot\frac{1}{2}\right)$

$= 2\sqrt{3} - 2i$

Yanıt / The Answer is E

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YÖS Soruları / YÖS Questions

1. $\left.\begin{matrix} z_1 = 2-3i \\ z_2 = 3+i \end{matrix}\right\} \Rightarrow \frac{z_2}{z_1} = ?$

A) $\frac{3}{13} - \frac{11}{13}i$ B) $\frac{3}{13} + \frac{11}{13}i$ C) $\frac{4}{13} - \frac{7}{13}i$

D) $\frac{4}{5} - \frac{11}{5}i$ E) $-\frac{4}{5} + \frac{11}{5}i$

[MARMARA ÜNİV. YÖS 2013]

2. $x, y \in R$,

$i = \sqrt{-1}$,

$z_1 = x - y + xi - 2yi$

$z_2 = 3 + 2y + 2xi - i$

$z_1 = z_2 \Rightarrow$ **y = ?**

A) $-\frac{2}{3}$ B) $-\frac{2}{5}$ C) 1

D) $\frac{2}{3}$ E) $\frac{3}{4}$

[ANKARA ÜNİV. YÖS 2013]

3. $i^2 = -1$

$i^4 + i^5 + i^6 + i^7 = ?$

A) i B) –i C) 0 D) 1 E) –1

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

4. $z_1 = 2(\cos 240° + i \sin 240°)$

$z_2 = 6(\cos 150° + i \sin 150°)$

$z_3 = 3(\cos 345° + i \sin 345°)$

$\Rightarrow \frac{z_1 z_2}{z_3} = ?$

A) $2\sqrt{2} - i2\sqrt{2}$ B) $2\sqrt{2} + 2i$

C) $2\sqrt{2}$ D) $2\sqrt{2}\,(1 + i)$

E) $2\sqrt{2} - 2i$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

5. $i^2 = -1$

$\Rightarrow \frac{256}{(1+i)^{14}} = ?$

A) $2i^5$ B) 2i C) 2 – i

D) i^8 E) $(1 + i)^{16}$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

6. $\frac{(i^{11} - 3i^{13})^2}{16i^3} = ?$

A) 0 B) –i C) i

D) –1 E) 1

[ANADOLU ÜNİV. YÖS 2011]

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7. $i^2 = -1$

$$\frac{3}{\sqrt{2}-i} + \frac{2}{i^{17}} = ?$$

A) $2\sqrt{2}+i$ B) $2\sqrt{2}-i$ C) $\sqrt{2}-i$

D) $\sqrt{2}+i$ E) $\sqrt{2}+2i$

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

8. $i^2 = -1$

$$\frac{1}{1+i} + x + yi = 2 + i$$

$$\frac{x}{y} = ?$$

A) -1 B) 1 C) $\frac{3}{2}$

D) $\frac{1}{2}$ E) 2

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

9. $i^2 = -1$

$$\frac{4}{\sqrt{3}-i} + \frac{2}{i} = ?$$

A) $\sqrt{3}+i$ B) $\sqrt{3}-i$ C) $2\sqrt{3}-i$

D) $\sqrt{3}-2i$ E) $\sqrt{3}+2i$

[YÖS 2010]

10. $i^2 = -1$

$z = 3i^{18} + i^{17} + i^{16}$

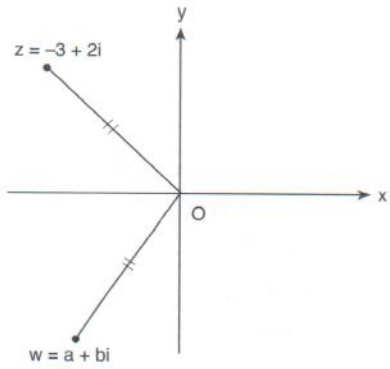
z = ?

A) $-2+i$ B) $3+i$ C) $2+3i$

D) $3-i$ E) $4-3i$

[YÖS 2009]

11. $a^2 + b^2 = ?$



A) 7 B) 9 C) 11 D) 13 E) 15

[YÖS 2009]

YANITLAR / ANSWERS

1. B	2. B	3. C	4. D	5. B	6. B	7. C	8. B
9. B	10. A	11. D					

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1. $\frac{z_2}{z_1} = \frac{3+i}{2-3i}$ $\quad (2+3i)$

$= \frac{(3+i)(2+3i)}{(2-3i)(2+3i)}$

$= \frac{3+11i}{13}$

$= \frac{3}{13} + \frac{11}{13}i$

Yanıt / The Answer is B

2. $z_1 = z_2$

$x - y + i(x-2y) = 3 + 2y + i(2x-1)$

$x - y = 3 + 2y \qquad x - 2y = 2x - 1$

$x - 3y = 3 \qquad x + 3y = 1$

$x - 3y = 3$

$+\ x + 3y = 1$

$-5y = 2 \Rightarrow y = -\frac{2}{5}$

Yanıt / The Answer is B

3. $i^4 + i^5 + i^6 + i^7 = 1 + i + (-1) + (-i)$

$= 0$

Yanıt / The Answer is C

4. $z_1 = 2(\cos 240° + i \sin 240°) = 2 \cdot \text{cis}240°$

$z_2 = 6(\cos 150° + i \sin 150°) = 6 \cdot \text{cis}150°$

$z_3 = 3(\cos 345° + i \sin 345°) = 3 \cdot \text{cis}345°$

$\frac{z_1 \cdot z_2}{z_3} = \frac{2\text{cis}240° \cdot 6\text{cis}150°}{3\text{cis}345°}$

$= 4 \cdot \text{cis}45°$

$= 4 \cdot (\cos 45° + i \sin 45°)$

$= 4 \cdot (\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})$

$= 2\sqrt{2} + 2\sqrt{2}\,i$

$= 2\sqrt{2}\,(1+i)$

Yanıt / The Answer is D

5. $\frac{256}{(1+i)^{14}} = \frac{256}{\left((1+i)^2\right)^7}$

$= \frac{256}{(2i)^7}$

$= \frac{256}{128 \cdot i^7}$

$= \frac{256}{128 \cdot (-i)} = 2i$

Yanıt / The Answer is B

6. $11 \,\big|\, \underline{4}\ 2$ $\quad$ $13 \,\big|\, \underline{4}\ 3$

$\quad 3 \qquad\qquad 1$

$\frac{(i^{11} - 3i^{13})^2}{16i^3} = \frac{(-i-3i)^2}{-16i}$

$= \frac{-16}{-16i}$

$= \frac{1}{i}$

$= -i$

Yanıt / The Answer is B

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7.

$$\begin{array}{r|l} 17 & 4 \\ -16 & 4 \\ \hline 1 & \end{array}$$

$$\frac{3}{\underset{(\sqrt{2}+i)}{\sqrt{2}-i}}+\frac{2}{\underset{(i)}{i}}=\frac{3(\sqrt{2}+i)}{2+1}+\frac{2i}{-1}$$

$$=\sqrt{2}+i-2i$$

$$=\sqrt{2}-i$$

Yanıt / The Answer is C

8. $\frac{1}{1+i}+x+yi=2+i$

$$\frac{1-i}{2}+x+yi=2+i$$

$$x+yi=\frac{4+2i-1+i}{2}=\frac{3+3i}{2} \Rightarrow \frac{x}{y}=\frac{\frac{3}{2}}{\frac{3}{2}}=1$$

Yanıt / The Answer is B

9. $\frac{4}{\underset{(\sqrt{3}+i)}{\sqrt{3}-i}}+\frac{2}{\underset{(i)}{i}}=\frac{4\sqrt{3}+4i}{4}+\frac{2i}{-1}$

$$=\sqrt{3}+i-2i$$

$$=\sqrt{3}-i$$

Yanıt / The Answer is B

10.

$$\begin{array}{r|l} 18 & 4 \\ -16 & 4 \\ \hline 2 & \end{array} \qquad \begin{array}{r|l} 17 & 4 \\ -16 & 4 \\ \hline 1 & \end{array} \qquad \begin{array}{r|l} 16 & 4 \\ -16 & 4 \\ \hline 0 & \end{array}$$

$$3\cdot i^{18}+i^{17}+i^{16}=3\cdot i^2+i+1$$

$$=-2+i$$

Yanıt / The Answer is A

11. $|z|=|w|$

$$\sqrt{(-3)^2+2^2}=\sqrt{a^2+b^2}$$

$$13=a^2+b^2$$

Yanıt / The Answer is D

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1. $i = \sqrt{-1} \Rightarrow i^{45} + i = ?$

Yanıt / The Answer is **2i**

2. $i^2 = -1 \Rightarrow (i^{175} - i^{86})^6 = ?$

Yanıt / The Answer is **8i**

3. $(1 + i)^{14} = ?$

Yanıt / The Answer is **−128i**

4. $z = a + bi$,
$3z - 5 = 1 + 3i \Rightarrow a - 2b = ?$

Yanıt / The Answer is **0**

5. $z = \dfrac{(1+i)(2-i)}{4+3i} \Rightarrow |z| = ?$

Yanıt / The Answer is $\dfrac{\sqrt{10}}{5}$

6. $z = 2 + i \Rightarrow |z^{-1}| = ?$

Yanıt / The Answer is $\dfrac{\sqrt{5}}{5}$

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7. $|z| - \bar{z} = 2 + 2\sqrt{3}\,i$
$\Rightarrow$ **z = ?**

Yanıt / The Answer is **$2(1 + \sqrt{3}i)$**

8. $i = \sqrt{-1}$,
$z = (1 - i)^{17} - (1 + i)^{17} \Rightarrow$ **z = ?**

Yanıt / The Answer is **$-512i$**

9. $4a + 6(2 - i) = 4b + 2 + 2(3 - b)i$
$\Rightarrow$ **a + b = ?**

Yanıt / The Answer is $\frac{19}{2}$

10. $z = (12 - 5i)\cdot(7 + 24i) \Rightarrow$ **|z| = ?**

Yanıt / The Answer is **325**

11. $z = 3 - 2i \Rightarrow$ **$|z \cdot \bar{z}|$ = ?**

Yanıt / The Answer is **13**

12. $z = (1 - 3i)\cdot(1 + 3i) \Rightarrow$ **|z| = ?**

Yanıt / The Answer is **10**

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13. $z = 3 - 3\sqrt{3}i \Rightarrow z^{12} = ?$

Yanıt / The Answer is 6^{12}

14. $z + 4i = iz - 8$

$\Rightarrow z = ?$

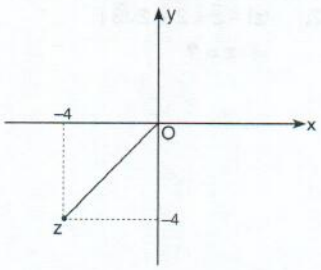
Yanıt / The Answer is $-2 - 6i$

15. $|z| + z = 8 - 4i$

$\Rightarrow z = ?$

Yanıt / The Answer is $3 - 4i$

16. $\Rightarrow z = ?$



Yanıt / The Answer is $4\sqrt{2}\,\text{cis}\,\frac{5\pi}{4}$

17. $z = 1 + i$

$\Rightarrow z^{29} = ?$

Yanıt / The Answer is $-2^{14}(1 + i)$

18. $z = (\sqrt{3} - \sqrt{3}i)^{60} = ?$

Yanıt / The Answer is -6^{30}

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1. $(1 - i^9)\cdot(1 + i^{17}) + (1 + i^{49})\cdot(1 + i^{23}) = ?$

A) 0
B) 4i
C) 4
D) –4
E) –4i

2. $(1 + i)^9 + (1 - i)^{11} = ?$

A) –48i
B) –48
C) $16\cdot(1 + i)$
D) $-16\cdot(1 + i)$
E) $16\cdot(1 - i)$

3. $i = \sqrt{-1}$
$z = i^{72} - i^{67} + i^{248} \Rightarrow z = ?$

A) i
B) 1
C) –1
D) 2 – i
E) 2 + i

4. $z = \frac{1+3i}{2i} \Rightarrow z^{-1} = ?$

A) $\frac{3}{5} + \frac{i}{5}$
B) $\frac{3}{5} - \frac{i}{5}$
C) $\frac{i}{5} - \frac{3}{5}$
D) $\frac{1}{5} + \frac{3i}{5}$
E) $-\frac{3}{5} - \frac{i}{5}$

5. $z \cdot (1 + i) = z + 4 + 3i \Rightarrow Re(z) \cdot Im(z) = ?$

A) –12
B) –10
C) –8
D) –4
E) –2

6. $z \cdot (2 + i) = 6 + 13i \Rightarrow Im(z) = ?$

A) 6
B) 5
C) 4
D) $\frac{3}{5}$
E) $\frac{2}{9}$

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7. $n \in N^+$,

$i^2 = -1 \Rightarrow i^{171} - i^{4n+2} = ?$

A) $1 - i$

B) 0

C) $-1 - i$

D) $-2i$

E) $2i$

8. $(1 + i)^{11} - (1 - i)^{11} = ?$

A) 64

B) $64i$

C) 0

D) -64

E) $-64i$

9. $i = \sqrt{-1}$,

$z = (1 + i^{15})^2 (1 - i^{23})^4 + (1 - i)(1 + i^{17}) \Rightarrow z = ?$

A) $6i$

B) $4i + 1$

C) $2(1 + 4i)$

D) $8 + i$

E) $16 - i$

10. $z_1 = 3 - 4i$, $z_2 = 2 + i \Rightarrow \frac{z_1}{z_2} = ?$

A) $\frac{4+7i}{6}$

B) $\frac{5-9i}{5}$

C) $\frac{2-11i}{5}$

D) $\frac{1-5i}{4}$

E) $\frac{4-11i}{5}$

11. $\left(\frac{1-i}{1+i}\right)^{23} \cdot i = ?$

A) $2i$

B) -1

C) $-2i$

D) 1

E) 2

12. $z \cdot (2 + i) + \bar{z} = 5 + 7i \Rightarrow z = ?$

A) $3 + 2i$

B) $4 + 3i$

C) $6 + 5i$

D) $3 + 4i$

E) $3 - 4i$

YANITLAR / ANSWERS

1. C	2. D	3. E	4. A	5. A	6. C	7. A	8. B
9. C	10. C	11. B	12. D				

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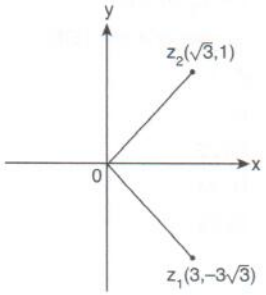
1. $z_1 = \text{cis}\frac{2\pi}{3}$ ve(and) $z_2 = \text{cis}\frac{4\pi}{3} \Rightarrow z_1 \cdot z_2 = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

2. $z_1(3, -3\sqrt{3})$,
$z_2(\sqrt{3}, 1) \Rightarrow \frac{z_1}{z_2} = ?$



A) 3 + i
B) –3 –i
C) 3i
D) –3i
E) –2i

3. $i^2 = -1 \Rightarrow \left(\frac{1+i}{1-i}\right)^{42} = ?$

A) 4i
B) –2i
C) –i
D) 1
E) –1

4. $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, $z_2 = 2 - 2\sqrt{3}i \Rightarrow |z_1 z_2|^2 = ?$

A) 4
B) 16
C) 64
D) 36
E) 216

5. $z_1 = 2 + i$, $z_2 = 7 + 3i \Rightarrow \text{im}\left(\frac{z_2}{z_1}\right) = ?$

A) $\frac{1}{5}$
B) $\frac{17}{5}$
C) $\frac{16}{5}$
D) 3
E) $-\frac{1}{5}$

6. $\left.\begin{array}{l} f : C \to C,\ f(z) = z^2 + \bar{z} \\ g : C \to C,\ g(z) = 1 + iz \end{array}\right\} \Rightarrow (fog)(1 + i) = ?$

A) –1 – i
B) –1 + i
C) 1 + i
D) 1 – 2i
E) 1 + 2i

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7. $z = -4 + 4\sqrt{3}\,i = ?$

A) $4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

B) $8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

C) $4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

D) $8\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

E) $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

8. $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow w = z^{59} = ?$

A) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

B) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

C) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

D) $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

E) $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

9. $z_1 = 1 - i,$
$z_2 = -1 + \sqrt{3}\,i,$
$z_3 = -3 - 3\sqrt{3}\,i \Rightarrow \text{Arg}(z_1 \cdot z_2 \cdot z_3) = ?$

A) $\frac{7\pi}{4}$

B) $\frac{5\pi}{3}$

C) $\frac{5\pi}{4}$

D) $\frac{7\pi}{6}$

E) $\frac{3\pi}{5}$

10. $z_1 = 2 - 2\sqrt{3}\,i,\ z_2 = 1 + i,\ z_3 = 3i$
$\Rightarrow \text{Arg}\left(\frac{z_1}{z_2 \cdot z_3}\right) = ?$

A) $\frac{\pi}{4}$

B) $\frac{5\pi}{3}$

C) $\frac{5\pi}{4}$

D) $\frac{7\pi}{12}$

E) $\frac{11\pi}{12}$

11. $z \in C,\ |z| \le 5$
$\Rightarrow \max(|z + 8 - 15i|) = ?$

A) 12

B) 17

C) 22

D) 24

E) 26

12. $z = 1 + \sqrt{3}\,i \Rightarrow \left(\frac{z - \bar{z}}{z + \bar{z}}\right)^8 = ?$

A) 81 B) 27 C) 9

D) $\frac{1}{27}$ E) $\frac{1}{81}$

YANITLAR / ANSWERS							
1. A	2. D	3. E	4. B	5. E	6. A	7. E	8. A
9. A	10. E	11. C	12. A				

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BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Briggs, Henry [1561- 1630]

İngiliz matematikçi. Briggsian logaritması olarak adlandırılan logaritmanın temelini oluşturmuştur. Napier'in yayımlamış olduğu logaritma tablosuna başvurmuş, başka alternatif önermiştir. 1617'de, Napier öldüğünde, ilk 1000 sayının logaritmasını tanımlamış ve 1624'de 14 tane ondalık basamağa sahip 30000 sayının logaritma tablosunu yapmıştır.

English mathematician who was responsible for the introduction of common logarithms (base 10), at one time called Briggsian logarithms. Following the publication of tables of logarithms by Napier, Briggs consulted him and proposed an alternative definition using base 10. In 1617,the year of Napier's death, Briggs published his logarithms of the first 1000 numbers and, in 1624, tables including 30000 logarithms to 14 decimal places.

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LOGARİTMA
LOGARITHM

BÖLÜM 2
CHAPTER

LOGARİTMİK FONKSİYONLAR

$a > 0$, $a \neq 1$ $f(x) = a^x \Rightarrow f(x)$ üstel fonksiyondur.

Tanım: $a > 0$, $a \neq 1$ ve $x > 0$ olmak üzere, $f(x) = a^x$ fonksiyonunun tersi x'in a tabanında logaritması olarak adlandırılır. $y = \log_a x$ ile gösterilir.

ÖRNEK 1

$x = \log_{\sqrt[4]{2}} 16 \Rightarrow x = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 2 D) 4 E) 16

ÇÖZÜM

Yanıt **E**

$\log_{\sqrt[4]{2}} 16 = x$

$\log_{2^{\frac{1}{4}}} 16 = x$

$(2^{\frac{1}{4}})^x = 16 = 2^4$

$\frac{x}{4} = 4 \Rightarrow x = 16$

LOGARİTMANIN ÖZELLİKLERİ

Özellik 1: 1'in herhangi a tabanındaki logaritması 0'dır.

$\log_a 1 = 0$

KANIT

$\log_a 1 = 0$ olduğundan, $a^0 = 1$ ($a > 0$, $a \neq 1$) dir.

ÖRNEK 2

$\log_2 (x^2 - x - 5) = 0 \Rightarrow \sum x = ?$

A) –1 B) 0 C) 1 D) 2 E) 3

ÇÖZÜM

Yanıt **C**

$\log_2 (x^2 - x - 5) = 0 \Rightarrow x^2 - x - 5 = 2^0 = 1$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x_1 = 3$, $x_2 = -2$

$\Rightarrow \sum x = 3 + (-2) = 1$

ETKİNLİK

a. $\log_4 1 = 0$

b. $\log_{\frac{1}{2}} 1 = 0$

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LOGARITHMIC FUNCTIONS

If $a > 0$, $a \neq 1$, $f(x) = a^x$, $f(x)$ is an exponential function.

Definition: The inverse of $f(x) = a^x$ is called the logarithm of x with the base a, where $a > 0$, $a \neq 1$ and $x > 0$. It is denoted by $y = \log_a x$.

EXAMPLE 1

$x = \log_{\sqrt{2}} 16 \Rightarrow x = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 2 D) 4 E) 16

SOLUTION

The answer is E

$\log_{\sqrt{2}} 16 = x$

$\log_{2^{\frac{1}{4}}} 16 = x$

$(2^{\frac{1}{4}})^x = 16 = 2^4$

$\frac{x}{4} = 4 \Rightarrow x = 16$

PROPERTIES of LOGARITHM

Property 1: The logarithm of 1 to any base, a, is zero.

$\log_a 1 = 0$

PROOF

$\log_a 1 = 0$ since $a^0 = 1$ ($a > 0$, $a \neq 1$)

ACTIVITY

a. $\log_4 1 = 0$

b. $\log_{\frac{1}{2}} 1 = 0$

EXAMPLE 2

$\log_2 (x^2 - x - 5) = 0 \Rightarrow \sum x = ?$

A) –1 B) 0 C) 1 D) 2 E) 3

SOLUTION

The answer is C

$\log_2 (x^2 - x - 5) = 0 \Rightarrow x^2 - x - 5 = 2^0 = 1$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x_1 = 3$, $x_2 = -2$

$\Rightarrow \sum x = 3 + (-2) = 1$

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Özellik 2: Sayının, aynı tabanındaki logaritması 1'dir.

$\log_a a = 1$

KANIT

$a^1 = a$ olduğundan, $\log_a a^1 = 1$ dir.

Etkinlik

a. $\log_4 4 = 1$

c. $\log_{0,3} 0,3 = 1$

b. $\log_{\frac{1}{2}} \frac{1}{2} = 1$

d. $\log_k k = 1$

ÖRNEK 3

$\log_3(a-2) = 1 \Rightarrow a = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt **E**

$\log_3(a-2) = 1$

$a - 2 = 3^1 = 3 \Rightarrow a = 5$

Özellik 3: Bir sayının kuvvetinin logaritması, sayının logaritması ile kuvvetinin çarpımına eşittir.

$\log_a x^n = n \cdot \log_a x$

Etkinlik

a. $\log_2 3^2 = 2 \cdot \log_2 3$

b. $\log_3 2^4 = 4 \cdot \log_3 2$

c. $\log_2 x^t = t \cdot \log_2 x$

ÖRNEK 4

$\log_3 9 + \log_3 \frac{1}{27} + \log_3 \left(3^{\frac{4}{5}}\right) = ?$

A) -1 B) $-\frac{1}{2}$ C) $-\frac{1}{3}$ D) $-\frac{1}{4}$ E) $-\frac{1}{5}$

ÇÖZÜM

Yanıt **E**

$\log_3 9 = \log_3 3^2 = 2\log_3 3 = 2$

$\log_3 \frac{1}{27} = \log_3 3^{-3} = -3\log_3 3 = -3$

$\log_3 3^{\frac{4}{5}} = \frac{4}{5}\log_3 3 = \frac{4}{5}$

$\Rightarrow \log_3 9 + \log_3 \frac{1}{27} + \log_3 3^{\frac{4}{5}} = 2 - 3 + \frac{4}{5} = -\frac{1}{5}$

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PROOF

$\log_a a = 1$ since $\log_a a^1 = a$.

Property 2: The logarithm of a number to the same number as its base is equal to one.

$$\log_a a = 1$$

Activity

a. $\log_4 4 = 1$

b. $\log_{\frac{1}{2}} \frac{1}{2} = 1$

c. $\log_{0,3} 0,3 = 1$

d. $\log_k k = 1$

EXAMPLE 3

$\log_3(a-2) = 1 \Rightarrow a = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION The answer is E

$\log_3(a-2) = 1$

$a - 2 = 3^1 = 3 \Rightarrow a = 5$

Property 3: The logarithm of a number raised to a power is the power multiplied by the logarithm of the number.

$$\log_a x^n = n \cdot \log_a x$$

Activity

a. $\log_2 3^2 = 2 \cdot \log_2 3$

b. $\log_3 2^4 = 4 \cdot \log_3 2$

c. $\log_2 x^t = t \cdot \log_2 x$

EXAMPLE 4

$$\log_3 9 + \log_3 \frac{1}{27} + \log_3 \left(3^{\frac{4}{5}}\right) = ?$$

A) -1 B) $-\frac{1}{2}$ C) $-\frac{1}{3}$ D) $-\frac{1}{4}$ E) $-\frac{1}{5}$

SOLUTION The answer is E

$\log_3 9 = \log_3 3^2 = 2\log_3 3 = 2$

$\log_3 \frac{1}{27} = \log_3 3^{-3} = -3\log_3 3 = -3$

$\log_3 3^{\frac{4}{5}} = \frac{4}{5}\log_3 3 = \frac{4}{5}$

$\Rightarrow \log_3 9 + \log_3 \frac{1}{27} + \log_3 3^{\frac{4}{5}} = 2 - 3 + \frac{4}{5} = -\frac{1}{5}$

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Özellik 4: $\log_{a^n} x^m = \frac{m}{n} \cdot \log_a x$

Etkinlik

a. $\log_{2^2} 3^2 = \frac{2}{2}\log_2 3 = \log_2 3$ **b.** $\log_3 4^2 = 2 \cdot \log_3 4$ **c.** $\log_{2^3} 3^4 = \frac{4}{3}\log_2 3$

ÖRNEK 5

$\log_{\frac{1}{2}} 16 = x$ ve $\log_3(\frac{1}{27}) = y \Rightarrow x \cdot y = ?$

A) 3 B) 6 C) 8 D) 9 E) 12

ÇÖZÜM

Yanıt **E**

$x = \log_{2^{-1}} 2^4 = \frac{4}{-1}\log_2 2 = -4$

$y = \log_3 3^{-3} = \frac{-3}{1}\log_3 3 = -3$

$\Rightarrow x \cdot y = (-4) \cdot (-3) = 12$

Özellik 5: $a^{\log_a x} = x$, $a^{\log_x b} = b^{\log_x a}$

KANIT

$y = \log_a x$

$a^y = x \Rightarrow x = a^{\log_a x}$

Etkinlik

a. $2^{\log_2 3} = 3$ **b.** $3^{2\log_3 4} = 3^{\log_3 4^2} = 16$ **c.** $2^{\log_3 5} = 5^{\log_3 2}$

ÖRNEK 6

$2^{\log x} = 3^{\log 2} \Rightarrow x = ?$

A) 3 B) 5 C) 7 D) 8 E) 9

ÇÖZÜM

Yanıt **A**

$\log 2^{\log x} = \log(3^{\log 2})$

$\log x \cdot \log 2 = \log 2 \cdot \log 3$

$\log x = \log 3$

$x = 3$

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Property 4: $\log_{a^n} x^m = \frac{m}{n} \cdot \log_a x$

Activity

a. $\log_{2^2} 3^2 = \frac{2}{2}\log_2 3 = \log_2 3$ b. $\log_3 4^2 = 2 \cdot \log_3 4$ c. $\log_{2^3} 3^4 = \frac{4}{3}\log_2 3$

EXAMPLE 5

$\log_{\frac{1}{2}} 16 = x$ and $\log_3(\frac{1}{27}) = y \Rightarrow x \cdot y = ?$

A) 3 B) 6 C) 8 D) 9 E) 12

SOLUTION The answer is **E**

$x = \log_{2^{-1}} 2^4 = \frac{4}{-1}\log_2 2 = -4$

$y = \log_3 3^{-3} = \frac{-3}{1}\log_3 3 = -3$

$\Rightarrow x \cdot y = (-4) \cdot (-3) = 12$

PROOF

Let $y = \log_a x$

$a^y = x \Rightarrow x = a^{\log_a x}$

Property 5: $a^{\log_a x} = x$, $a^{\log_x b} = b^{\log_x a}$

Etkinlik

a. $2^{\log_2 3} = 3$ b. $3^{2\log_3 4} = 3^{\log_3 4^2} = 16$ c. $2^{\log_3 5} = 5^{\log_3 2}$

EXAMPLE 6

$2^{\log x} = 3^{\log 2} \Rightarrow x = ?$

A) 3 B) 5 C) 7 D) 8 E) 9

SOLUTION The answer is **A**

$\log 2^{\log x} = \log(3^{\log 2})$

$\log x \cdot \log 2 = \log 2 \cdot \log 3$

$\log x = \log 3$

$x = 3$

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Özellik 6: İki sayının çarpımının logaritması, sayıların logaritmalarının toplamıdır.

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

Etkinlik

a. $\log_2 21 = \log_2 3 + \log_2 7$

b. $\log_3 10 = \log_3 2 + \log_3 5$

ÖRNEK 7

$\log_5 2 = m \Rightarrow \log_5 160 = ?$

A) 4m B) 5m C) 5m – 1 D) 5m + 1 E) m^5

ÇÖZÜM Yanıt **D**

$\log_5 2 = m$

$\log_5 160 = \log_5(16 \cdot 10) = \log_5 32 + \log_5 5$

$= \log_5 2^5 + 1$

$= 5 \log_5 2 + 1$

$= 5m + 1$

PEKİŞTİRME

$\log_3 6 = x \Rightarrow \log_3 2 = ?$

YANIT : x – 1

Özellik 7: İki sayının bölümünün logaritması, bölünenin logaritmasından bölenin logaritması çıkarılarak bulunur.

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

ÖRNEK 8

$\log xy^3 = 3$ ve $\log \frac{x^2}{y} = -8 \Rightarrow x \cdot y = ?$

A) 10 B) 10^{-1} C) 10^2 D) 1 E) 10^{-2}

ÇÖZÜM Yanıt **B**

$$\left.\begin{array}{l} \log x + 3\log y = 3 \\ 2\log x - \log y = -8 \end{array}\right\} \Rightarrow \begin{array}{l} \log x + 3\log y = 3 \\ 6\log x - 3\log y = -24 \end{array}$$

$7 \log x = -21$

$\log x = -3 \Rightarrow x = 10^{-3}$

$-3 + 3\log y = 3 \Rightarrow \log y = 2$

$y = 10^2$

$\Rightarrow x \cdot y = 10^{-3} \cdot 10^2 = 10^{-1}$

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Property 6: The logarithm of the product of two numbers is the sum of the logarithms of the two numbers.

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

Activity

a. $\log_2 21 = \log_2 3 + \log_2 7$

b. $\log_3 10 = \log_3 2 + \log_3 5$

REINFORCE

$\log_3 6 = x \Rightarrow \log_3 2 = ?$

ANSWER : x – 1

EXAMPLE 7

$\log_5 2 = m \Rightarrow \log_5 160 = ?$

A) 4m B) 5m C) 5m – 1 D) 5m + 1 E) m^5

SOLUTION The answer is **D**

$\log_5 2 = m$

$\log_5 160 = \log_5(16 \cdot 10) = \log_5 32 + \log_5 5$

$= \log_5 2^5 + 1$

$= 5 \log_5 2 + 1$

$= 5m + 1$

Property 7: The logarithm of the quotient of two numbers is the logarithm of the dividend minus the logarithm of the divisor.

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

EXAMPLE 8

$\log xy^3 = 3$ and $\log \frac{x^2}{y} = -8 \Rightarrow x \cdot y = ?$

A) 10 B) 10^{-1} C) 10^2 D) 1 E) 10^{-2}

SOLUTION The answer is **B**

$$\left.\begin{array}{l} \log x + 3\log y = 3 \\ 2\log x - \log y = -8 \end{array}\right\} \Rightarrow \begin{array}{l} \log x + 3\log y = 3 \\ 6\log x - 3\log y = -24 \end{array}$$

$7 \log x = -21$

$\log x = -3 \Rightarrow x = 10^{-3}$

$-3 + 3\log y = 3 \Rightarrow \log y = 2$

$y = 10^2$

$\Rightarrow x \cdot y = 10^{-3} \cdot 10^2 = 10^{-1}$

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Özellik 8: Taban Değiştirme

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Etkinlik

a. $\log_3 5 = \frac{\log_2 5}{\log_2 3}$

b. $\log_5 2 = \frac{\log_6 2}{\log_6 5}$

ÖRNEK 9

$\log_2 3 = a$,

$\log_2 7 = b$

$\Rightarrow \log_6 21 = ?$

A) $\frac{a-b}{a+1}$ B) $\frac{a+b}{a}$ C) $\frac{a-b}{a}$

D) $\frac{a+b}{a+1}$ E) $\frac{a+b}{a-1}$

ÇÖZÜM

Yanıt **D**

$$\log_6 21 = \frac{\log_2 21}{\log_2 6}$$

$$= \frac{\log_2 3 + \log_2 7}{\log_2 2 + \log_2 3}$$

$$= \frac{a+b}{a+1}$$

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Property 8: Changing a Base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Activity

a. $\log_3 5 = \frac{\log_2 5}{\log_2 3}$

b. $\log_5 2 = \frac{\log_6 2}{\log_6 5}$

EXAMPLE 9

$\log_2 3 = a$,

$\log_2 7 = b$

$\Rightarrow \log_6 21 = ?$

A) $\frac{a-b}{a+1}$ B) $\frac{a+b}{a}$ C) $\frac{a-b}{a}$

D) $\frac{a+b}{a+1}$ E) $\frac{a+b}{a-1}$

SOLUTION

The answer is **D**

$$\log_6 21 = \frac{\log_2 21}{\log_2 6}$$

$$= \frac{\log_2 3 + \log_2 7}{\log_2 2 + \log_2 3}$$

$$= \frac{a+b}{a+1}$$

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Özellik 9: $\log_a x = \frac{1}{\log_x a}$

Etkinlik

a. $\log_3 4 = \frac{1}{\log_4 3}$

b. $\log_a 5 = \frac{1}{\log_5 a}$

ÖRNEK 10

$A = \frac{2}{\log_{11} 385} + \frac{2}{\log_7 385} + \frac{2}{\log_5 385} \Rightarrow A = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt B

$A = 2\left(\frac{1}{\frac{1}{\log_{385} 11}} + \frac{1}{\frac{1}{\log_{385} 7}} + \frac{1}{\frac{1}{\log_{385} 5}}\right)$

$A = 2(\log_{385} 11 + \log_{385} 7 + \log_{385} 5)$

$A = 2(\log_{385} 11 \cdot 7 \cdot 5) = 2\log_{385} 385$

$= 2 \cdot 1$

$= 2$

PEKİŞTİRME

$\frac{1}{\log_3 5} + \frac{1}{\log_4 5} + \frac{1}{\log_6 5} = ?$

YANIT : $\log_5 72$

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Property 9: $\log_a x = \frac{1}{\log_x a}$

Activity

a. $\log_3 4 = \frac{1}{\log_4 3}$

b. $\log_a 5 = \frac{1}{\log_5 a}$

REINFORCE

$\frac{1}{\log_3 5} + \frac{1}{\log_4 5} + \frac{1}{\log_6 5} = ?$

ANSWER : $\log_5 72$

EXAMPLE 10

$A = \frac{2}{\log_{11} 385} + \frac{2}{\log_7 385} + \frac{2}{\log_5 385} \Rightarrow$ **A = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION The answer is **B**

$$A = 2\left(\frac{1}{\frac{1}{\log_{385} 11}} + \frac{1}{\frac{1}{\log_{385} 7}} + \frac{1}{\frac{1}{\log_{385} 5}}\right)$$

$A = 2(\log_{385} 11 + \log_{385} 7 + \log_{385} 5)$

$A = 2(\log_{385} 11 \cdot 7 \cdot 5) = 2 \log_{385} 385$

$= 2 \cdot 1$

$= 2$

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Özellik 10: $\log_{10} x = \log x$

ÖRNEK 11

$\log_5(\frac{1}{5}) + \log 0{,}001 = ?$

A) –5 B) –4 C) –3 D) –2 E) –1

ÇÖZÜM

Yanıt B

$$\log_5(\frac{1}{5}) + \log 0{,}001 = \log_5 5^{-1} + \log_{10} 10^{-3}$$
$$= -\log_5 5 - 3 \cdot \log_{10} 10$$
$$= -1 - 3$$
$$= -4$$

Özellik 11: Doğal Logaritma

$\log_e x = \ln x, \quad e = 2,71\ldots$

ÖRNEK 12

$\log_4 [\log_5 (\ln x)] = 0 \Rightarrow x = ?$

A) 0 B) 1 C) e^3 D) e^4 E) e^5

ÇÖZÜM

Yanıt E

$\log_4 [\log_5 (\ln x)] = 0$

$\log_5(\ln x) = 4^0 = 1$

$\ln x = 5^1 = 5$

$\log_e x = 5$

$x = e^5$

PEKİŞTİRME

$\log 100! = a \Rightarrow \log 99! = ?$

YANIT : a – 2

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Property 10: $\log_{10} x = \log x$

EXAMPLE 11

$\log_5(\frac{1}{5}) + \log 0{,}001 = ?$

A) –5 B) –4 C) –3 D) –2 E) –1

SOLUTION

The answer is **B**

$$\log_5(\frac{1}{5}) + \log 0{,}001 = \log_5 5^{-1} + \log_{10} 10^{-3}$$
$$= -\log_5 5 - 3 \cdot \log_{10} 10$$
$$= -1 - 3$$
$$= -4$$

Property 11: Natural Logarithm

$\log_e x = \ln x, \quad e = 2, 71\ldots$

REINFORCE

$\log 100! = a \Rightarrow \log 99! = ?$

ANSWER : a – 2

EXAMPLE 12

$\log_4 [\log_5 (\ln x)] = 0 \Rightarrow x = ?$

A) 0 B) 1 C) e^3 D) e^4 E) e^5

SOLUTION

The answer is **E**

$\log_4 [\log_5 (\ln x)] = 0$

$\log_5(\ln x) = 4^0 = 1$

$\ln x = 5^1 = 5$

$\log_e x = 5$

$x = e^5$

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Özellik 12: $\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$

Etkinlik

a. $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 = \log_2 5$

b. $\log_2 a \cdot \log_a 5 \cdot \log_5 2 = \log_2 2 = 1$

ÖRNEK 13

$\log_2 3 \cdot \log_3 5 \cdot \log_5 9 \cdot \log_9 16 = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt **D**

$$\log_2 3 \cdot \log_3 5 \cdot \log_5 9 \cdot \log_9 16 = \log_2 16$$
$$= \log_2 2^4$$
$$= 4\log_2 2$$
$$= 4$$

ÖRNEK 14

$\ln 4 \cdot \log_4 9 \cdot \log_3 e = ?$

A) 1 B) 2 C) 4 D) ln5 E) ln7

ÇÖZÜM

Yanıt **B**

$$\ln 4 \cdot \log_4 9 \cdot \log_3 e = \log_e 4 \cdot \log_4 9 \cdot \log_3 e$$
$$= \log_e 4 \cdot 2\log_4 3 \cdot \log_3 e$$
$$= 2$$

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Property 12: $\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$

Activity

a. $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 = \log_2 5$

b. $\log_2 a \cdot \log_a 5 \cdot \log_5 2 = \log_2 2 = 1$

EXAMPLE 13

$\log_2 3 \cdot \log_3 5 \cdot \log_5 9 \cdot \log_9 16 = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION

The answer is **D**

$$\log_2 3 \cdot \log_3 5 \cdot \log_5 9 \cdot \log_9 16 = \log_2 16$$
$$= \log_2 2^4$$
$$= 4\log_2 2$$
$$= 4$$

EXAMPLE 14

$\ln 4 \cdot \log_4 9 \cdot \log_3 e = ?$

A) 1 B) 2 C) 4 D) ln5 E) ln7

SOLUTION

The answer is **B**

$$\ln 4 \cdot \log_4 9 \cdot \log_3 e = \log_e 4 \cdot \log_4 9 \cdot \log_3 e$$
$$= \log_e 4 \cdot 2\log_4 3 \cdot \log_3 e$$
$$= 2$$

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LOGARİTMA FONKSİYONUNUN TERSİ

$$f(x) = \log_a x \Leftrightarrow f^{-1}(x) = a^x$$

ÖRNEK 15

$f(x) = \log_5(3x - 2) \Rightarrow f^{-1}(2) = ?$

A) 3 B) 6 C) 9 D) 12 E) 15

ÇÖZÜM

Yanıt C

$f(x) = \log_5 (3x - 2) \Rightarrow y = \log_5 (3x - 2)$

$5^y = 3x - 2 \Rightarrow x = \dfrac{5^y + 2}{3}$

$f^{-1}(x) = \dfrac{5^x + 2}{3} \Rightarrow f^{-1}(2) = \dfrac{5^2 + 2}{3} = \dfrac{27}{3}$

$f^{-1}(2) = 9$

ÖRNEK 16

$f(x) = 2^{5x-3} - 28 \Rightarrow f^{-1}(100) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM

Yanıt E

$y = 2^{5x-3} - 28$

$2^{5x-3} = y + 28$

$5x - 3 = \log_2(y + 28)$

$x = \dfrac{\log_2 (y + 28) + 3}{5}$

$f^{-1}(x) = \dfrac{\log_2 8(x + 28)}{5}$

$f^{-1}(100) = \dfrac{\log_2 (2^3 \cdot 128)}{5}$

$= \dfrac{\log_2 2^{10}}{5}$

$= \dfrac{10}{5} = 2$

PEKİŞTİRME

$f(x) = \log_2 (x - 4) \Rightarrow f^{-1}(x) = ?$

YANIT : $2^x + 4$

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INVERSE of a LOGARITHM FUNCTION

$$f(x) = \log_a x \Leftrightarrow f^{-1}(x) = a^x$$

EXAMPLE 15

$f(x) = \log_5(3x - 2) \Rightarrow f^{-1}(2) = ?$

A) 3 B) 6 C) 9 D) 12 E) 15

SOLUTION

The answer is C

$f(x) = \log_5 (3x - 2) \Rightarrow y = \log_5 (3x - 2)$

$5^y = 3x - 2 \Rightarrow x = \dfrac{5^y + 2}{3}$

$f^{-1}(x) = \dfrac{5^x + 2}{3} \Rightarrow f^{-1}(2) = \dfrac{5^2 + 2}{3} = \dfrac{27}{3}$

$f^{-1}(2) = 9$

REINFORCE

$f(x) = \log_2 (x - 4) \Rightarrow f^{-1}(x) = ?$

ANSWER : $2^x + 4$

EXAMPLE 16

$f(x) = 2^{5x-3} - 28 \Rightarrow f^{-1}(100) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

SOLUTION

The answer is E

$y = 2^{5x-3} - 28$

$2^{5x-3} = y + 28$

$5x - 3 = \log_2(y + 28)$

$x = \dfrac{\log_2 (y + 28) + 3}{5}$

$f^{-1}(x) = \dfrac{\log_2 8(x + 28)}{5}$

$f^{-1}(100) = \dfrac{\log_2 (2^3 \cdot 128)}{5}$

$= \dfrac{\log_2 2^{10}}{5}$

$= \dfrac{10}{5} = 2$

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LOGARİTMİK DENKLEMLER

Logaritma terimi içeren ve logaritma özellikleri kullanılarak çözülen denklemlere *logaritmik denklemler* denir.

Bir logaritmik denklemi çözmek için, denklem üstel biçimde yazılır ve değişken için çözüm yapılır.

NOT

$\log_a f(x)$ fonksiyonunda,

$a > 0$

$a \neq 1$

$f(x) > 0$ dır.

ÖRNEK 17

$\log_2 (x-1) + \log_2 (3x+1) = 6 \Rightarrow$ **ÇK = ?**

A) {3} B) {4} C) {5} D) {6} E) {7}

ÇÖZÜM

Yanıt **C**

$$\left.\begin{array}{l} x-1>0 \Rightarrow x>1, \\ 3x+1>0 \Rightarrow x>-\frac{1}{3} \end{array}\right\} \Rightarrow x>1$$

$\log_2 (x-1)\cdot(3x+1) = 6$

$(x-1)\cdot(3x+1) = 2^6$

$3x^2 - 2x - 65 = 0$

$(3x+13)(x-5) = 0 \Rightarrow x = -\frac{13}{3}, \quad x = 5$

ÇK = {5}

ÖRNEK 18

$(\log_3 x)^2 - 5(\log_3 x) + 6 = 0 \Rightarrow$ **$x_1 \cdot x_2$ = ?**

A) 3^{-7} B) 3^{-3} C) 3^2 D) 3^5 E) 3^8

ÇÖZÜM

Yanıt **D**

$t = \log_3 x \Rightarrow \quad t^2 - 5t + 6 = 0$

$(t-3)\cdot(t-2) = 0$

$t = 3 \quad , \quad t = 2$

$\log_3 x = 3 \quad , \quad \log_3 x = 2$

$x = 27 \qquad x = 9$

$\Rightarrow x_1 \cdot x_2 = 9 \cdot 27 = 3^2 \cdot 3^3 = 3^5$

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NOTE

In the function $\log_a f(x)$,

a is bigger than 0

a is not equal to 1

f(x) is bigger than 0.

LOGARITHMIC EQUATIONS

The equations involving a logarithmic term are called *logarithmic equations* that are solved by using the properties of logarithm.

To solve a logarithmic equation, rewrite the equation in exponential form and solve for the variable.

EXAMPLE 17

$\log_2 (x-1) + \log_2 (3x+1) = 6 \Rightarrow$ **SS = ?**

A) {3} B) {4} C) {5} D) {6} E) {7}

SOLUTION

The answer is **C**

$$\left.\begin{array}{l} x-1>0 \Rightarrow x>1, \\ 3x+1>0 \Rightarrow x>-\frac{1}{3} \end{array}\right\} \Rightarrow x>1$$

$$\log_2 (x-1)\cdot(3x+1) = 6$$

$$(x-1)\cdot(3x+1) = 2^6$$

$$3x^2 - 2x - 65 = 0$$

$$(3x+13)(x-5) = 0 \Rightarrow x = -\frac{13}{3},\ x = 5$$

SS = {5}

EXAMPLE 18

$(\log_3 x)^2 - 5(\log_3 x) + 6 = 0 \Rightarrow$ **$x_1 \cdot x_2$ = ?**

A) 3^{-7} B) 3^{-3} C) 3^2 D) 3^5 E) 3^8

SOLUTION

The answer is **D**

$t = \log_3 x \Rightarrow t^2 - 5t + 6 = 0$

$(t-3)\cdot(t-2) = 0$

$t = 3,\ t = 2$

$\log_3 x = 3,\ \log_3 x = 2$

$x = 27,\ x = 9$

$\Rightarrow x_1 \cdot x_2 = 9 \cdot 27 = 3^2 \cdot 3^3 = 3^5$

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LOGARİTMİK EŞİTSİZLİKLER

$a > 1,\ \log_a f(x) < \log_a g(x) \Rightarrow 0 < f(x) < g(x)$

$0 < a < 1,\ \log_a f(x) < \log_a g(x) \Rightarrow f(x) > g(x) > 0$

ÖRNEK 19

$\log_3(x^2 - 6x) > 3 \Rightarrow$ **ÇK = ?**

A) $x < 3$, $x > 9$ B) $x < -3$, $x > 9$ C) $x < -3$, $x > -9$ D) $x > 3$, $x < 9$ E) $x > -3$, $x < -9$

ÇÖZÜM

Yanıt **B**

$\log_3(x^2 - 6x) > 3$

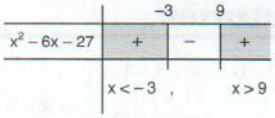
$x^2 - 6x > 3^3$, $x^2 - 6x > 0$

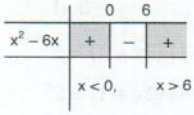
$x^2 - 6x > 27$ $x(x - 6) > 0$

$x^2 - 6x - 27 > 0$ $x_1 = 0$

$(x - 9)(x + 3) > 0$ $x_2 = 6$

$x_1 = 9,\ x_2 = -3$





İki koşulun kesişimi

$x < -3,\ x > 9$

ÖRNEK 20

$x \in Z$,

$\log_3(x + 1) + 1 < \log_3 25 \Rightarrow \sum x = ?$

A) 25 B) 26 C) 27 D) 28 E) 29

ÇÖZÜM

Yanıt **D**

$\log_3(x + 1) + 1 < \log_3 25$

$\log_3(x + 1) + \log_3 3 < \log_3 25$

$\log_3(3x + 3) < \log_3 25$

$3x + 3 < 25$

$x < \frac{22}{3}$

$x + 1 > 0 \Rightarrow x > -1,\ x < \frac{22}{3}$

$\Rightarrow \sum x = 0 + 1 + 2 + \ldots + 7 = 28$

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LOGARITHMIC INEQUALITIES

$a > 1,\ \log_a f(x) < \log_a g(x) \Rightarrow 0 < f(x) < g(x)$

$0 < a < 1,\ \log_a f(x) < \log_a g(x) \Rightarrow f(x) > g(x) > 0$

EXAMPLE 19

$\log_3(x^2 - 6x) > 3 \Rightarrow$ **SS = ?**

A) $x < 3$, $x > 9$ B) $x < -3$, $x > 9$ C) $x < -3$, $x > -9$ D) $x > 3$, $x < 9$ E) $x > -3$, $x < -9$

SOLUTION

The answer is **B**

$\log_3(x^2 - 6x) > 3$

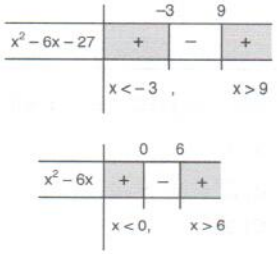
$x^2 - 6x > 3^3$, $x^2 - 6x > 0$

$x^2 - 6x > 27$ $x(x - 6) > 0$

$x^2 - 6x - 27 > 0$ $x_1 = 0$

$(x - 9)(x + 3) > 0$ $x_2 = 6$

$x_1 = 9,\ x_2 = -3$



Intersection of the two conditions

$x < -3,\ x > 9$

EXAMPLE 20

$x \in Z$,

$\log_3(x + 1) + 1 < \log_3 25 \Rightarrow \sum x = ?$

A) 25 B) 26 C) 27 D) 28 E) 29

SOLUTION

The answer is **D**

$$\log_3(x + 1) + 1 < \log_3 25$$

$$\log_3(x + 1) + \log_3 3 < \log_3 25$$

$$\log_3(3x + 3) < \log_3 25$$

$$3x + 3 < 25$$

$$x < \frac{22}{3}$$

$x + 1 > 0 \Rightarrow x > -1,\ x < \frac{22}{3}$

$\Rightarrow \sum x = 0 + 1 + 2 + \ldots + 7 = 28$

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1. $\log_2 (x-5) = 4 \Rightarrow x = ?$

A) 21
B) 16
C) 8
D) –8
E) –16

2. $\log_2 (\log_3 x) = 3 \Rightarrow x = ?$

A) 2^3
B) 2^6
C) 3^8
D) 3^6
E) -3^4

3. $\log x^2 + \log x^3 = 15 \Rightarrow x = ?$

A) 10^3
B) 10^5
C) 6^{15}
D) 2^{15}
E) 3^{10}

4. $\log_2 (x+2) + \log_2 (x-2) = 3 \Rightarrow x = ?$

A) $-2\sqrt{3}$
B) $\frac{\sqrt{3}}{2}$
C) $\sqrt{3}$
D) $2\sqrt{3}$
E) $3\sqrt{3}$

5. $\log_3 x - \log_3 (x-1) = 2 \Rightarrow x = ?$

A) $\frac{8}{9}$
B) $\frac{9}{8}$
C) $\frac{2}{3}$
D) $-\frac{2}{3}$
E) $-\frac{9}{10}$

6. $\log_3 5 = a \Rightarrow \log_5 9 = ?$

A) a
B) 2a
C) $\frac{2}{a}$
D) $-a$
E) $-\frac{a}{2}$

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7. $\log_6 2 = a \Rightarrow \log_6 9 = ?$

A) 3a

B) 6 – 3a

C) –2a

D) 2a – 4

E) 2 – 2a

8. log 2 = m ve (and) log 3 = n ⇒ **log 720 = ?**

A) 3n – 1

B) 2n + m

C) 2n + 3m +1

D) 2n + 1

E) m^5

9. $x^{\ln x} - e^6 \cdot x = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{e^3}, e^2\right\}$

B) $\left\{e^2, \frac{1}{e^2}\right\}$

C) $\left\{\frac{2}{e^2}, e^2\right\}$

D) $\left\{\frac{1}{e^2}, e^3\right\}$

E) $\left\{e^3, \frac{1}{e^3}\right\}$

10. $e^x - 12e^{-x} - 4 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {ln2}

B) {ln4}

C) {ln5}

D) {ln6}

E) {ln8}

11. $\log_3 x + \log_9 x = 5 \Rightarrow$ **x = ?**

A) $\sqrt[3]{3}$

B) $3\sqrt[3]{9}$

C) $3\sqrt[3]{3}$

D) $27\sqrt{3}$

E) $27\sqrt[3]{3}$

12. $\frac{2}{3}\log(x^2-y^2) - \frac{1}{2}[\log(x-y)+\log(x+y)] = ?$

A) $\log\sqrt{x-y}$

B) $\log\sqrt[3]{x^2+y^2}$

C) $\log\sqrt[6]{x^2+y^2}$

D) $\log\sqrt[6]{x^2-y^2}$

E) $\log\sqrt[3]{x^2-y^2}$

YANITLAR / ANSWERS

1. A	2. C	3. A	4. D	5. B	6. C	7. E	8. C
9. D	10. D	11. E	12. D				

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1. $\log_2 (x-5) = 4 \Rightarrow x - 5 = 2^4$

$x - 5 = 16$

$x = 16 + 5$

$x = 21$

Yanıt / The Answer is A

2. $\log_2 (\log_3 x) = 3 \Rightarrow \log_3 x = 2^3$

$\log_3 x = 8$

$x = 3^8$

Yanıt / The Answer is C

3. $\log x^2 + \log x^3 = \log(x^2 \cdot x^3) = \log x^5$

$\log x^5 = 15 \Rightarrow x^5 = 10^{15}$

$x = 10^3$

Yanıt / The Answer is A

4. $\log_2 (x+2) + \log_2 (x-2) = 3$

$\log_2 [(x+2)\cdot(x-2)] = 3$

$\log_2 (x^2 - 4) = 3$

$x^2 - 4 = 2^3$

$x^2 = 12$

$x = 2\sqrt{3}$

Yanıt / The Answer is D

5. $\log_3 x - \log_3 (x-1) = 2 \Rightarrow \log_3\left(\frac{x}{x-1}\right) = 2$

$\frac{x}{x-1} = 3^2 \Rightarrow \frac{x}{x-1} = 9$

$x = 9x - 9$

$x = \frac{9}{8}$

Yanıt / The Answer is B

6. $\log_3 5 = a$

$\log_5 9 = \frac{1}{\log_9 5} = \frac{1}{\log_{3^2} 5} = \frac{2}{\log_3 5} = \frac{2}{a}$

Yanıt / The Answer is C

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7. $\log_6 2 = \frac{1}{\log_2 6} = \frac{1}{\log_2 3 + 1} = a$

$\log_2 3 = \frac{1}{a} - 1 = \frac{1-a}{a} \Rightarrow \log_3 2 = \frac{a}{1-a}$

$\log_6 9 = \frac{1}{\log_9 6} = \frac{2}{\log_3 6} = \frac{2}{\log_3 2 + 1}$

$= \frac{2}{\frac{a}{1-a} + 1}$

$= \frac{2}{\frac{1}{1-a}}$

$= 2\cdot(1-a)$

$= 2 - 2a$

Yanıt / The Answer is E

8. $\log 2 = m$, $\log 3 = n$

$\log 720 = \log (72 \cdot 10) = \log 72 + \log 10$

$= \log (9 \cdot 8) + 1$

$= \log 9 + \log 8 + 1$

$= \log 3^2 + \log 2^3 + 2$

$= 2 \log 3 + 3 \log 2 + 1$

$= 2n + 3m + 1$

Yanıt / The Answer is C

9. $x^{\ln x} - e^6 \cdot x = 0 \Rightarrow x^{\ln x} = e^6 \cdot x$

$\ln(x^{\ln x}) = \ln e^6 \cdot x$

$\ln^2 x - \ln x - 6 = 0$

$(\ln x - 3)\cdot(\ln x + 2) = 0$

$\ln x - 3 = 0$, $\ln x = -2$

$x = e^3$, $x = e^{-2}$

$ÇK(SS) = \left\{\frac{1}{e^2}, e^3\right\}$

Yanıt / The Answer is D

10. $e^x - \frac{12}{e^x} - 4 = 0 \Rightarrow e^{2x} - 4e^x - 12 = 0$

$e^x = t \Rightarrow t^2 - 4t - 12 = 0$

$(t+2)\cdot(t-6) = 0$

$t + 2 = 0 \Rightarrow t = -2$, $t - 6 = 0 \Rightarrow t = 6$

$e^x = -2 \Rightarrow ÇK(SS) = Ø$

$e^x = 6 \Rightarrow x = \ln 6$

$ÇK(SS) = \{\ln 6\}$

Yanıt / The Answer is D

11. $\log_3 x + \log_9 x = \log_3 x + \frac{1}{2}\log_3 x$

$= \log_3 x + \log_3 x^{\frac{1}{2}}$

$= \log_3 (x \cdot x^{\frac{1}{2}})$

$= \log_3 x^{\frac{3}{2}} = 5$

$x^{\frac{3}{2}} = 3^5$

$x = 3^{\frac{10}{3}}$

$x = 27\sqrt[3]{3}$

Yanıt / The Answer is E

12. $\frac{2}{3}\log(x^2 - y^2) - \frac{1}{2}[\log(x-y) + \log(x+y)]$

$= \log(x^2 - y^2)^{\frac{2}{3}} - \log(x^2 - y^2)^{\frac{1}{2}}$

$= \log \frac{(x^2 - y^2)^{\frac{2}{3}}}{(x^2 - y^2)^{\frac{1}{2}}}$

$= \log \sqrt[6]{x^2 - y^2}$

Yanıt / The Answer is D

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YÖS Soruları / YÖS Questions

1. $3^{3x} = 10$

$\Rightarrow$ **x = ?**

A) log3 B) $3\log3^{10}$ C) $2\log3^{2}$

D) $\log_3 10^{1/3}$ E) $\log 3^{1/10}$

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

2. $\log a + \log b = 1$,

$a^2 + b^2 = 44 \Rightarrow$ **|a + b| = ?**

A) 5 B) 6 C) 7

D) 8 E) 9

[ANKARA ÜNİV. YÖS 2013]

3. $\log_x (\ln y) = 3$

$x = \frac{1}{3}\ln y \Rightarrow$ **y = ?**

A) e B) $e^{\sqrt{3}}$ C) e^2

D) $e^{2\sqrt{3}}$ E) $e^{3\sqrt{3}}$

[ANKARA ÜNİV. YÖS 2013]

4. $\log_2 3 = x$

$\Rightarrow$ **$\log_{12}$ 24 = ?**

A) $\frac{5}{x+2}$ B) $\frac{x+3}{x+2}$ C) $\frac{3x+1}{x+2}$

D) $\frac{2x+3}{x+3}$ E) $\frac{2x+1}{x+2}$

[İSTANBUL ÜNİV. YÖS 2013]

5. $\log_2 [2 + \log_3 (2x - 1)] = 2$

x = ?

A) 1 B) 2 C) 3 D) 4 E) 5

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

6. $\log_{\sqrt{2}} x - \log_2 x - \log_4 x = \log 0,01 \Rightarrow$ **x = ?**

A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{1}{2}$

D) 4 E) 8

[MARMARA ÜNİV. YÖS 2013]

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7. $\log_a 25 = 3$

$\log_5 a = b$

$\Rightarrow$ **b = ?**

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) $\frac{5}{2}$ E) $\frac{7}{2}$

[MARMARA ÜNİV. YÖS 2012]

8. $\log_2 3 = a$, $\log_2 5 = b$ ise, **$\log_{30}$ 48 ifadesinin a ve b'ye bağlı değeri aşağıdakilerden hangisidir?**

If $\log_2 3 = a$, $\log_2 5 = b$, which of the following is equal to the expression $\log_{30}$ 48 in terms of a and b ?

A) $\frac{a+4}{a+b}$ B) $\frac{a+4}{a+b+1}$ C) $\frac{a+4}{a+b-1}$

D) $\frac{a+4}{a+b+2}$ E) $\frac{a+3}{a+b+1}$

(DOKUZ EYLÜL ÜNİV. YÖS 2011)

9. $\log_3 (\log_4 16) = \log_{27} x^3 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 4

D) 9 E) 12

(ANADOLU ÜNİV. YÖS 2011)

10. $x > 0$

$2\log 5 + \log x = 2$

x = ?

A) 4 B) 5 C) 10

D) 25 E) 50

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

11. $\log_3 2 = m$

$\log_2$ 18 = ?

A) 2m B) 1+3m C) $\frac{m+2}{m}$

D) $\frac{2}{m}$ E) m+3

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

12. $\log_3 5 = a \Rightarrow \log_{\sqrt{27}} \left(\frac{1}{125}\right) = ?$

A) –2a B) –a C) a D) 2a E) 3a

[YÖS 2010]

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13. $\log_9 3 + \log_{16} 8 = ?$

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{5}{4}$ D) $\frac{7}{4}$ E) $\frac{9}{4}$

[YÖS 2010]

14. $x > 0$

$\log_2(x + 2) - \log_4 4x^2 = 2 \Rightarrow$ **x = ?**

A) $\frac{2}{5}$ B) $\frac{1}{6}$ C) $\frac{2}{7}$ D) $\frac{3}{8}$ E) $\frac{4}{9}$

[YÖS 2009]

15. $2^{x-2} = 5$

x = ?

A) $-2 + \log_2 5$ B) $2 + \log_2 5$ C) $5 - \log_2 5$

D) $2 - \log_5 2$ E) $5 - \log_5 2$

[YÖS 2008]

16. $x \in R^+,\ x \neq 1$

$\log_{0,001} x^2 = p\log_{10} x$

p = ?

A) $-\frac{2}{5}$ B) $-\frac{2}{3}$ C) $-\frac{1}{3}$ D) $\frac{3}{2}$ E) $\frac{5}{2}$

[YÖS 2008]

17. $\log_2 (x - 3) + \log_2 (x + 3) \leq 4$

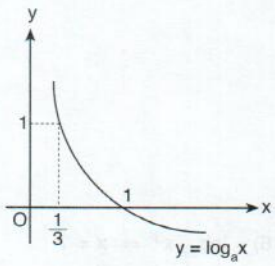
$\Rightarrow$ **? < x < ?**

A) $-5 < x < 0$ B) $-3 < x < 0$ C) $-3 < x < 5$

D) $2 \leq x < 3$ E) $3 < x \leq 5$

[YÖS 2007]

18.



$\Rightarrow \log_a 81 = ?$

A) -4 B) -3 C) -2 D) 1 E) 3

[YÖS 2006]

YANITLAR / ANSWERS							
1. D	2. D	3. E	4. B	5. E	6. A	7. B	8. B
9. B	10. A	11. C	12. A	13. C	14. C	15. B	16. B
17. E	18. A						

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1. $3^{3x} = 10 \Rightarrow 3x \cdot \log 3 = \log 10$

$$x = \frac{\log 10}{3 \cdot \log 3}$$

$$x = \log_3 10^{\frac{1}{3}}$$

Yanıt / The Answer is D

2.

$\log a + \log b = 1$	$a^2 + b^2 = (a+b)^2 - 2ab$
$\log(a \cdot b) = 1$	$44 = (a+b)^2 - 20$
$a \cdot b = 10$	$64 = (a+b)^2$
	$8 = \lvert a+b \rvert$

Yanıt / The Answer is D

3. $x = \frac{1}{3}\ln y \Rightarrow 3x = \ln y$

$\log_x(\ln y) = \log_x(3x) = 3$

$x^3 = 3x$

$x^2 = 3$

$x = \sqrt{3}$

$x = \sqrt{3} \Rightarrow \ln y = 3\sqrt{3}$

$y = e^{3\sqrt{3}}$

Yanıt / The Answer is E

4. $$\frac{\log_2 24}{\log_2 12} = \frac{\log_2 8 + \log_2 3}{\log_2 4 + \log_2 3}$$

$$= \frac{3+x}{2+x}$$

Yanıt / The Answer is B

5. $2 + \log_3 (2x - 1) = 2^2 = 4$

$\log_3 (2x - 1) = 2$

$2x - 1 = 3^2 = 9$

$2x = 10$

$x = 5$

Yanıt / The Answer is E

6. $\log_{\sqrt{2}} x - \log_2 x - \log_4 x = \log 0,01$

$\log_{\frac{1}{2^{\frac{1}{2}}}} x - \log_2 x - \log_{2^2} x = \log 10^{-2}$

$\frac{1}{\frac{1}{2}} \log_2 x - \log_2 x - \frac{1}{2}\log_2 x = -2 \cdot \log 10$

$\log_2 x \left(2 - 1 - \frac{1}{2}\right) = -2$

$(\log_2 x) \cdot \frac{1}{2} = -2$

$\log_2 x = -4 \Rightarrow x = 2^{-4} = \frac{1}{16}$

Yanıt / The Answer is A

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7. $\log_a 25 = 3 \Rightarrow a^3 = 25$

$$a = \sqrt[3]{25} = 5^{\frac{2}{3}}$$

$$\log_5 a = b \Rightarrow \log_5 5^{\frac{2}{3}} = \frac{2}{3} = b$$

Yanıt / The Answer is B

8. $$\log_{30} 48 = \frac{\log_2 48}{\log_2 30} = \frac{\log_2 16 + \log_2 3}{\log_2 2 + \log_2 3 + \log_2 5}$$

$$= \frac{4+a}{1+a+b}$$

Yanıt / The Answer is B

9. $\log_3 (\log_4 16) = \log_{27} x^3$

$\log_3 2 = \log_3 x$

$2 = x$

Yanıt / The Answer is B

10. $\log 25 + \log x = 2$

$\log x = 2 - \log 25$

$\log x = \log 100 - \log 25$

$$= \log\left(\frac{100}{25}\right)$$

$\log x = \log 4$

$x = 4$

Yanıt / The Answer is A

11. $\log_2 18 = \log_2 2 + \log_2 3^2$

$= 1 + 2\log_2 3$

$$= 1 + \frac{2}{m}$$

$$= \frac{m+2}{m}$$

Yanıt / The Answer is C

12. $$\log_{\sqrt{27}}\left(\frac{1}{125}\right) = \log_{3^{\frac{3}{2}}} 5^{-3} = \frac{-3}{\frac{3}{2}} \log_3 5$$

$$= -2\log_3 5$$

$\log_3 5 = a \Rightarrow -2 \cdot \log_3 5 = -2a$

Yanıt / The Answer is A

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13. $\log_9 3+\log_{16} 8=\frac{1}{2}\log_3 3+\frac{3}{4}\log_2 2$

$=\frac{1}{2}+\frac{3}{4}$

$=\frac{5}{4}$

Yanıt / The Answer is C

14. $\log_2 (x+2)-\log_{2^2} (2x)^2=2$

$\log_2\left(\frac{x+2}{2x}\right)=2$

$\Rightarrow \frac{x+2}{2x}=2^2 \Rightarrow 8x=x+2 \Rightarrow x=\frac{2}{7}$

Yanıt / The Answer is C

15. $2^{x-2}=5 \Rightarrow x-2=\log_2 5$

$x=\log_2 5+2$

Yanıt / The Answer is B

16. $\log_{0,001} x^2=\log_{10^{-3}} x^2=p\cdot\log_{10} x$

$\frac{2}{-3}\cdot\log_{10} x=p\cdot\log_{10} x$

$p=-\frac{2}{3}$

Yanıt / The Answer is B

17. $\log_2 (x-3)+\log_2 (x+3)\le 4$

$\Rightarrow x-3>0\ ,\ x+3>0$

$x>3\ ,\ x>-3$

$\Rightarrow x>3$ ..... **(1)**

$\log_2 (x-3)+\log_2 (x+3)\le 4$

$\log_2 (x-3)\cdot(x+3)\le 4$

$x^2-9<2^4$

$x^2-9<16 \Rightarrow x^2\le 25$

$-5\le x\le 5$ .....**(2)**

1 ve 2 denklemlerinden / *from equation 1 and 2*

$3<x\le 5$ bulunur. / *obtained.*

Yanıt / The Answer is E

18. $\log_a \frac{1}{3}=1 \Rightarrow a=\frac{1}{3}$

$\log_{\frac{1}{3}} 81=\log_{3^{-1}} 3^4=-4\log_3 3=-4$

Yanıt / The Answer is A

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1. $\log_2 (x + 1) = 2 \Rightarrow x = ?$

Yanıt / The Answer is **3**

2. $\log_2 (\log_{10} x) = 3 \Rightarrow x = ?$

Yanıt / The Answer is 10^8

3. $x > 0,\ \log_2 [\log_3 (x^2+17)\ ] = 2 \Rightarrow x = ?$

Yanıt / The Answer is **8**

4. $\left.\begin{array}{l} \ln(xy) = 3 \\ \ln x - \ln y = 1 \end{array}\right\} \Rightarrow x = ?$

Yanıt / The Answer is e^2

5. $\log_2 (x - 4) = \log_5(\log_3 3) \Rightarrow x = ?$

Yanıt / The Answer is **5**

6. $\ln e^6 + \log_{0,1} 0{,}001 = ?$

Yanıt / The Answer is **9**

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7. $\log_{\sqrt{3}} 9 + \log_5(\log_3 243) = ?$

Yanıt / The Answer is **5**

8. $2^{\log_2 5} + 5^{2\log_5 2} = ?$

Yanıt / The Answer is **9**

9. $125^{\log_5 2} + \log_5 0{,}008 = ?$

Yanıt / The Answer is **5**

10. $\log_4[\log_3(\ln x)] = 0 \Rightarrow x = ?$

Yanıt / The Answer is e^3

11. $\log_4 8 \cdot \log_8 32 = ?$

Yanıt / The Answer is $\frac{5}{2}$

12. $\log_2\left(\frac{1}{16}\right) = x \Rightarrow x = ?$

Yanıt / The Answer is **–4**

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13. $3 + \log_5 10 - \log_5 50 = ?$

Yanıt / The Answer is **2**

14. $\log (a + 3) + \log a = 1 \Rightarrow$ **ÇK (SS) = ?**

Yanıt / The Answer is **{2}**

15. $\log_3 \left(\frac{1}{3^a} \right) = 2^{\log_2 3} \Rightarrow$ **a = ?**

Yanıt / The Answer is **–3**

16. $\log_2 x + \log_2 (x + 8) = 7 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is **{8}**

17. $f(x) = \log_3 (4x - 1) + 2 \Rightarrow f^{-1}(5) = ?$

Yanıt / The Answer is **7**

18. $\log_5 (3x - 15) \leq 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is $5 < x \leq \frac{16}{3}$

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1. $\log_{\frac{1}{8}}\left[\log_9(\log_3 x)\right]=\frac{1}{3} \Rightarrow$ **x = ?**

A) 2
B) 3
C) 9
D) 27
E) 81

2. $\log_3 \sqrt[3]{a\sqrt[3]{a\sqrt[3]{a\ldots}}} = 2 \Rightarrow$ **a = ?**

A) 9
B) 27
C) 81
D) 243
E) 729

3. $x \in \left(0, \frac{\pi}{2}\right)$,

$\log_{\frac{1}{3}}(\sin x) = 2 \Rightarrow$ **cosx = ?**

A) $\frac{4\sqrt{5}}{9}$
B) $\frac{2\sqrt{5}}{3}$
C) $\frac{\sqrt{5}}{9}$
D) $\frac{\sqrt{5}}{3}$
E) $\frac{2\sqrt{5}}{5}$

4. $\log_a 3 + \log_a 4 = \frac{1}{2} \Rightarrow$ **a = ?**

A) 81
B) 100
C) 121
D) 144
E) 169

5. $\log_3(x-2)+\log_3 6 = 2 \Rightarrow$ **x = ?**

A) $\frac{7}{2}$
B) $\frac{2}{7}$
C) $\frac{3}{4}$
D) 3
E) 7

6. $\log_a \sqrt[3]{0{,}04} = -\frac{2}{3} \Rightarrow$ **$\log_a$ 625 = ?**

A) 2
B) 4
C) 6
D) 9
E) 12

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7. $\log_2(\log_3 16 \cdot \log_2 81) = ?$

A) 3
B) 4
C) 5
D) 10
E) 20

8. $3^{\log_2 x} = 5^{\log_2 3} \Rightarrow x = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

9. $3^{2x} - 5 \cdot 3^x + 6 = 0 \Rightarrow$ ÇK(SS) = ?

A) $\{4, \log_3 2\}$
B) $\{3, \log_3 2\}$
C) $\{2, \log_3 2\}$
D) $\{1, \log_3 2\}$
E) $\{\log_3 2\}$

10. $\log_a 2 + \log_a 4 + \log_a 8 = 24 \Rightarrow a = ?$

A) $\sqrt[4]{2}$
B) $\sqrt{2}$
C) $\sqrt[3]{2}$
D) $2\sqrt{2}$
E) $\sqrt[4]{3}$

11. $\log_{10}(\log_8 x) + \log_{10}(\log_x 8) = ?$

A) 0
B) 1
C) 2
D) 3
E) 5

12. $\log_5(-x) + \log_5(4 - x) = \log_5 12 \Rightarrow x = ?$

A) –2
B) –1
C) 0
D) 1
E) 2

YANITLAR / ANSWERS							
1. D	2. C	3. A	4. D	5. A	6. B	7. B	8. E
9. D	10. A	11. A	12. A				

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1. $a \in Z$,

$(\log_{a-1} 9)^{\log_2 16} = 16 \Rightarrow$ **a = ?**

A) 3

B) 4

C) 5

D) 7

E) 9

2. $\log_5 (x-2) + \log_5 (x+2) = 1 \Rightarrow$ **ÇK (SS) = ?**

A) {3}

B) {4}

C) {6}

D) {8}

E) {9}

3. $\log x + \log(2x+1) = 0 \Rightarrow$ **x = ?**

A) 0

B) $\frac{1}{2}$

C) $\frac{2}{5}$

D) $\frac{3}{7}$

E) $\frac{5}{7}$

4. $\log_7 9 \cdot \log_3 5 \cdot \log_{25} 7 = ?$

A) 1

B) $\frac{5}{3}$

C) 2

D) $\frac{7}{3}$

E) 3

5. $\log_7 10 = a \Rightarrow \log_{100} 7 = ?$

A) $\frac{1}{a}$

B) $\frac{2}{a}$

C) $\frac{1}{2a}$

D) a^2

E) 2a

6. $\log 2 = a$ ve (and) $\log 3 = b \Rightarrow \log 108 = ?$

A) a + b

B) a + 2b

C) 2a + b

D) 2a + 3b

E) 3a + 4b

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7. $\log_5 7 = x$ ve (and) $\log_5 2 = y \Rightarrow \log_{49} 40 = ?$

A) $\frac{1+3y}{2x}$

B) $\frac{1+2y}{3x}$

C) $\frac{1+2x}{3y}$

D) $\frac{1+3x}{2y}$

E) $\frac{x+3y}{2}$

8. $\log_2 3 = a$ ve (and) $\log_8 3 = b \Rightarrow a = ?$

A) $\frac{b}{3}$

B) $\frac{b}{2}$

C) b

D) 2b

E) 3b

9. $\log_3 2x \cdot \log_2 3 = 3 \Rightarrow x = ?$

A) 2

B) 3

C) 4

D) 5

E) 6

10. $4^a = 27 \Rightarrow \log_6 12 = ?$

A) $\frac{2a+5}{a+3}$

B) $\frac{2a+3}{2a+5}$

C) $\frac{2a+4}{a+6}$

D) $\frac{2a+3}{3a+2}$

E) $\frac{2a+6}{2a+3}$

11. $\left.\begin{matrix}\log_3 5 = a \\ \log_3 2 = b\end{matrix}\right\} \Rightarrow \log_6 30 = ?$

A) $\frac{a+b}{1+b}$

B) $\frac{a+b+1}{1+a}$

C) $\frac{a+b+1}{1+b}$

D) $\frac{a-b+1}{1+a}$

E) $\frac{a-b+1}{1+b}$

12. $\log_a b = 6 \Rightarrow \log_a bc + \log_a \frac{b}{c} = ?$

A) 15

B) 14

C) 13

D) 12

E) 11

YANITLAR / ANSWERS							
1. B	2. A	3. B	4. A	5. C	6. D	7. A	8. E
9. C	10. E	11. C	12. D				

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1. $\left.\begin{array}{l}\log_b x = 2\\ \log_{\frac{1}{b}} y = -4\end{array}\right\} \Rightarrow \log_b (x \cdot y) = ?$

A) 2
B) 3
C) 4
D) 5
E) 6

2. $\sqrt[3]{\log_2 (x+2)+2} = 2 \Rightarrow x = ?$

A) 54
B) 62
C) 96
D) 124
E) 128

3. $\dfrac{(\log_2 20)^2 - (\log_2 5)^2}{\log_2 10} = ?$

A) 1
B) 2
C) 3
D) 4
E) 6

4. $3^n = a,$
$\log_a 81^2 = n^2 \Rightarrow n = ?$

A) 5
B) 4
C) 3
D) 2
E) 1

5. $\left.\begin{array}{l}\log_2 3 = a\\ \log_2 5 = b\end{array}\right\} \Rightarrow \log 60 = ?$

A) $\dfrac{2+a+b}{1+b}$
B) $\dfrac{2+a-b}{1+b}$
C) $\dfrac{2-a-b}{1+b}$
D) $\dfrac{2(a+b)}{1+b}$
E) $\dfrac{2(a-b)}{1+b}$

6. $\log_3 5 = a \Rightarrow \log_{81} 15 = ?$

A) $\dfrac{a+1}{2}$
B) $a+1$
C) $\dfrac{4}{a+1}$
D) $\dfrac{a-1}{4}$
E) $\dfrac{a+1}{4}$

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7. $\log_3 4 = x \Rightarrow \log_3 162 = ?$

A) $\frac{x-8}{2}$

B) $\frac{x+8}{2}$

C) $x+4$

D) $x-4$

E) $\frac{x-4}{2}$

8. $\frac{1}{\log_2 18} + \frac{1}{\log_6 18} + \frac{1}{\log_{27} 18} = ?$

A) 2

B) 3

C) 4

D) 5

E) 6

9. $f(x) = 3 + 2 \cdot \log_{16}(3x - 2) \Rightarrow f^{-1}(3) = ?$

A) 1

B) 3

C) 4

D) 5

E) 6

10. $x \in Z$

$\log_4(2x - 5) < \log_2 3 \Rightarrow \sum x = ?$

A) 10

B) 14

C) 15

D) 18

E) 22

11. $\log_{\frac{1}{3}}(x-2) > 3 \Rightarrow$ ÇK(SS) = ?

A) $1 < x < \frac{55}{27}$

B) $2 < x < \frac{55}{27}$

C) $3 < x < \frac{55}{27}$

D) $4 < x < \frac{55}{27}$

E) $5 < x < \frac{55}{27}$

12. $a = \log_4 5,\ b = \log_{\frac{1}{5}} 4,\ c = \log_5 4 \Rightarrow ? < ? < ?$

A) $a < b < c$

B) $c < b < a$

C) $a < c < b$

D) $b < c < a$

E) $b < a < c$

YANITLAR / ANSWERS							
1. E	2. B	3. D	4. D	5. A	6. E	7. B	8. A
9. A	10. D	11. B	12. D				

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ÜNİTE 4

UNIT 4

Bu ünitede; tümevarım, diziler ve seriler; permütasyon, kombinasyon ve binom açılımı; matris ve determinant ayrıntılı açıklanmıştır.

Bu ünite; 117 çözümlü örnek, 54 alıştırma ve 84 test sorusundan oluşmaktadır.

In this unit, induction, sequences and series; permutation, combination and binomial expansion; matrix and determinant are explained in detailed.

This unit includes; 117 examples, 54 exercises, and 84 test questions.

- **TÜMEVARIM, DİZİLER ve SERİLER** . . . . . 361 – 404
- **PERMÜTASYON, KOMBİNASYON ve BİNOM AÇILIMI** . . . . . 405 – 422
- **MATRİS ve DETERMİNANT** . . . . . 423 – 478

- **INDUCTION, SEQUENCES, and SERIES** . . . . . 361 – 404
- **PERMUTATION, COMBİNATION, and BINOMIAL EXPANSION** . . . . . 405 – 422
- **MATRIX and DETERMINANT** . . . . . 423 – 478

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BÖLÜM 1
CHAPTER

TARİHSEL NOT / HISTORICAL NOTE

Cauchy, Augustin –Louis [1789 – 1857]

19. yüzyılda yaşamış önemli Fransız matematikçilerindendir. Limitin tanımını kullanarak süreklilik ve yakınsaklık tanımlarını geliştirmiştir. Kompleks fonksiyonlar teorisinin öncülerindendir.

One of the most important mathematicians of the early nineteenth century and a dominating figure in French mathematics. Using the definition of limit as it is now known, he developed sound definitions of continuity and convergence. he was also a pioneer in the theory of functions of a complex variable.

TÜMEVARIM, DİZİLER ve SERİLER
INDUCTION, SEQUENCES, and SERIES

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- ✔ Toplam Sembolü . . . . . 362 – 374
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- ✔ Konu Testleri . . . . . 399 – 404

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  - ■ Properties of Summation Notation
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- ✔ Sequences and Series . . . . . 383 – 389
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TÜMEVARIM, DİZİLER ve SERİLER
INDUCTION, SEQUENCES, and SERIES

BÖLÜM 1
CHAPTER

TÜMEVARIM

Tanım: Tümevarım matematikte kullanılan ispat yöntemlerinden biridir.

$\forall n \in N$ olmak üzere P(n) açık önermesinin doğruluğunu tümevarım yöntemiyle kanıtlamak için,

1. $n = 1$ için P(1) önermesinin doğruluğu gösterilir.
2. $k \in N^+$ olmak üzere, P(k) önermesi doğru varsayılır.
3. P(k + 1) önermesinin doğruluğu gösterilir.

Bu durumda, $\forall n \in N^+$ için P(n) doğrudur denir.

TOPLAM ve ÇARPIM SEMBOLLERİ

A. TOPLAM SEMBOLÜ (Σ)

$\forall n \in N^+$ ve $a_k \in R$ olmak üzere;

$a_1 + a_2 + a_3 + ... + a_n$ toplamı daha kısa olarak $\sum_{k=1}^{n} a_k$ biçiminde gösterilir ve 1'den n'e kadar a_k ların toplamı şeklinde okunur.

$\sum$ gösterimi sigma diye okunur. k ye de *toplama indisi* denir.

NOT

$\sum_{k=1}^{5} a_k$ ifadesinde k harfi yerine başka harfler kullanılabilir ve alt sınır 1 olmayabilir.

$$\sum_{i=5}^{20} i, \quad \sum_{i=-3}^{30} i, \quad \sum_{p=-7}^{13} 3^p$$

Etkinlikler

✓ $\sum_{k=1}^{6} a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

✓ $\sum_{k=3}^{7} a_k = a_3 + a_4 + a_5 + a_6 + a_7$

✓ $\sum_{k=3}^{5} (3k-8) = (3\cdot3-8) + (3\cdot4-8) + (3\cdot5-8) = 1+4+7 = 12$

✓ $\sum_{i=1}^{4} 2^i = 2^1 + 2^2 + 2^3 + 2^4 = 2+4+8+16 = 30$

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INDUCTION

Definition: Mathematical induction is a method of proving statements.

Let P(n) be an open sentence involving $\forall \in N$

1. Prove that P(1) is true for n = 1.
2. Assume that P(k) is true for n = k.
3. Show that P(k + 1) is true for n = k + 1

In this situation, for $\forall n \in N^+$, P(n) is true.

SUMMATION and MULTIPLICATION SYMBOLS

A. SUMMATION SYMBOL (Σ)

NOTE

The letter k can be replaced by any other symbols and upper limit cannot be 1.

$$\sum_{i=5}^{20} i, \quad \sum_{i=-3}^{30} i, \quad \sum_{p=-7}^{13} 3^p$$

For all $\forall n \in N^+$, $a_k \in R$;
the sum $a_1 + a_2 + a_3 + ... + a_n$ can be represented by the symbol $\sum_{k=1}^{n} a_k$ and read "the sum of a_k for k running from 1 to n."

Representation $\sum$ read "sigma." k is called the *index of summation.*

Etkinlikler

✓ $\sum_{k=1}^{6} a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

✓ $\sum_{k=3}^{7} a_k = a_3 + a_4 + a_5 + a_6 + a_7$

✓ $\sum_{k=3}^{5} (3k-8) = (3\cdot 3-8) + (3\cdot 4-8) + (3\cdot 5-8) = 1+4+7 = 12$

✓ $\sum_{i=1}^{4} 2^i = 2^1 + 2^2 + 2^3 + 2^4 = 2+4+8+16 = 30$

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ÖRNEK 1

$\sum_{i=1}^{5} 2i = ?$

A) 26 B) 28 C) 30 D) 32 E) 48

ÇÖZÜM

Yanıt C

$$\sum_{i=1}^{5} 2i = 2\cdot1+2\cdot2+2\cdot3+2\cdot4+2\cdot5$$
$$= 2+4+6+8+10$$
$$= 30$$

ÖRNEK 2

$\sum_{k=3}^{6} k^2 = ?$

A) 86 B) 82 C) 42 D) 38 E) 36

ÇÖZÜM

Yanıt A

$$\sum_{k=3}^{6} k^2 = 3^2+4^2+5^2+6^2$$
$$= 9+16+25+36$$
$$= 86$$

PEKİŞTİRME

$\sum_{t=3}^{4} 2t^2 = ?$

YANIT : 50

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EXAMPLE 1

$\sum_{i=1}^{5} 2i = ?$

A) 26 B) 28 C) 30 D) 32 E) 48

SOLUTION

The answer is C

$$\sum_{i=1}^{5} 2i = 2\cdot1+2\cdot2+2\cdot3+2\cdot4+2\cdot5$$
$$= 2+4+6+8+10$$
$$= 30$$

REINFORCE

$\sum_{t=3}^{4} 2t^2 = ?$

EXAMPLE 2

$\sum_{k=3}^{6} k^2 = ?$

A) 86 B) 82 C) 42 D) 38 E) 36

ANSWER : 50

SOLUTION

The answer is A

$$\sum_{k=3}^{6} k^2 = 3^2+4^2+5^2+6^2$$
$$= 9+16+25+36$$
$$= 86$$

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ÖRNEK 3

$$\sum_{k=1}^{78} \log_3\left(\frac{k+2}{k+3}\right) = ?$$

A) -6 B) -5 C) -4 D) -3 E) -2

ÇÖZÜM

Yanıt **D**

$$\sum_{k=1}^{78} \log_3\left(\frac{k+2}{k+3}\right) = \log_3\frac{3}{4} + \log_3\frac{4}{5} + \ldots + \log_3\frac{80}{81}$$

$$= \log_3\left(\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{80}{81}\right)$$

$$= \log_3\frac{1}{27}$$

$$= \log_3 3^{-3}$$

$$= -3\log_3 3 = -3$$

ÖRNEK 4

$$\sum_{n=1}^{2} \frac{((n-1)(n+1))!}{n^2!} = ?$$

A) $\frac{5}{4}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) $\frac{5}{2}$ E) 1

ÇÖZÜM

Yanıt **A**

$$\sum_{n=1}^{2} \frac{((n-1)(n+1))!}{n^2!} = \sum_{n=1}^{2} \frac{(n^2-1)!}{n^2(n^2-1)!}$$

$$= \sum_{n=1}^{2} \frac{1}{n^2}$$

$$= \frac{1}{1^2} + \frac{1}{2^2} = \frac{5}{4}$$

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EXAMPLE 3

$$\sum_{k=1}^{78} \log_3\left(\frac{k+2}{k+3}\right) = ?$$

A) –6 B) –5 C) –4 D) –3 E) –2

SOLUTION

The answer is D

$$\sum_{k=1}^{78} \log_3\left(\frac{k+2}{k+3}\right) = \log_3\frac{3}{4} + \log_3\frac{4}{5} + \ldots + \log_3\frac{80}{81}$$

$$= \log_3\left(\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{80}{81}\right)$$

$$= \log_3\frac{1}{27}$$

$$= \log_3 3^{-3}$$

$$= -3\log_3 3 = -3$$

EXAMPLE 4

$$\sum_{n=1}^{2} \frac{((n-1)(n+1))!}{n^2!} = ?$$

A) $\frac{5}{4}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) $\frac{5}{2}$ E) 1

SOLUTION

The answer is A

$$\sum_{n=1}^{2} \frac{((n-1)(n+1))!}{n^2!} = \sum_{n=1}^{2} \frac{(n^2-1)!}{n^2(n^2-1)!}$$

$$= \sum_{n=1}^{2} \frac{1}{n^2}$$

$$= \frac{1}{1^2} + \frac{1}{2^2} = \frac{5}{4}$$

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Toplam Sembolünün Özellikleri

1. $\sum_{k=1}^{n} c = n \cdot c$

2. $\sum_{k=1}^{n} (c \cdot a_k) = c \cdot \sum_{k=1}^{n} a_k$

3. $\sum_{k=1}^{n} (a_k \mp b_k) = \sum_{k=1}^{n} a_k \mp \sum_{k=1}^{n} b_k$

4. $1 < m < n$ için, $\sum_{k=1}^{n} a_k = \sum_{k=1}^{m} a_k + \sum_{k=m+1}^{n} a_k$

5. $\sum_{k=r}^{n} a_k = \sum_{k=r\mp p}^{n\mp p} a_{k\pm p}$

Etkinlikler

✓ $\sum_{k=1}^{9} 7 = 9 \cdot 7 = 63$

✓ $\sum_{k=3}^{6} 2k = \sum_{k=1}^{4} 2(k+2) = 2(1+2)+2(2+2)+2(3+2)+2(4+2) = 36$

✓ $\sum_{k=-2}^{1} (3k+2) = \sum_{k=1}^{4} (3(k-3)+2) = \sum_{k=1}^{4} (3k-7) = (3\cdot1-7)+(3\cdot2-7)+(3\cdot3-7)+(3\cdot4-7)$

$= -4-1+2+5 = 2$

ÖRNEK 5

$\sum_{n=5}^{9} a = 45 \Rightarrow$ **a = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

ÇÖZÜM

Yanıt **E**

$\sum_{n=5}^{9} a = 45 \Rightarrow \sum_{n=5-4}^{9-4} a = \sum_{n=1}^{5} a = 5a = 45$

$a = 9$

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Properties of Summation Notation

1. $\sum_{k=1}^{n} c = n \cdot c$

2. $\sum_{k=1}^{n} (c \cdot a_k) = c \cdot \sum_{k=1}^{n} a_k$

3. $\sum_{k=1}^{n} (a_k \mp b_k) = \sum_{k=1}^{n} a_k \mp \sum_{k=1}^{n} b_k$

4. For $1 < m < n$, $\sum_{k=1}^{n} a_k = \sum_{k=1}^{m} a_k + \sum_{k=m+1}^{n} a_k$

5. $\sum_{k=r}^{n} a_k = \sum_{k=r\mp p}^{n\mp p} a_{k\pm p}$

Activities

✓ $\sum_{k=1}^{9} 7 = 9 \cdot 7 = 63$

✓ $\sum_{k=3}^{6} 2k = \sum_{k=1}^{4} 2(k+2) = 2(1+2)+2(2+2)+2(3+2)+2(4+2) = 36$

✓ $\sum_{k=-2}^{1} (3k+2) = \sum_{k=1}^{4} (3(k-3)+2) = \sum_{k=1}^{4} (3k-7) = (3\cdot 1-7)+(3\cdot 2-7)+(3\cdot 3-7)+(3\cdot 4-7)$

$= -4-1+2+5 = 2$

EXAMPLE 5

$\sum_{n=5}^{9} a = 45 \Rightarrow$ **a = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

SOLUTION

The answer is **E**

$\sum_{n=5}^{9} a = 45 \Rightarrow \sum_{n=5-4}^{9-4} a = \sum_{n=1}^{5} a = 5a = 45$

$a = 9$

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Bazı Önemli Toplam Formülleri

1. $\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

2. $\sum_{k=1}^{n} (2k-1) = 1+3+5+\ldots+(2n-1) = n^2$

3. $\sum_{k=1}^{n} 2k = 2+4+6+8+\ldots+2n = n(n+1)$

4. $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

5. $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+4^3+\ldots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

6. $\sum_{k=0}^{n-1} r^k = 1+r+r^2+\cdots+r^{n-1} = \frac{1-r^n}{1-r} \quad (r \neq 1)$

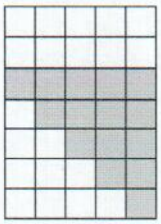
7. $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)} = \frac{n}{n+1}$

8. $\sum_{k=1}^{n} k\cdot k! = (n+1)! - 1$

9. $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

10. $\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Düşünün



Taralı karelerin sayısı kaçtır?

ÖRNEK 6

$\sum_{k=1}^{n} (2k+3) = 140 \Rightarrow n = ?$

A) 6 B) 8 C) 10 D) 12 E) 14

ÇÖZÜM

Yanıt C

$$\sum_{k=1}^{n} (2k+3) = 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$= 2\cdot\frac{n(n+1)}{2} + 3n = 140$$

$$= n^2 + 4n - 140 = 0 \Rightarrow n = 10$$

(n ve n; $+14$ ve -10)

Yanıt : 15

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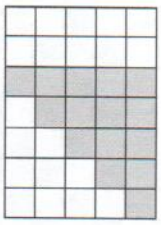
Think about it



What is the number of shaded square?

Answer : 15

Some Important Summation Formulas

1. $\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

2. $\sum_{k=1}^{n} (2k-1) = 1+3+5+\ldots+(2n-1) = n^2$

3. $\sum_{k=1}^{n} 2k = 2+4+6+8+\ldots+2n = n(n+1)$

4. $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

5. $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+4^3+\ldots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

6. $\sum_{k=0}^{n-1} r^k = 1+r+r^2+\cdots+r^{n-1} = \frac{1-r^n}{1-r} \quad (r \neq 1)$

7. $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)} = \frac{n}{n+1}$

8. $\sum_{k=1}^{n} k\cdot k! = (n+1)! - 1$

9. $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

10. $\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

EXAMPLE 6

$\sum_{k=1}^{n} (2k+3) = 140 \Rightarrow n = ?$

A) 6 B) 8 C) 10 D) 12 E) 14

SOLUTION

The answer is C

$$\sum_{k=1}^{n} (2k+3) = 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$= 2\cdot\frac{n(n+1)}{2} + 3n = 140$$

$$= \underset{n}{n^2} + \underset{+14}{4n} \underset{-10}{-140} = 0 \Rightarrow n = 10$$

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ÖRNEK 7

$$\sum_{k=-5}^{9} 2k = ?$$

A) 30 B) 40 C) 50 D) 60 E) 70

ÇÖZÜM

Yanıt **D**

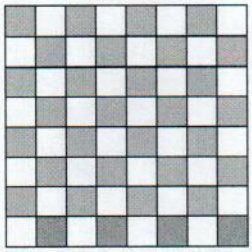
$$\sum_{k=-5}^{9} 2k = \sum_{k=1}^{15} 2(k-6) = 2\sum_{k=1}^{15} k - \sum_{k=1}^{15} 12$$

$$= 2 \cdot \frac{15 \cdot 16}{2} - 12 \cdot 15$$

$$= 240 - 180$$

$$= 60$$

Düşünün



Satranç tahtasında kaç tane kare vardır?

ÖRNEK 8

$f(x) = x + 1,\ x_1 = 2,\ x_2 = 3,\ x_3 = 5$

$$\Rightarrow \sum_{i=1}^{3} (x_i - 1)\, f(x_i) = ?$$

A) 35 B) 30 C) 27 D) 26 E) 25

ÇÖZÜM

Yanıt **A**

$$\sum_{i=1}^{3} (x_i - 1)\, f(x_i) = (x_1 - 1)f(x_1) + (x_2 - 1)f(x_2) + (x_3 - 1)f(x_3)$$

$$= (2-1)f(2) + (3-1)f(3) + (5-1)f(5)$$

$$= 1 \cdot 3 + 2 \cdot 4 + 4 \cdot 6$$

$$= 35$$

Yanıt : 204

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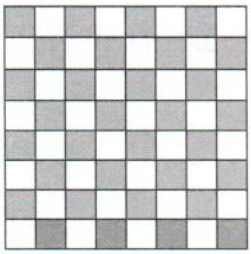
Think about it



What is the number of squares on a chessboard?

EXAMPLE 7

$$\sum_{k=-5}^{9} 2k = ?$$

A) 30 B) 40 C) 50 D) 60 E) 70

SOLUTION

The answer is **D**

$$\sum_{k=-5}^{9} 2k = \sum_{k=1}^{15} 2(k-6) = 2\sum_{k=1}^{15} k - \sum_{k=1}^{15} 12$$

$$= 2 \cdot \frac{15 \cdot 16}{2} - 12 \cdot 15$$

$$= 240 - 180$$

$$= 60$$

EXAMPLE 8

$f(x) = x + 1,\ x_1 = 2,\ x_2 = 3,\ x_3 = 5$

$$\Rightarrow \sum_{i=1}^{3} (x_i - 1)\, f(x_i) = ?$$

A) 35 B) 30 C) 27 D) 26 E) 25

SOLUTION

The answer is **A**

$$\sum_{i=1}^{3} (x_i - 1)\, f(x_i) = (x_1 - 1)f(x_1) + (x_2 - 1)f(x_2) + (x_3 - 1)f(x_3)$$

$$= (2-1)f(2) + (3-1)f(3) + (5-1)f(5)$$

$$= 1 \cdot 3 + 2 \cdot 4 + 4 \cdot 6$$

$$= 35$$

Answer : 204

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ÖRNEK 9

$f(x)=2x+1 \Rightarrow \sum_{n=1}^{10}[f(n)]^2 = ?$

A) 1670 B) 1680 C) 1710 D) 1740 E) 1770

ÇÖZÜM

Yanıt **E**

$$\sum_{n=1}^{10}[f(n)]^2 = \sum_{n=1}^{10}(4n^2+4n+1) = 4\sum_{n=1}^{10}n^2 + 4\sum_{n=1}^{10}n + \sum_{n=1}^{10}1$$

$$= 4\cdot\frac{10\cdot11\cdot21}{6} + 4\cdot\frac{10\cdot11}{2} + 10$$

$$= 4\cdot5\cdot11\cdot7 + 220 + 10$$

$$= 20\cdot77 + 230$$

$$= 1540 + 230$$

$$= 1770$$

ÖRNEK 10

$\sum_{i=1}^{3}\left(\sum_{j=-2}^{3}(2k+3j+2)\right) = ?$

A) 100 B) 115 C) 125 D) 135 E) 145

ÇÖZÜM

Yanıt **D**

$$\sum_{i=1}^{3}\left(\sum_{j=-2}^{3}(2k+3j+2)\right) = \sum_{i=1}^{3}\left(\sum_{j=1}^{6}(2k+3j-9+2)\right)$$

$$= \sum_{i=1}^{3}\left(12k+3\cdot\frac{6\cdot7}{2}-42\right)$$

$$= \sum_{i=1}^{3}(12k+21)$$

$$= 12\cdot\frac{3\cdot4}{2} + 63$$

$$= 72+63 = 135$$

Düşünün

$(n+1)! + \sum_{k=1}^{5}(n+k)!\,(n+k) = ?$

Yanıt : (n + 6)!

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Think about it

$(n+1)! + \sum_{k=1}^{5} (n+k)!\,(n+k) = ?$

EXAMPLE 9

$f(x) = 2x+1 \Rightarrow \sum_{n=1}^{10} [f(n)]^2 = ?$

A) 1670 B) 1680 C) 1710 D) 1740 E) 1770

SOLUTION

The answer is **E**

$$\sum_{n=1}^{10} [f(n)]^2 = \sum_{n=1}^{10} (4n^2+4n+1) = 4\sum_{n=1}^{10} n^2 + 4\sum_{n=1}^{10} n + \sum_{n=1}^{10} 1$$

$$= 4 \cdot \frac{10 \cdot 11 \cdot 21}{6} + 4 \cdot \frac{10 \cdot 11}{2} + 10$$

$$= 4 \cdot 5 \cdot 11 \cdot 7 + 220 + 10$$

$$= 20 \cdot 77 + 230$$

$$= 1540 + 230$$

$$= 1770$$

EXAMPLE 10

$\sum_{i=1}^{3} \left(\sum_{j=-2}^{3} (2k+3j+2) \right) = ?$

A) 100 B) 115 C) 125 D) 135 E) 145

SOLUTION

The answer is **D**

$$\sum_{i=1}^{3} \left(\sum_{j=-2}^{3} (2k+3j+2) \right) = \sum_{i=1}^{3} \left(\sum_{j=1}^{6} (2k+3j-9+2) \right)$$

$$= \sum_{i=1}^{3} \left(12k + 3 \cdot \frac{6 \cdot 7}{2} - 42 \right)$$

$$= \sum_{i=1}^{3} (12k+21)$$

$$= 12 \cdot \frac{3 \cdot 4}{2} + 63$$

$$= 72 + 63 = 135$$

Answer : (n + 6)!

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B. ÇARPIM SEMBOLÜ (Π)

Tanım: $\forall k \in N^+$ ve $a_k \in R$ olmak üzere,

$a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot \ldots \cdot a_n$ çarpımı kısa olarak $\prod_{k=1}^{n} a_k$ yazılır. $\prod$ gösterimi pi diye okunur. k ye ise, *çarpma indisi* denir.

Etkinlikler

✓ $\prod_{k=2}^{3} 2k = (2\cdot 2)\cdot(2\cdot 3) = 24$

✓ $\prod_{k=2}^{5} 2^k = 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^5 = 2^{14}$

Çarpım Sembolünün Özellikleri

1. $\prod_{k=1}^{n} c = c^n \quad (c \in R)$

2. $\prod_{k=1}^{n} c\cdot a_k = c^n \cdot \prod_{k=1}^{n} a_k \quad (c \in R)$

3. $\prod_{k=1}^{n} (a_k \cdot b_k) = \prod_{k=1}^{n} a_k \cdot \prod_{k=1}^{n} b_k$

4. $1 < m < n \Rightarrow \prod_{k=1}^{n} a_k = \prod_{k=1}^{m} a_k \cdot \prod_{k=m+1}^{n} a_k$

5. $\prod_{k=1}^{n} \frac{a_k}{b_k} = \left(\frac{\prod_{k=1}^{n} a_k}{\prod_{k=1}^{n} b_k} \right) \quad (\forall b \neq 0)$

6. $\prod_{k=p}^{n} a_k = \prod_{k=p\mp r}^{n\mp r} (a_{k\pm r})$

Etkinlikler

✓ $\prod_{k=1}^{3} (k\cdot 3^k) = 1\cdot 3^1 \cdot 2\cdot 3^2 \cdot 3\cdot 3^3 = 3\cdot 18\cdot 81 = 4374$

✓ $\prod_{k=1}^{8} 2 = 2^8 = 256$

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B. MULTIPLICATION SYMBOL (Π)

Definition: For all $\forall\, k \in N^+$, $a_k \in R$

The product $a_1 \cdot a_2 \cdot a_3 \cdot a_4 \ldots\ldots \cdot a_n$ can be expressed as the notation $\prod_{k=1}^{n} a_k$. The notation π read "pi". k is called the *index of multiplication.*

Activities

✓ $\prod_{k=2}^{3} 2k = (2\cdot 2)\cdot(2\cdot 3) = 24$

✓ $\prod_{k=2}^{5} 2^k = 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^5 = 2^{14}$

Properties of Multiplication Symbol

1. $\prod_{k=1}^{n} c = c^n \quad (c \in R)$

2. $\prod_{k=1}^{n} c \cdot a_k = c^n \cdot \prod_{k=1}^{n} a_k \quad (c \in R)$

3. $\prod_{k=1}^{n} (a_k \cdot b_k) = \prod_{k=1}^{n} a_k \cdot \prod_{k=1}^{n} b_k$

4. $1 < m < n \Rightarrow \prod_{k=1}^{n} a_k = \prod_{k=1}^{m} a_k \cdot \prod_{k=m+1}^{n} a_k$

5. $\prod_{k=1}^{n} \frac{a_k}{b_k} = \left(\frac{\prod_{k=1}^{n} a_k}{\prod_{k=1}^{n} b_k} \right) \quad (\forall b \neq 0)$

6. $\prod_{k=p}^{n} a_k = \prod_{k=p\mp r}^{n\mp r} (a_{k\pm r})$

Activities

✓ $\prod_{k=1}^{3} (k \cdot 3^k) = 1\cdot 3^1 \cdot 2\cdot 3^2 \cdot 3\cdot 3^3 = 3\cdot 18\cdot 81 = 4374$

✓ $\prod_{k=1}^{8} 2 = 2^8 = 256$

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Bazı Önemli Çarpım Formülleri

1. $\prod_{k=1}^{n} k = n!$

2. $\prod_{k=p}^{n} \log_k (k+1) = \log_p (n+1)$

3. $\prod_{k=1}^{n} r^k = r^{\frac{n\cdot(n+1)}{2}}$

4. $\prod_{i=2}^{n} \left(\frac{i^2-1}{i^2}\right) = \frac{n+1}{2n}$

Etkinlikler

✓ $\prod_{k=5}^{10} \log_k (k+1) = \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9 \cdot \log_9 10 \cdot \log_{10} 11 = \log_5 11$

✓ $\prod_{k=3}^{80} \log_k (k+1) = \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \ldots\ldots\ldots \log_{79} 80 \cdot \log_{80} 81$

$= \log_3 81 = \log_3 3^4 = 4$

HATIRLATMA

$\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$

ÖRNEK 11

$\prod_{k=7}^{48} \frac{k-1}{k+1} = ?$

A) $\frac{1}{56}$ B) $\frac{1}{50}$ C) $\frac{1}{48}$ D) $\frac{1}{42}$ E) $\frac{1}{30}$

ÇÖZÜM

Yanıt **A**

$\prod_{k=7}^{48} \frac{k-1}{k+1} = \frac{6}{8} \cdot \frac{7}{9} \cdot \frac{8}{10} \cdot \ldots \cdot \frac{47}{49}$

$= \frac{6\cdot 7}{48\cdot 49} = \frac{1}{56}$

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Some Important Multiplication Formulas

1. $\prod_{k=1}^{n} k = n!$

2. $\prod_{k=p}^{n} \log_k (k+1) = \log_p (n+1)$

3. $\prod_{k=1}^{n} r^k = r^{\frac{n(n+1)}{2}}$

4. $\prod_{i=2}^{n} \left(\frac{i^2-1}{i^2}\right) = \frac{n+1}{2n}$

REMINDER

$\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$

Activities

✓ $\prod_{k=5}^{10} \log_k (k+1) = \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9 \cdot \log_9 10 \cdot \log_{10} 11 = \log_5 11$

✓ $\prod_{k=3}^{80} \log_k (k+1) = \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \ldots\ldots \log_{79} 80 \cdot \log_{80} 81$

$= \log_3 81 = \log_3 3^4 = 4$

EXAMPLE 11

$\prod_{k=7}^{48} \frac{k-1}{k+1} = ?$

A) $\frac{1}{56}$ B) $\frac{1}{50}$ C) $\frac{1}{48}$ D) $\frac{1}{42}$ E) $\frac{1}{30}$

SOLUTION

The answer is **A**

$\prod_{k=7}^{48} \frac{k-1}{k+1} = \frac{6}{8} \cdot \frac{7}{9} \cdot \frac{8}{10} \cdot \ldots \cdot \frac{47}{49}$

$= \frac{6 \cdot 7}{48 \cdot 49} = \frac{1}{56}$

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ÖRNEK 12

$$\sum_{k=1}^{3}\prod_{i=1}^{k} 2 = ?$$

A) 8 B) 10 C) 12 D) 14 E) 16

ÇÖZÜM

Yanıt D

$$\prod_{i=1}^{k} 2 = 2^k$$

$$\sum_{k=1}^{3} 2^k = 2^1 + 2^2 + 2^3 = 14$$

ÖRNEK 13

$$\prod_{k=8}^{19} 2^k = ?$$

A) 2^{155} B) 2^{158} C) 2^{160} D) 2^{162} E) 2^{165}

ÇÖZÜM

Yanıt D

$$\prod_{k=8}^{19} 2^k = 2^8 \cdot 2^9 \cdot \ldots \cdot 2^{19}$$

$$= 2^{8+9+\ldots+19}$$

$$= 2^{\frac{19\cdot 20}{2} - \frac{7\cdot 8}{2}}$$

$$= 2^{190-28}$$

$$= 2^{162}$$

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EXAMPLE 12

$$\sum_{k=1}^{3}\prod_{i=1}^{k} 2 = ?$$

A) 8 B) 10 C) 12 D) 14 E) 16

SOLUTION

The answer is D

$$\prod_{i=1}^{k} 2 = 2^k$$

$$\sum_{k=1}^{3} 2^k = 2^1 + 2^2 + 2^3 = 14$$

EXAMPLE 13

$$\prod_{k=8}^{19} 2^k = ?$$

A) 2^{155} B) 2^{158} C) 2^{160} D) 2^{162} E) 2^{165}

SOLUTION

The answer is D

$$\prod_{k=8}^{19} 2^k = 2^8 \cdot 2^9 \cdot \ldots \cdot 2^{19}$$

$$= 2^{8+9+\ldots+19}$$

$$= 2^{\frac{19\cdot 20}{2} - \frac{7\cdot 8}{2}}$$

$$= 2^{190-28}$$

$$= 2^{162}$$

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DİZİLER

Tanım: $f: N^+ \to R$, $f(n) = a_n$ biçiminde tanımlı her fonksiyona *dizi* denir.

$n = 1, 2, 3, 4 \ldots$ sayılarına karşılık gelen $f(1) = a_1$, $f(2) = a_2$, $f(3) = a_3$, ... $f(n) = a_n$ reel sayılarına *dizinin terimleri* denir. $f(n) = a_n$ reel sayısına da dizinin *genel terimi* denir.

$f(1) = a_1$, $f(2) = a_2$, ... $f(n) = a_n$ ise, f dizisi genel olarak $(a_1, a_2, a_3, \ldots a_n, \ldots)$ veya kısaca (a_n) ile gösterilir.

Sabit Dizi

Tanım: Her $n \in N^+$ için $k \in R$ olmak üzere, $a_n = k$ ise, (a_n) dizisine *sabit dizi* denir.

Örneğin;

$(a_n) = 3$

$(a_n) = ((-1)^n \cdot (-1)^{n+1})$ bir sabit dizidir.

ÖRNEK 14

$(a_n) = \dfrac{xn+5}{5n+1}$ bir sabit dizi ise, **x kaçtır?**

A) 12 B) 16 C) 18 D) 20 E) 25

ÇÖZÜM

Yanıt **E**

Çözüm 1

(a_n) sabit dizi ise, $\forall n \in R^+$ için (a_n) in değeri aynı olmalıdır.

$$n = 1 \Rightarrow (a_1) = \frac{x+5}{6}$$

$$n = 2 \Rightarrow (a_2) = \frac{2x+5}{11}$$

$$a_1 = a_2 \Rightarrow \frac{x+5}{6} = \frac{2x+5}{11}$$

$$11x + 55 = 12x + 30$$

$$x = 25$$

Çözüm 2

$$\frac{x}{5} = \frac{5}{1} \Rightarrow x = 25$$

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SEQUENCES

Definition: A function which is defined in the form $f: N^+ \to N$, $f(n) = a_n$ is called a sequence. The real numbers $f(1) = a_1 \ldots f(n) = a_n$ which correspond to the numbers. $n = 1, 2, 3, 4 \ldots$ are called the *terms of the sequence.*

$f(n) = a_n$ is called the *general term of the sequence.*

Let $f(1) = a_1$, $f(2) = a_2$, ... $f(n) = a_n$.

The sequence is denoted by $(a_1, a_2, a_3, \ldots a_n, \ldots)$ in general or it is denoted by (a_n), briefly.

Constant Sequence

Definition: A sequence (a_n) is said to be constant if $a_n = k$, $k \in R$, for every $n \in N^+$.

For example,

$(a_n) = 3$

$(a_n) = ((-1)^n \cdot (-1)^{n+1})$ is a constant sequence.

EXAMPLE 14

If $(a_n) = \dfrac{xn+5}{5n+1}$ is a constant sequence, then $x =$

A) 12 B) 16 C) 18 D) 20 E) 25

SOLUTION

The answer is **E**

Solution 1

If (a_n) is a constant sequence then for every $\forall\, n \in N^+$ the elements of (a_n) must be the same.

$$n = 1 \Rightarrow (a_1) = \frac{x+5}{6}$$

$$n = 2 \Rightarrow (a_2) = \frac{2x+5}{11}$$

$$a_1 = a_2 \Rightarrow \frac{x+5}{6} = \frac{2x+5}{11}$$

$$11x + 55 = 12x + 30$$

$$x = 25$$

Solution 2

$$\frac{x}{5} = \frac{5}{1} \Rightarrow x = 25$$

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Dizilerin Eşitliği

Tanım: $\forall n \in N^+$ için $a_n = b_n$ ise, a_n ile b_n dizileri eşittir.

ÖRNEK 15

$$\left.\begin{array}{l} a_n = \dfrac{6n+k}{2n+4} \\ b_n = \dfrac{3n+5}{n+2} \end{array}\right\}, a_n = b_n \Rightarrow k = ?$$

A) 3 B) 4 C) 6 D) 8 E) 10

ÇÖZÜM

Yanıt **E**

$a_n = b_n$

$$\frac{6n+k}{2(n+2)} = \frac{3n+5}{n+2}$$

$$6n+k = 6n+10$$

$$k = 10$$

Dizilerde İşlemler

a_n ve b_n herhangi iki dizi ve $a, b, c \in R$.

- ✓ $(a_n \mp b_n) = (a_n) \mp (b_n)$
- ✓ $(a_n \cdot b_n) = (a_n) \cdot (b_n)$
- ✓ $\left(\dfrac{a_n}{b_n}\right) = \dfrac{(a_n)}{(b_n)} \quad b_n \neq 0$

Etkinlikler

$a_n = n + 1,\ b_n = n - 1 \Rightarrow$

- ✓ $a_n + b_n = n + 1 + n - 1 = 2n$
- ✓ $a_n - b_n = n + 1 - n + 1 = 2$
- ✓ $a_n \cdot b_n = (n + 1) \cdot (n - 1) = n^2 - 1$
- ✓ $\dfrac{a_n}{b_n} = \dfrac{n+1}{n-1}$

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Equality of Sequences

Definition: Two sequences (a_n) and (b_n) are said to be equal if and only if $a_n = b_n$ for every $n \in N^+$.

EXAMPLE 15

$$\left.\begin{array}{l} a_n = \dfrac{6n+k}{2n+4} \\ b_n = \dfrac{3n+5}{n+2} \end{array}\right\}, a_n = b_n \Rightarrow k = ?$$

A) 3 B) 4 C) 6 D) 8 E) 10

SOLUTION

The answer is **E**

$$a_n = b_n$$

$$\frac{6n+k}{2(n+2)} = \frac{3n+5}{n+2}$$

$$6n+k = 6n+10$$

$$k = 10$$

Operations on Sequences

Let (a_n) and (b_n) be any two sequences and $a, b, c \in R$.

- ✓ $(a_n \mp b_n) = (a_n) \mp (b_n)$
- ✓ $(a_n \cdot b_n) = (a_n) \cdot (b_n)$
- ✓ $\left(\dfrac{a_n}{b_n}\right) = \dfrac{(a_n)}{(b_n)} \quad b_n \neq 0$

Activities

$a_n = n + 1,\ b_n = n - 1 \Rightarrow$

- ✓ $a_n + b_n = n + 1 + n - 1 = 2n$
- ✓ $a_n - b_n = n + 1 - n + 1 = 2$
- ✓ $a_n \cdot b_n = (n + 1) \cdot (n - 1) = n^2 - 1$
- ✓ $\dfrac{a_n}{b_n} = \dfrac{n+1}{n-1}$

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SERİLER

Tanım: (a_n) bir reel terimli dizi olmak üzere, genel terimi, $(S_n) = a_1 + a_2 + a_3 + \ldots + a_n$ olan (S_n) dizisi gözönüne alınırsa $((a_n), (S_n))$ sıralı ikilisine bir *seri* denir. (a_n) terimine serinin genel terimi, (S_n) dizisine de serinin *kısmi toplamlar dizisi* adı verilir.

ÖRNEK 16

$(a_n) = \left(\frac{n+5}{2n-4}\right) \Rightarrow a_1 + a_3 = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt **A**

$a_1 = \frac{1+5}{2\cdot 1-4} = \frac{6}{-2} = -3$

$a_3 = \frac{3+5}{2\cdot 3-4} = \frac{8}{2} = 4$

$\Rightarrow a_1 + a_3 = (-3) + 4 = 1$

ÖRNEK 17

$(a_n) = (n-1)! \cdot 2^n \Rightarrow \frac{a_{n+1}}{a_n} = ?$

A) n B) 2n C) n + 1 D) n – 1 E) 2^n

ÇÖZÜM

Yanıt **B**

$a_{n+1} = (n+1-1)! \cdot 2^{n+1} = n! \cdot 2^{n+1}$

$a_n = (n-1)! \cdot 2^n$

$\Rightarrow \frac{a_{n+1}}{a_n} = \frac{n! \cdot 2^{n+1}}{(n-1)! \cdot 2^n} = 2n$

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SERIES

Definition: Provided that (a_n) is a sequence, the terms of which is real number, let (S_n) be the sequence with the general term. $(S_n) = a_1 + a_2 + a_3 + \ldots + a_n$.
Then the ordered pair $((a_n), (S_n))$ is called a *series*. (a_n) is called the *general term of the series* and (S_n) is called the *partial sum sequence of the series*.

EXAMPLE 16

$$(a_n)=\left(\frac{n+5}{2n-4}\right) \Rightarrow a_1+a_3=?$$

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION

The answer is **A**

$$a_1=\frac{1+5}{2\cdot 1-4}=\frac{6}{-2}=-3$$

$$a_3=\frac{3+5}{2\cdot 3-4}=\frac{8}{2}=4$$

$$\Rightarrow a_1+a_3=(-3)+4=1$$

EXAMPLE 17

$$(a_n)=(n-1)!\cdot 2^n \Rightarrow \frac{a_{n+1}}{a_n}=?$$

A) n B) 2n C) n + 1 D) n – 1 E) 2^n

SOLUTION

The answer is **B**

$$a_{n+1}=(n+1-1)!\cdot 2^{n+1}=n!\cdot 2^{n+1}$$

$$a_n=(n-1)!\cdot 2^n$$

$$\Rightarrow \frac{a_{n+1}}{a_n}=\frac{n!\cdot 2^{n+1}}{(n-1)!\cdot 2^n}=2n$$

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ÖRNEK 18

$$(a_n)=\begin{cases} n-1, & n\equiv 0 \pmod 3 \\ 2n+4, & n\equiv 1 \pmod 3 \\ n^2-n, & n\equiv 2 \pmod 3 \end{cases} \Rightarrow \frac{a_5}{a_6}\cdot a_{13}=?$$

A) 60 B) 80 C) 90 D) 120 E) 150

Düşünün

$a_n \cdot a_{n+3} = a_{n+2} \cdot a_{n+5}$

$a_1 \cdot a_2 + a_3 \cdot a_4 + a_5 \cdot a_6 + a_7 \cdot a_8 = 8$

$\Rightarrow a_1a_2 + a_3a_4 + \ldots + a_{18} \cdot a_{19} = ?$

ÇÖZÜM

Yanıt D

$5 \equiv 2 \pmod 3 \Rightarrow a_5 = 5^2 - 5 = 20$

$6 \equiv 0 \pmod 3 \Rightarrow a_6 = 6 - 1 = 5$

$13 \equiv 1 \pmod 3 \Rightarrow a_{13} = 2\cdot 13 + 4 = 30$

$$\Rightarrow \frac{a_5}{a_6}\cdot a_{13} = \frac{20}{5}\cdot 30 = 120$$

ÖRNEK 19

$$\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k = ?$$

A) $\frac{1}{2}$ B) 0 C) $\frac{2}{3}$ D) 1 E) 2

ÇÖZÜM

Yanıt D

$$\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k = \frac{1}{2}+\left(\frac{1}{2}\right)^2+\ldots$$

$$=\frac{1}{2}\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\ldots\right)$$

$$=\frac{1}{2}\cdot\frac{1}{1-\frac{1}{2}}=1$$

Yanıt : 19

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Think about it

$a_n \cdot a_{n+3} = a_{n+2} \cdot a_{n+5}$

$a_1 \cdot a_2 + a_3 \cdot a_4 + a_5 \cdot a_6 + a_7 \cdot a_8 = 8$

$\Rightarrow a_1a_2 + a_3a_4 + \ldots + a_{18} \cdot a_{19} = ?$

EXAMPLE 18

$$(a_n) = \begin{cases} n-1, & n \equiv 0 \pmod 3 \\ 2n+4, & n \equiv 1 \pmod 3 \\ n^2-n, & n \equiv 2 \pmod 3 \end{cases} \Rightarrow \frac{a_5}{a_6} \cdot a_{13} = ?$$

A) 60 B) 80 C) 90 D) 120 E) 150

SOLUTION

The answer is D

$5 \equiv 2 \pmod 3 \Rightarrow a_5 = 5^2 - 5 = 20$

$6 \equiv 0 \pmod 3 \Rightarrow a_6 = 6 - 1 = 5$

$13 \equiv 1 \pmod 3 \Rightarrow a_{13} = 2 \cdot 13 + 4 = 30$

$$\Rightarrow \frac{a_5}{a_6} \cdot a_{13} = \frac{20}{5} \cdot 30 = 120$$

EXAMPLE 19

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = ?$$

A) $\frac{1}{2}$ B) 0 C) $\frac{2}{3}$ D) 1 E) 2

SOLUTION

The answer is D

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \ldots$$

$$= \frac{1}{2}\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \ldots\right)$$

$$= \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2}} = 1$$

Answer : 19

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1. $\sum_{p=3}^{6}\left(\sum_{n=0}^{3}(p\cdot n+2)\right)=?$

A) 148
B) 144
C) 142
D) 140
E) 136

2. $\sum_{n=1}^{10}(a\cdot n-3a)=150 \Rightarrow a=?$

A) 4
B) 5
C) 6
D) 7
E) 8

3. $\sum_{k=1}^{24}\frac{1}{(2k-1)(2k+1)}=?$

A) $\frac{1}{2}$
B) $\frac{24}{49}$
C) $\frac{25}{49}$
D) $\frac{25}{47}$
E) $\frac{3}{4}$

4. $\sum_{i=1}^{4}\left(\sum_{j=-1}^{3}(3j+2i)\right)=?$

A) 240
B) 220
C) 180
D) 170
E) 160

5. $\sum_{k=1}^{16}\frac{1}{(2k-1)\cdot(2k+1)}=?$

A) $\frac{16}{33}$
B) $\frac{17}{33}$
C) $\frac{19}{33}$
D) $\frac{23}{33}$
E) $\frac{25}{33}$

6. $\sum_{k=12}^{33}(-1)^k\cdot(2k+1)=?$

A) –29
B) –22
C) –15
D) –9
E) –2

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7. $\sum_{n=1}^{3}\left(\prod_{k=1}^{4} 2^k\right) = ?$

A) 2496
B) 2966
C) 3072
D) 4642
E) 5048

8. $\prod_{n=-9}^{12}\left(\frac{1}{3}\right)^n = ?$

A) 3^{-33}
B) 3^{-32}
C) 3^{+32}
D) 3^{+33}
E) 3^{-34}

9. $\prod_{k=1}^{9} 9^{\log_3\left(1+\frac{1}{k}\right)} = ?$

A) 376
B) 304
C) 196
D) 156
E) 100

10. $$a_n = \begin{cases} 2n, & n \equiv 0 \pmod 3 \\ n-1, & n \equiv 1 \pmod 3 \\ \frac{n+1}{3}, & n \equiv 2 \pmod 3 \end{cases} \Rightarrow a_{14} + a_9 + a_{22} = ?$$

A) 22
B) 26
C) 28
D) 42
E) 44

11. $a_n = \frac{n!}{2^n}$, $\frac{a_n}{a_{n-1}} = 2 \Rightarrow n = ?$

A) 5
B) 4
C) 3
D) 2
E) 1

12. $a_n = \frac{2n^2+5n+6}{n}$,

$a_n \in Z \Rightarrow \sum n = ?$

A) 1
B) 2
C) 3
D) 6
E) 12

YANITLAR / ANSWERS							
1. D	2. C	3. B	4. E	5. A	6. B	7. C	8. A
9. E	10. E	11. B	12. E				

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1. $\sum_{n=0}^{3}(p\cdot n+2)=2+p+2+2p+2+3p+2=6p+8$

$\sum_{p=3}^{6}(6p+8)=26+32+38+44=140$

Yanıt / The Anwers is **D**

2. $\sum_{n=1}^{10}(a\cdot n-3a)=150$

$a\sum_{n=1}^{10}n-\sum_{n=1}^{10}3a=150$

$\frac{a\cdot 10\cdot 11}{2}-3a\cdot 10=150$

$25a=150$

$a=6$

Yanıt / The Anwers is **C**

3. $\frac{1}{(2k-1)(2k+1)}=\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)\cdot\frac{1}{2}$

$\sum_{k=1}^{24}\frac{1}{(2k-1)(2k+1)}=\frac{1}{2}\sum_{k=1}^{24}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$

$=\frac{1}{2}\left[1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\ldots+\frac{1}{47}-\frac{1}{49}\right]$

$=\frac{1}{2}\left[1-\frac{1}{49}\right]$

$=\frac{1}{2}\cdot\frac{48}{49}=\frac{24}{49}$

Yanıt / The Anwers is **B**

4. $\sum_{j=-1}^{3}(3j+2i)=2i-3+2i+2i+3+2i+6+2i+9=10i+15$

$\sum_{i=1}^{4}(10i+15)=10\cdot\frac{4\cdot 5}{2}+4\cdot 15=100+60=160$

Yanıt / The Anwers is **E**

5. $\sum_{k=1}^{16}\frac{1}{(2k-1)\cdot(2k+1)}=\sum_{k=1}^{16}\frac{1}{2}\cdot\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$

$=\frac{1}{2}\cdot\left[1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\ldots+\frac{1}{31}-\frac{1}{33}\right]$

$=\frac{1}{2}\left[1-\frac{1}{33}\right]=\frac{16}{33}$

Yanıt / The Anwers is **A**

6. $k=12\Rightarrow(-1)^k(2k+1)=25$

$k=13\Rightarrow(-1)^k(2k+1)=-27$

$\vdots$

$k=32\Rightarrow(-1)^k(2k+1)=65$

$k=33\Rightarrow(-1)^k(2k+1)=-67$

$\sum_{k=12}^{33}(-1)^k\cdot(2k+1)=\underbrace{25-27}_{-2}+\underbrace{29-31}_{-2}+\ldots+\underbrace{65-67}_{-2}$

$=\left(\frac{65-25}{4}+1\right)(-2)=-22$

Yanıt / The Anwers is **B**

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7. $\prod_{k=1}^{4} 2^k = 2\cdot 2^2\cdot 2^3\cdot 2^4 = 2\cdot 4\cdot 8\cdot 16 = 1024$

$\sum_{n=1}^{3} 1024 = 3\cdot 1024 = 3072$

Yanıt / The Anwers is C

8. $\prod_{n=-9}^{12}\left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^{-9}\cdot\left(\frac{1}{3}\right)^{-8}\cdot\ldots\cdot\left(\frac{1}{3}\right)^{10}\cdot\left(\frac{1}{3}\right)^{11}\cdot\left(\frac{1}{3}\right)^{12}$

$= 3^{-10}\cdot 3^{-11}\cdot 3^{-12}$

$= 3^{-33}$

Yanıt / The Anwers is A

9. $9^{\log_3\left(1+\frac{1}{k}\right)} = 3^{2\log_3\frac{k+1}{k}}$

$= 3^{\log_3\left(\frac{k+1}{k}\right)^2}$

$= \left(\frac{k+1}{k}\right)^2$

$\prod_{k=1}^{9} 9^{\log_3\left(1+\frac{1}{k}\right)} = \prod_{k=1}^{9}\left(\frac{k+1}{k}\right)^2$

$= \left(\frac{2}{1}\right)^2\cdot\left(\frac{3}{2}\right)^2\cdot\ldots\cdot\left(\frac{10}{9}\right)^2$

$= \left(\frac{2}{1}\cdot\frac{3}{2}\cdot\ldots\cdot\frac{10}{9}\right)^2$

$= 10^2 = 100$

Yanıt / The Anwers is E

10. $14 \mid 3$, $12 \mid 4$, 2 — $9 \mid 3$, $9 \mid 3$, 0 — $22 \mid 3$, $21 \mid 7$, 1

$a_{14} = \frac{14+1}{3} = 5$

$a_9 = 2\cdot 9 = 18$

$a_{22} = 22-1 = 21$

$a_{14} + a_9 + a_{22} = 5+18+21 = 44$

Yanıt / The Anwers is E

11. $a_n = \frac{n!}{2^n} \Rightarrow a_{n-1} = \frac{(n-1)!}{2^{n-1}}$

$\frac{a_n}{a_{n-1}} = \frac{\frac{n!}{2^n}}{\frac{(n-1)!}{2^{n-1}}} = \frac{n!}{2^n}\cdot\frac{2^{n-1}}{(n-1)!}$

$= \frac{n(n-1)!}{2^n}\cdot\frac{2^n}{2(n-1)!}$

$= \frac{n}{2}$

$\frac{n}{2} = 2 \Rightarrow n = 4$

Yanıt / The Anwers is B

12. $a_n = \frac{2n^2+5n+6}{n} \in Z$

$\frac{2n^2}{n} + \frac{5n}{n} + \frac{6}{n} = 2n+5+\frac{6}{n} \in Z$

$n = 1, 2, 3, 6 \Rightarrow \sum n = 1+2+3+6 = 12$

Yanıt / The Anwers is E

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YÖS Soruları / YÖS Questions

1. $\sum_{k=-6}^{5} k^3 = ?$

A) 0 B) –216 C) –125 D) –64 E) –8

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

2. $\sum_{k=1}^{198} \log \frac{k+2}{k+1} = ?$

A) $\frac{1}{2}$ B) 2 C) 3 D) 4 E) 6

[MARMARA ÜNİV. YÖS 2013]

3. $n > 1, a_1 = 1$

$a_n = na_{n-1}$

$\Rightarrow a_{20} = ?$

A) 20! B) 20n C) 34110 D) 19×20 E) 20^{20}

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

4. $|x| < 1 \Rightarrow \sum_{n=1}^{\infty} x^n = ?$

A) $\frac{1}{1-x}$ B) $\frac{1}{1+x}$ C) $\frac{x}{1-x}$ D) 1 E) 10

[İSTANBUL ÜNİV. YÖS 2013]

5. $\prod_{k=2}^{31} \log_k (k+1) = ?$

A) 3 B) 4 C) 11 D) 7 E) 5

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

6. $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

$(a_n) = \frac{n^n}{n!} \Rightarrow \frac{a_{n+1}}{a_n} = ?$

A) $\left(\frac{1}{n+1}\right)^n$ B) $\frac{1}{n!}$ C) $\left(\frac{n+1}{n}\right)^n$ D) $\left(\frac{n}{n-1}\right)^n$ E) $\left(\frac{n-2}{n+1}\right)^n$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

YANITLAR / ANSWERS

1. B	2. B	3. A	4. C	5. E	6. C

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1. $\sum_{k=-6}^{5} k^3 = (-6)^3 + (-5)^3 + (-4)^3 + \ldots + (4)^3 + (5)^3$

$= -6^3$

$= -216$

Yanıt / The Answer is B

3. $\sum_{k=1}^{198} \log \frac{k+2}{k+1} = \log \frac{3}{2} + \log \frac{4}{3} + \ldots + \log \frac{200}{199}$

$= \log_{10} \frac{3}{2} \cdot \frac{4}{3} \cdot \ldots \cdot \frac{200}{199}$

$= \log_{10} 100$

$= 2\log_{10} 10 = 2$

Yanıt / The Answer is B

3. $n > 1, a_1 = 1$

$a_n = na_{n-1} = \frac{a_n}{a_{n-1}} = n$

$\frac{a_2}{a_1} \cdot \frac{a_3}{a_2} \cdot \ldots \cdot \frac{a_{20}}{a_{19}} = 2 \cdot 3 \cdot \ldots \cdot 20$

$\frac{a_{20}}{a_1} = 20!$

$a_{20} = 20!$

Yanıt / The Answer is A

4. $\sum_{n=1}^{\infty} x^n = \frac{a_1}{1-r} = \frac{x}{1-x}$

Yanıt / The Answer is C

5. $\prod_{k=2}^{31} \log_k (k+1) = \log_2 3 \cdot \log_3 4 \cdot \ldots \cdot \log_{31} 32$

$= \log_2 32 = 5$

Yanıt / The Anwers is E

6. $\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}}$

$= \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n}$

$= \frac{(n+1)^{n+1}}{(n+1)} \cdot \frac{1}{n^n}$

$= \frac{(n+1)^n}{n^n} = \left(\frac{(n+1)}{n}\right)^n$

Yanıt / The Anwers is C

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1. $\sum_{k=3}^{21} (3k+4) = ?$

Yanıt / The Answer is **760**

2. $\sum_{k=3}^{27} k = ?$

Yanıt / The Answer is **375**

3. $\sum_{k=-1}^{18} 5k = ?$

Yanıt / The Answer is **850**

4. $\sum_{k=-2}^{9} (3k+4) = ?$

Yanıt / The Answer is **174**

5. $\sum_{k=2}^{243} \log_3\left(1-\frac{1}{k}\right) = ?$

Yanıt / The Answer is **–5**

6. $\sum_{k=1}^{n+2} \log_3 (1+\frac{1}{k}) = 4 \Rightarrow n = ?$

Yanıt / The Answer is **78**

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7. $\sum_{k=1}^{624} \log_5\left(1+\frac{1}{k}\right) = ?$

Yanıt / The Answer is **4**

8. $\sum_{k=1}^{4}\left[\prod_{n=1}^{3}(k\cdot n!)\right] = ?$

Yanıt / The Answer is **1200**

9. $\prod_{k=1}^{5} 4^{k-2} = 32^{x} \Rightarrow x = ?$

Yanıt / The Answer is **2**

10. $\prod_{n=1}^{120} 5^{\log_{11}\left(1+\frac{1}{n}\right)} = ?$

Yanıt / The Answer is **25**

11. $\prod_{k=5}^{124} \log_k (k+1) = ?$

Yanıt / The Answer is **3**

12. $\prod_{k=1}^{2}\left(\sum_{n=1}^{4}(n+k)\right) = ?$

Yanıt / The Answer is **252**

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13. $\prod_{n=-2}^{5} 2^{\log_2(n+3)} = ?$

Yanıt / The Answer is **8!**

14. $\prod_{k=77}^{85} 2 = ?$

Yanıt / The Answer is **512**

15. $\dfrac{\prod_{k=4}^{n+3}(3k)}{(n+2)!\cdot 3^n} = ?$

Yanıt / The Answer is $\dfrac{n+3}{6}$

16. $(a_n) = 3n+2 \Rightarrow a_5 = ?$

Yanıt / The Answer is **17**

17. $a_1 = 1,$

$\dfrac{a_{n+1}}{a_n} = n+1 \Rightarrow a_{17} = ?$

Yanıt / The Answer is **17!**

18. $a_n = 3n + 1 \Rightarrow a_{2n} = ?$

Yanıt / The Answer is **6n + 1**

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1. $\sum_{k=3}^{32}(2k-3)=?$

A) 950
B) 960
C) 970
D) 980
E) 990

2. $\sum_{k=1}^{15}(k-3a)=30 \Rightarrow a=?$

A) 2
B) 3
C) 4
D) 5
E) 6

3. $f(x)=x^2-2x+5 \Rightarrow \sum_{k=1}^{2} f(k)=?$

A) 5
B) 6
C) 7
D) 8
E) 9

4. $\sum_{k=5}^{19}(2k+1)=?$

A) 375
B) 350
C) 325
D) 275
E) 250

5. $\sum_{n=1}^{70}(3+n\cdot a)=1275 \Rightarrow a=?$

A) $\frac{2}{3}$
B) $\frac{3}{5}$
C) $\frac{5}{6}$
D) $\frac{5}{7}$
E) $\frac{3}{7}$

6. $\sum_{n=1}^{3}\left(\sum_{m=-1}^{3}\left(\sum_{p=1}^{4}(n+2m+p)\right)\right)=?$

A) 280
B) 340
C) 390
D) 450
E) 490

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7. $\sum_{x=4}^{12}(x-3)^2 = ?$

A) 265
B) 275
C) 285
D) 295
E) 315

8. $\sum_{x=-2}^{3}\sum_{y=3}^{5}(2y-3x+1) = ?$

A) 90
B) 110
C) 125
D) 130
E) 135

9. $\forall x \in N^+$,
$f(x-3) = x \Rightarrow \sum_{n=1}^{9}[f(n)]^2 = ?$

A) 576
B) 596
C) 600
D) 624
E) 636

10. $\sum_{k=n+1}^{3n+2}\frac{1}{k^2+k} = ?$

A) $\frac{2}{3(n+1)}$
B) $\frac{1}{n+3}$
C) $\frac{n}{n+1}$
D) $\frac{n}{3(n+1)}$
E) $\frac{n+1}{3n+1}$

11. $\frac{6}{12\cdot13}+\frac{6}{13\cdot14}+\frac{6}{14\cdot15}+\ldots\frac{6}{24\cdot25} = ?$

A) $\frac{1}{50}$
B) $\frac{3}{50}$
C) $\frac{11}{50}$
D) $\frac{13}{50}$
E) $\frac{17}{50}$

12. $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+\ldots+\frac{17}{18!} = ?$

A) $\frac{1}{18!}$ B) $1-\frac{1}{18!}$ C) $1+\frac{1}{18!}$
D) $2-\frac{1}{18!}$ E) $2+\frac{1}{18!}$

YANITLAR / ANSWERS

1. B	2. A	3. E	4. A	5. E	6. C	7. C	8. E
9. E	10. A	11. D	12. B				

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1. $\prod_{n=2}^{255} \log_n (n+1) = ?$

A) 2
B) 4
C) 6
D) 8
E) 10

2. $\prod_{k=2}^{63} \log_k (k+1) = ?$

A) 2
B) 3
C) 5
D) 6
E) 8

3. $\prod_{n=1}^{96} 2^{\log_7\left(\frac{n+1}{n+2}\right)} = ?$

A) $\frac{1}{16}$
B) $\frac{1}{8}$
C) $\frac{1}{4}$
D) 4
E) 8

4. $\prod_{n=1}^{4} \sum_{k=1}^{n} \left(\frac{k}{n}\right) = ?$

A) 16
B) 12
C) $\frac{10}{3}$
D) $\frac{15}{2}$
E) $\frac{21}{2}$

5. $\prod_{n=-1}^{179} 5^{\cos(n+1)} = ?$

A) 0
B) $\frac{1}{25}$
C) $\frac{1}{5}$
D) 1
E) 5

6. $\prod_{k=1}^{20} 2^{3k-2} = 2^x \Rightarrow x = ?$

A) 580
B) 590
C) 600
D) 610
E) 620

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7. $\prod_{k=1}^{n} 3^k = 3^{300} \Rightarrow n = ?$

A) 36
B) 32
C) 25
D) 24
E) 20

8. $\sum_{k=1}^{3} \prod_{n=-1}^{5} n = ?$

A) 15
B) 5
C) 3
D) 0
E) –1

9. $\prod_{k=1}^{n} \left(1 - \frac{3}{3k+2}\right) = \frac{1}{55} \Rightarrow n = ?$

A) 18
B) 24
C) 27
D) 32
E) 36

10. $\prod_{k=1}^{23} \sin\left[\frac{1}{24}(k+1)\pi\right] = ?$

A) –2
B) –1
C) 0
D) 1
E) 2

11. $f : N^+ \to N^+, g : N^+ \to N^+$

$f(x) = \sum_{k=19}^{x} (2k-32), g(x) = \prod_{k=1}^{x} (3k-2)$

$\Rightarrow$ **(fog) (3) = ?**

A) 70
B) 90
C) 120
D) 150
E) 175

12. $f : N^+ \to N^+,$

$f(x) = \prod_{n=1}^{x} 3^n \Rightarrow$ **(fof) (2) = ?**

A) 3^{27}
B) 3^{36}
C) 3^{108}
D) 3^{296}
E) 3^{378}

YANITLAR / ANSWERS

1. D	2. D	3. C	4. D	5. D	6. B	7. D	8. D
9. E	10. C	11. D	12. E				

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1. $a_n = 5n + 1 \Rightarrow a_2 + a_4 = ?$

A) 30
B) 31
C) 32
D) 33
E) 34

2. $a_n = \frac{2^{n+1}}{5^n} \Rightarrow \frac{a_{n+1}}{a_{n-1}} = ?$

A) $\frac{4}{25}$
B) $\frac{3}{25}$
C) $\frac{2}{5}$
D) $\frac{1}{3}$
E) 1

3. $a_n = 3n - 1 \Rightarrow \sum_{n=1}^{10} a_n = ?$

A) 65
B) 72
C) 88
D) 121
E) 155

4. $a_n = \sum_{k=1}^{n} (k^2 + k) \Rightarrow a_5 = ?$

A) 70
B) 65
C) 60
D) 55
E) 50

5. $a_{2n+3} = \frac{5n+1}{2n-1} \Rightarrow a_{11} = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

6. $n \geq 1$,
$a_1 = 3$,
$3a_n = 2 + 3a_{n+1} \Rightarrow a_{40} = ?$

A) –13
B) –15
C) –17
D) –19
E) –23

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7. $a_n = \sum_{k=1}^{n} \frac{3}{2k\cdot(k+1)}$, $a_m = \frac{15}{11} \Rightarrow m = ?$

A) 20
B) 16
C) 14
D) 12
E) 10

8. $a_n = \left(\sum_{k=1}^{n} \log\left(\frac{k+1}{k}\right)\right)$,

$b_n = \prod_{k=10}^{n} \log_{k+1} k \Rightarrow a_n \cdot b_n = ?$

A) 1
B) $\log n + 1$
C) $2\log n + 1$
D) $\log_{11} n$
E) 0

9. $a_1 = 4$,

$\sqrt[3]{2}\, a_n = a_{n-1}$, $n \geq 2 \Rightarrow a_{13} = ?$

A) $\frac{1}{\sqrt[3]{16}}$
B) $\frac{1}{4}$
C) $\frac{1}{\sqrt[3]{32}}$
D) $\frac{1}{8}$
E) $\frac{1}{\sqrt[3]{128}}$

10. $\left.\begin{array}{l} a_n = \frac{3n+1}{2n-3} \\ b_n = \frac{2n-5}{n+1} \end{array}\right\} \Rightarrow 2(a_7) - b_6 = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

11. $a_n = \begin{cases} \frac{3n+2}{n}, & n = 2k-1 \\ \frac{2n+6}{n}, & n = 2k \end{cases} \Rightarrow a_1 + a_6 = ?$

A) 2
B) 5
C) 6
D) 8
E) 12

12. $a_{n+2} = \left(\frac{3n+8}{2n-6}\right) \Rightarrow a_n = ?$

A) $\frac{3n+2}{2n-10}$ B) $\frac{3n+2}{2n+10}$ C) $\frac{3n-2}{2n+10}$

D) $\frac{3n}{2n+10}$ E) $\frac{3n}{2n-10}$

YANITLAR / ANSWERS							
1. C	2. A	3. E	4. A	5. C	6. E	7. E	8. A
9. B	10. C	11. D	12. A				

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BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Cox, Sir David FRS [1924 -]

Matematiksel istatistik, olasılık ve yöneylem araştırması alanlarında çalışmış İngiliz istatistikçi. Deney tasarımı, ikili dataların analizi, stokastik süreçler, kalite kontrolünde ve yöneylem araştırmasında yeni yöntemlerin geliştirilmesi, kuyruk ve yenilenme teorisine olan katkıları ile tanınır.

British statistician distinguished for his work in mathematical statistics and applied probability, especially in relation to industrial and operational research. His varied contributions include the design and analysis of statistical experiments, analysis of binary data and of point stochastic process and the development of new methods in quality control and operational research particularly in relation to queuing and renewal theory.

PERMÜTASYON, KOMBİNASYON ve BİNOM AÇILIMI
PERMUTATION, COMBINATION, and BINOMIAL EXPANSION

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PERMÜTASYON, KOMBİNASYON ve BİNOM AÇILIMI
PERMUTATION, COMBINATION, and BINOMIAL EXPANSION

BÖLÜM / CHAPTER **2**

PERMÜTASYON

Tanım: A kümesi, **n** elemanlı bir küme ve $r \le n$ olmak üzere, A kümesinin birbirinden farklı r elemanının sıralanışına A kümesinin *r'li permütasyonu* denir. Permütasyon *P* ile gösterilir.

Permütasyon sayısı: $P(n, r) = \frac{n!}{(n-r)!}$

NOTLAR

✓ $P(n, n) = \frac{n!}{(n-n)!} = n!$

✓ $P(n, 0) = \frac{n!}{(n-0)!} = 1$

ÖRNEK 1

$3 \cdot P(n, 3) + P(n, 3) = 240 \Rightarrow n = ?$

A) 10 B) 9 C) 6 D) 5 E) 3

ÇÖZÜM

Yanıt **D**

$3 \cdot P(n, 3) + P(n, 3) = 240$ [Verilen denklem]

$4 \cdot P(n, 3) = 240$ [Benzer terimleri topla.]

$P(n, 3) = 60$ [Her iki tarafı 4'e böl.]

$\frac{n!}{(n-3)!} = 60$ [Permütasyon formulü.]

$\frac{n(n-1)\cdot(n-2)\cdot(n-3)!}{(n-3)!} = 60$ [Sadeleştir.]

$n(n-1)\cdot(n-2) = 60$ [İndirge.]

$n(n-1)\cdot(n-2) = 5\cdot4\cdot3$ [60 yerine 5·4·3 yaz.]

$n = 5$ [Sonuç]

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NOTES

✓ $P(n, n) = \frac{n!}{(n-n)!} = n!$

✓ $P(n, 0) = \frac{n!}{(n-0)!} = 1$

PERMUTATION

Definition: Provided that the set A has n elements and $r \leq n$, an ordering of *r distinct elements* of the set A is called a *permutation with r* of the set A. Permutation is denoted by *P*.

Number of permutations: $P(n, r) = \frac{n!}{(n-r)!}$

EXAMPLE 1

$3 \cdot P(n, 3) + P(n, 3) = 240 \Rightarrow$ **n = ?**

A) 10 B) 9 C) 6 D) 5 E) 3

SOLUTION

The answer is **D**

$3 \cdot P(n, 3) + P(n, 3) = 240$ [Given equation.]

$4 \cdot P(n, 3) = 240$ [Combine like terms.]

$P(n, 3) = 60$ [Divide by 4 both sides]

$\frac{n!}{(n-3)!} = 60$ [Permutation formula.]

$\frac{n(n-1)\cdot(n-2)\cdot(n-3)!}{(n-3)!} = 60$ [Simplify.]

$n(n-1)\cdot(n-2) = 60$ [Reduce.]

$n(n-1)\cdot(n-2) = 5 \cdot 4 \cdot 3$ [Substitute 5·4·3 for 60.]

$n = 5$ [Conclusion.]

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KOMBİNASYON

Tanım: **n** elemanlı bir kümenin *r* elemanlı alt kümelerinin sayısına n'nin *r'li kombinasyonu* denir.

$$\text{Kombinasyon sayısı} : C(n, r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

ÖRNEK 2

P(8, 2) + C(6, 3) = ?

A) 74 B) 76 C) 78 D) 80 E) 82

ÇÖZÜM

Yanıt **B**

$$P(8,2) = \frac{8!}{(8-2)!} = \frac{8\cdot 7\cdot 6!}{6!} = 56$$

$$C(6,3) = \frac{6!}{(6-3)!\cdot 3!} = \frac{6\cdot 5\cdot 4\cdot 3!}{3!\cdot 3!} = 20$$

$$P(8,2) + C(6,3) = 56 + 20 = 76$$

Kombinasyonun Özellikleri

1. $\binom{n}{0} = \binom{n}{n} = 1$
2. $\binom{n}{1} = \binom{n}{n-1} = n$
3. $\binom{n}{r} = \binom{n}{k} \Rightarrow r = k$ veya $n = r + k$
4. $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$
5. $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$

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COMBINATION

Definition: The number of subsets with *r* elements of a set with n elements is called the *combination with r of n.*

Number of combinations: $C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

EXAMPLE 2

P(8, 2) + C(6, 3) = ?

A) 74 B) 76 C) 78 D) 80 E) 82

SOLUTION

The answer is **B**

$$P(8,2) = \frac{8!}{(8-2)!} = \frac{8 \cdot 7 \cdot 6!}{6!} = 56$$

$$C(6,3) = \frac{6!}{(6-3)! \cdot 3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3!} = 20$$

$$P(8,2) + C(6,3) = 56 + 20 = 76$$

Properties of Combination

1. $\binom{n}{0} = \binom{n}{n} = 1$
2. $\binom{n}{1} = \binom{n}{n-1} = n$
3. $\binom{n}{r} = \binom{n}{k} \Rightarrow r = k \text{ or } n = r + k$
4. $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$
5. $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$

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ÖRNEK 3

$C(n, 5) = C(n, 2) \Rightarrow n = ?$

A) 8 B) 7 C) 6 D) 5 E) 4

ÇÖZÜM

Yanıt B

$\binom{n}{5} = \binom{n}{2} \Rightarrow n = 5 + 2 = 7$

BİNOM AÇILIMI ve PASCAL ÜÇGENİ

1 → $(x+y)^0$

1 1 → $(x+y)^1$

1 2 1 → $(x+y)^2$

1 3 3 1 → $(x+y)^3$

1 4 6 4 1 → $(x+y)^4$

1 5 10 10 5 1 → $(x+y)^5$

1 6 15 20 15 6 1 → $(x+y)^6$

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$$

NOT

Pascal üçgeninin her bir satırındaki ilk ve son sayı 1'dir. Her satırdaki diğer sayılar, üzerinde yer alan iki sayının toplamından elde edilir.

Binom Açılımının Özellikleri

1. $(x + y)^n$ nin açılımında, n + 1 tane terim vardır.
2. $(x + y)^n$ açılımında, her terimdeki x ve y nin üslerinin toplamı n'ye eşittir.
3. Baştan (r + 1). terim, $\binom{n}{r}x^{n-r}y^r$ dir.

NOT

Her bir açılımda, x ve y simetriktir. x'in kuvveti 1 azalırken, y'nin kuvveti 1 artar.

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EXAMPLE 3

$C(n, 5) = C(n, 2) \Rightarrow n = ?$

A) 8 B) 7 C) 6 D) 5 E) 4

SOLUTION

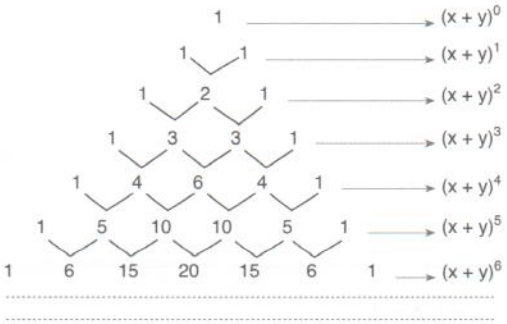
The answer is B

$$\binom{n}{5} = \binom{n}{2} \Rightarrow n = 5 + 2 = 7$$

BINOMIAL EXPANSION and PASCAL'S TRIANGLE

NOTE

The first and last number in each row of Pascal's triangle is 1. Every other number in each row is formed by adding the two numbers immediately above the number.



$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$$

NOTE

In each expansion, x and y have symmetrical roles. The powers of x decrease by 1 in successive terms, whereas the powers of y increase by 1.

Properties of Binomial Expansion

1. There are $(n + 1)$ terms in the expansion $(x + y)^n$.
2. In each terms the sum of the power of x and y is equal to n in the expansion $(x + y)^n$.
3. The $(r + 1)$. term from beginning is $\binom{n}{r}x^{n-r}y^r$.

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ÖRNEK 4

$(x+5)^6 = \ldots\ldots + Ax^4 \Rightarrow A = ?$

A) 125 B) 250 C) 325 D) 375 E) 425

ÇÖZÜM

Yanıt D

$Ax^4 = \binom{6}{r} \cdot (x)^{6-r} \cdot (5)^r$

$\Rightarrow 6-r=4$

$r=2$

$A = \binom{6}{2} \cdot 5^2 = \frac{6\cdot 5}{2} \cdot 25 = 375$

PEKİŞTİRME

$(x-2)^6 = \ldots\ldots + Ax^5 \Rightarrow A = ?$

YANIT : –12

ÖRNEK 5

$(x^2+y)^6 = \ldots\ldots + Ax^4y^n \Rightarrow n + A = ?$

A) 12 B) 15 C) 16 D) 18 E) 19

ÇÖZÜM

Yanıt E

$A\cdot(x^2)^2 \cdot y^n \Rightarrow 2+n=6$

$n=4$

$Ax^4y^4 = \binom{6}{r}(x^2)^{6-r}(y)^r$

$2(6-r)=4$

$6-r=2$

$r=4$

$\Rightarrow A = \binom{6}{4} = \frac{6\cdot5\cdot4\cdot3}{4\cdot3\cdot2} = 15$

$A+n=15+4=19$

PEKİŞTİRME

$(x^4+y^3)^n = \ldots\ldots + Ax^8y^6 \Rightarrow A+n=?$

YANIT : 8

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REINFORCE

$(x + y)^4 = \ldots\ldots + Ax^2y^2 \Rightarrow A = ?$

ANSWER : –12

EXAMPLE 4

$(x + 5)^6 = \ldots\ldots + Ax^4 \Rightarrow \mathbf{A = ?}$

A) 125 B) 250 C) 325 D) 375 E) 425

SOLUTION

The answer is **D**

$$Ax^4 = \binom{6}{r} \cdot (x)^{6-r} \cdot (5)^r$$

$$\Rightarrow 6 - r = 4$$

$$r = 2$$

$$A = \binom{6}{2} \cdot 5^2 = \frac{6 \cdot 5}{2} \cdot 25 = 375$$

REINFORCE

$(x^4 + y^3)^n = \ldots\ldots + Ax^8y^6 \Rightarrow A + n = ?$

ANSWER : 8

EXAMPLE 5

$(x^2 + y)^6 = \ldots\ldots + Ax^4y^n \Rightarrow \mathbf{n + A = ?}$

A) 12 B) 15 C) 16 D) 18 E) 19

SOLUTION

The answer is **E**

$$A \cdot (x^2)^2 \cdot y^n \Rightarrow 2 + n = 6$$

$$n = 4$$

$$Ax^4y^4 = \binom{6}{r}(x^2)^{6-r}(y)^r$$

$$2(6 - r) = 4$$

$$6 - r = 2$$

$$r = 4$$

$$\Rightarrow A = \binom{6}{4} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2} = 15$$

$$A + n = 15 + 4 = 19$$

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ÖRNEK 6

$\left(x+\frac{1}{x}\right)^5 = \ldots\ldots + Ax \Rightarrow A = ?$

A) 6 B) 8 C) 10 D) 12 E) 14

ÇÖZÜM

Yanıt C

$Ax^1 = \binom{5}{r} x^{5-r} \cdot \left(\frac{1}{x}\right)^r$

$5-r-r=1$

$2r=4$

$r=2 \Rightarrow \binom{5}{2} = 10$

▼ p, q, r ∈ N,

$p+q+r=n \Rightarrow$

$(ax+by+cz)^n = (ax)^p \cdot (by)^q \cdot (cz)^r \cdot \frac{n!}{p!\cdot q!\cdot r!}$

ÖRNEK 7

$(x + 2y + z)^5 = \ldots\ldots + Ax^3yz \Rightarrow A = ?$

A) 10 B) 20 C) 30 D) 40 E) 50

ÇÖZÜM

Yanıt D

Çözüm 1

$p=3$

$q=1$

$r=1$

$n=p+q+r=3+1+1=5$

$A = \frac{5!}{3!\cdot 1!\cdot 1!}\cdot (2)^1 = 40$

Çözüm 2

$2y+z=t \Rightarrow (x+t)^5 = \ldots\ldots + Kx^3t^2$

$Kx^3t^2 = \binom{5}{r} x^{5-r}\, t^r$

$r=2$

$K = \binom{5}{2} = 10$

$t = 2y+z \Rightarrow (2y+z)^2 = \ldots\ldots + Ly^1z^1$

$Lyz = \binom{2}{r} (2y)^{2-r} (z)^r$

$r=1 \Rightarrow L = \binom{2}{1}\cdot 2^1 = 4$

$A = K\cdot L = 10\cdot 4 = 40$

PEKİŞTİRME

$\left(x^2+\frac{y}{3}\right)^5 = \ldots\ldots + Ax^8y \Rightarrow A = ?$

YANIT : 5/3

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REINFORCE

$\left(x^2+\frac{y}{3}\right)^5 = \ldots\ldots + Ax^8y \Rightarrow A = ?$

ANSWER : 5/3

EXAMPLE 6

$\left(x+\frac{1}{x}\right)^5 = \ldots\ldots + Ax \Rightarrow \mathbf{A = ?}$

A) 6 B) 8 C) 10 D) 12 E) 14

SOLUTION

The answer is **C**

$Ax^1 = \binom{5}{r} x^{5-r} \cdot \left(\frac{1}{x}\right)^r$

$5-r-r=1$

$2r=4$

$r=2 \Rightarrow \binom{5}{2} = 10$

▼ p, q, r ∈ N,

$p+q+r=n \Rightarrow$

$(ax+by+cz)^n = (ax)^p \cdot (by)^q \cdot (cz)^r \cdot \frac{n!}{p!\,q!\,r!}$

EXAMPLE 7

$(x+2y+z)^5 = \ldots\ldots + Ax^3yz \Rightarrow \mathbf{A = ?}$

A) 10 B) 20 C) 30 D) 40 E) 50

SOLUTION

The answer is **D**

Solution 1

$p=3$

$q=1$

$r=1$

$n=p+q+r=3+1+1=5$

$A = \frac{5!}{3!\cdot 1!\cdot 1!}\cdot (2)^1 = 40$

Solution 2

$2y+z=t \Rightarrow (x+t)^5 = \ldots\ldots + Kx^3t^2$

$Kx^3t^2 = \binom{5}{r} x^{5-r}\, t^r$

$r=2$

$K = \binom{5}{2} = 10$

$t = 2y+z \Rightarrow (2y+z)^2 = \ldots\ldots + Ly^1z^1$

$Lyz = \binom{2}{r} (2y)^{2-r} (z)^r$

$r=1 \Rightarrow L = \binom{2}{1}\cdot 2^1 = 4$

$A = K\cdot L = 10\cdot 4 = 40$

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1. $P(5, n) = 20 \Rightarrow$ **n = ?**

A) 1
B) 2
C) 3
D) 4
E) 5

2. $P(n, 3) = k \cdot P(n, 2) \Rightarrow$ **k = ?**

A) n
B) n – 1
C) n – 2
D) n – 3
E) n – 4

3. $C(n + 1, 2) + C(n, 3) = 63 \Rightarrow$ **C(n, 4) = ?**

A) 35
B) 42
C) 48
D) 56
E) 64

4. $(x^3 - 3x)^{10} = \ldots\ldots + Ax^{24} + \ldots\ldots \Rightarrow$ **A = ?**

A) 3240
B) 3400
C) –3200
D) –3240
E) –3400

5. $(x + y + z)^8 = \ldots\ldots + Ax^2y^3z^3 + \ldots\ldots \Rightarrow$ **A = ?**

A) 440
B) 460
C) 480
D) 520
E) 560

6. $(a + b + c)^9 = \ldots\ldots + K \cdot a^4b^3c^2 + \ldots\ldots \Rightarrow$ **K = ?**

A) 1180
B) 1200
C) 1240
D) 1260
E) 1280

YANITLAR / ANSWERS

1. B	2. C	3. A	4. D	5. E	6. D

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1. $P(5, n) = 20$

$$\frac{5!}{(5-n)!} = 20$$

$$(5-n)! = 3!$$

$$5-n = 3$$

$$n = 2$$

Yanıt / The Answer is B

2. $P(n, 3) = k \cdot P(n, 2)$

$$\frac{n!}{(n-3)!} = k \cdot \frac{n!}{(n-2)!}$$

$$\frac{1}{(n-3)!} = \frac{k}{(n-2)\cdot(n-3)!}$$

$$k = n-2$$

Yanıt / The Answer is C

3. $C(n+1, 2) + C(n, 3) = 63$

$$\frac{(n+1)!}{(n-1)!\cdot 2!} + \frac{n!}{(n-3)!\cdot 3!} = 63$$

$$\frac{(n+1)\cdot n}{2} + \frac{n(n-1)\cdot(n-2)}{6} = 63$$

$$n(3n+3+n^2-3n+2) = 378$$

$$n(n^2+5) = 378$$

$$n = 7$$

$$C(n, 4) = \binom{7}{4} = \frac{7\cdot 6\cdot 5}{3\cdot 2} = 35$$

Yanıt / The Answer is A

4. $Ax^{24} = \binom{10}{r}(x^3)^{10-r}\cdot(-3x)^r$

$$3(10-r)+r = 24$$

$$30-2r = 24$$

$$r = 3$$

$$\Rightarrow A = \binom{10}{3}(-3)^3 = \frac{10\cdot 9\cdot 8}{3\cdot 2}\cdot(-27) = 120\cdot(-27) = -3240$$

Yanıt / The Answer is D

5. $A = \frac{8!}{2!\cdot 3!\cdot 3!}\cdot(1)^2(1)^3(1)^3$

$$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3!}{2\cdot 3\cdot 2\cdot 3!}$$

$$= 560$$

Yanıt / The Answer is E

6. $K = \frac{9!}{4!\cdot 3!\cdot 2!}(1)^4\cdot(1)^3\cdot(1)^2$

$$= \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!\cdot 3\cdot 2\cdot 2}$$

$$= 1260$$

Yanıt / The Answer is D

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YÖS Soruları / YÖS Questions

1. $a = 2013$

$b = 2015$

$\Rightarrow a^5 - 5a^4b + 10a^2b^3 + 5ab^4 - b^5 = ?$

A) –32 B) –21 C) 0 D) 16 E) 20

[MARMARA ÜNİV. YÖS 2013]

2. $\left(x - \frac{2}{x}\right)^5 = Ax^5 + Bx^3 + Cx + Dx^{-1} + Ex^{-3} + Fx^{-5}$

$\Rightarrow C = ?$

A) –36 B) –20 C) 24 D) 40 E) 64

[YÖS 2009]

3. $C(n, r) = \frac{n!}{r!\,(n-r)!}$

$C(5, 3) = C(k, 2)$

$\Rightarrow k = ?$

A) 7 B) 6 C) 5 D) 4 E) 3

[YÖS 2002]

4. $P(n, r) = \frac{n!}{(n-r)!}$

$P(4, 2) = 3 \cdot P(k, 1) \Rightarrow k = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

[YÖS 2001]

YANITLAR / ANSWERS

1. A 2. D 3. C 4. E

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1. $a^5 - 5a^4b + 10a^2b^3 + 5ab^4 - b^5 = (a - b)^5$

$= (2013 - 2015)^5$

$= (-2)^5$

$= -32$

Yanıt / The Answer is A

2. $C \cdot x = \binom{5}{3} \cdot x^3 \cdot \left(\frac{-2}{x}\right)^2$

$C \cdot x = \frac{5!}{(5-3)! \cdot 3!} \cdot x^3 \cdot \frac{4}{x^2}$

$C \cdot x = 40x \Rightarrow C = 40$

Yanıt / The Answer is D

3. $C(5,3) = C(k,2)$

$\frac{5!}{3!(5-3)!} = \frac{k!}{2!(k-2)!}$

$\frac{3! \cdot 4 \cdot 5}{3! \cdot 2!} = \frac{(k-2)!(k-1)k}{2! \cdot (k-2)!}$

$10 = \frac{k(k-1)}{2}$

$k(k-1) = 20 \Rightarrow k = 5$

Yanıt / The Answer is C

4. $P(4, 2) = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot 2!}{2!} = 12$

$\Rightarrow 12 = 3 \cdot \frac{k!}{(k-1)!} \Rightarrow 4 = \frac{k \cdot (k-1)!}{(k-1)!} \Rightarrow k = 4$

Yanıt / The Answer is E

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1. $P(4, 1) + P(5, 2) + P(6, 3) = ?$

Yanıt / The Answer is **144**

2. $\frac{(5n-4)!}{(5n-5)!} = 46 \Rightarrow P(n, 3) = ?$

Yanıt / The Answer is **720**

3. $P(n, 3) = 6n^2 + 3n - 9 \Rightarrow P(n, 2) = ?$

Yanıt / The Answer is **72**

4. $P(4n, 2) - 32C(n, n-2) = 96 \Rightarrow P(n, 2) = ?$

Yanıt / The Answer is **56**

5. $(2 + 3x)^5 = \ldots\ldots + Ax^3 + \ldots\ldots \Rightarrow A = ?$

Yanıt / The Answer is **1080**

6. $\left(x^3 - \frac{1}{x^2}\right)^{10} = \ldots\ldots + Ax^5 + \ldots\ldots \Rightarrow A = ?$

Yanıt / The Answer is **–252**

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1. **P(7, 4) + C(8, 2) = ?**

A) 850
B) 856
C) 860
D) 864
E) 868

2. C(n, 2) + P(4, 2) = 48 ⇒ **n = ?**

A) 5
B) 6
C) 8
D) 9
E) 10

3. P(7, n) + C(7, 3) = 77 ⇒ **n = ?**

A) 2
B) 3
C) 4
D) 5
E) 6

4. $\binom{n}{2}$ + P(n, 2) = 63 ⇒ **n = ?**

A) 4
B) 5
C) 6
D) 7
E) 8

5. $(x + 3)^7 = \ldots + a \cdot x^5 \ldots$ ⇒ **a = ?**

A) 189
B) 199
C) 209
D) 219
E) 229

6. $(x - 1)^9 = \ldots + a \cdot x^2 \ldots$ ⇒ **a = ?**

A) –36
B) –18
C) 0
D) 18
E) 36

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7. $(x+2)^6 = \ldots + a\cdot x^4 \ldots \Rightarrow$ **a = ?**

A) 60
B) 62
C) 64
D) 66
E) 68

8. $(x-y)^n = \ldots + a\cdot x^2\cdot y^5 \ldots \Rightarrow$ **a + n = ?**

A) –35
B) –28
C) –21
D) –14
E) –7

9. $\left(x-\frac{2}{x}\right)^8 = \ldots + a\cdot x^6 \ldots \Rightarrow$ **a = ?**

A) –18
B) –16
C) –14
D) –12
E) –10

10. $\left(x-\frac{3}{x}\right)^5 = \ldots + a\cdot x \ldots \Rightarrow$ **a = ?**

A) 80
B) 85
C) 90
D) 95
E) 100

11. $x = 101 \Rightarrow$ **$x^3 - 3x^2 + 3x$ = ?**

A) $10^3 + 1$
B) $10^3 - 1$
C) $10^6 + 1$
D) $10^6 - 1$
E) $10^2 + 1$

12. $\left(\sqrt{x}+\frac{x}{3}\right)^6 = \ldots\ldots + Ax^4 \Rightarrow$ **A = ?**

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{4}{3}$ E) $\frac{5}{3}$

YANITLAR / ANSWERS							
1. E	2. D	3. A	4. D	5. A	6. A	7. A	8. D
9. B	10. C	11. C	12. E				

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BÖLÜM 3
CHAPTER 3

TARİHSEL NOT / HISTORICAL NOTE

Cayley, Arthur [1821 – 1895]

19. yüzyılda Britanya'da pür matematiğin canlanmasına büyük katkıları olmuştur. Geometri ve cebirle ilgli 900e yakın makale yazmıştır. Matrisler teorisini geliştirmiştir.

British mathematician who contributed greatly to the resurgence of pure mathematics in Britain in the nineteenth century. He published over 900 papers on many aspects of geometry and algebra. He developed the theory of matrices.

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MATRIX and DETERMINANT

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MATRİS ve DETERMİNANT
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BÖLÜM 3
CHAPTER

MATRİS

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

(→ m satır; ↓ n sütun)

Yukarıdaki n tane sütun, m tane satırdan oluşan dikdörtgen tabloya *matris* denir.

$a_{ij} \in R\ (1 \le i \le m,\ 1 \le j \le n)$

m : Satır sayısı

n : Sütun sayısı

Örneğin;

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & 1 & -3 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

A bir 3 x 4 matristir.

NOT

Bir matris, [aij] şeklinde köşeli parantezle gösterilir.

MATRİS ÇEŞİTLERİ

KARE MATRİS

Tanım: Satır ve sütun sayısı aynı olan matrise *kare matris* denir.

Kare Matrislere Örnekler:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

(köşegen)

$$A=\begin{bmatrix} 2 & 3 & -4 & 1 \\ 0 & 5 & -1 & 5 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$ (köşegen)

$$B=\begin{bmatrix} -1 & 0 & 0 \\ 3 & 4 & 0 \\ -1 & -7 & 2 \end{bmatrix}$$ (köşegen)

$$C=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$ (köşegen)

NOT

Tek satıra sahip olan matrise satır matrisi, tek sütuna sahip matrise ise sütün matrisi denir

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MATRIX

NOTE

A matrix can be denoted by a representative element enclosed in brackets, such as [aij].

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

(→ m rows; ↓ n columns)

The rectangle array composed of n columns (vertical lines) and m rows (horizantal lines) is called a *matrix*.

$a_{ij} \in R\ (1 \le i \le m,\ 1 \le j \le n)$

m : Number of rows

n : Number of columns

For example;

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & 1 & -3 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

A is a 3 x 4 matrix.

KINDS of MATRIX

NOTE

A matrix that has only one row is called a row matris, and a matris that has only one column is called a column matrix.

SQUARE MATRIX

Definition: A matrix having equal number of rows and columns is called a *square matrix.*

Examples for square matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$ → diagonal

$$A = \begin{bmatrix} 2 & 3 & -4 & 1 \\ 0 & 5 & -1 & 5 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$ → diagonal

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 3 & 4 & 0 \\ -1 & -7 & 2 \end{bmatrix}$$ → diagonal

$$C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$ → diagonal

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BİRİM MATRİS

Tanım: Köşegen (esas köşegen) üzerindeki elemanları 1 ve diğer elemanları 0 olan kare matrise *birim matris* denir. I veya I_n ile gösterilir.

Örneğin;

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I_2 ve I_3 birim matrislerdir.

MATRİSLERİN EŞİTLİĞİ

$$\begin{bmatrix} x & y & z \\ m & n & t \\ a & b & c \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow \begin{cases} x = a_{11}, & y = a_{12}, & z = a_{13} \\ m = a_{21}, & n = a_{22}, & t = a_{23} \\ a = a_{31}, & b = a_{32}, & c = a_{33} \end{cases}$$

NOT

$1 \le i \le m$ ve $1 \le j \le n$ olmak üzere, m x n boyutlu matrisler için $a_{ij} = b_{ij}$ ise, $A = [a_{ij}]$ ve $B = [b_{ij}]$ matrisleri eşittir. Başka bir deyişle, 2 matrisin uygun hücreleri eşitse bu matrisler eşittir.

ÖRNEK 1

$$K = \begin{bmatrix} x+1 & 2 \\ y-4 & z+5 \end{bmatrix}, \quad L = \begin{bmatrix} 4 & t-5 \\ 6 & 2 \end{bmatrix}$$

$K = L \Rightarrow x + y + z + t = ?$

A) 8 B) 10 C) 13 D) 15 E) 17

ÇÖZÜM

Yanıt **E**

$$\begin{bmatrix} x+1 & 2 \\ y-4 & z+5 \end{bmatrix} = \begin{bmatrix} 4 & t-5 \\ 6 & 2 \end{bmatrix}$$

$K = L \Rightarrow x + 1 = 4 \qquad 2 = t - 5$

$x = 3 \qquad t = 7$

$y - 4 = 6 \qquad z + 5 = 2$

$y = 10 \qquad z = -3$

$x + y + z + t = 3 + 10 + (-3) + 7 = 17$

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IDENTITY MATRIX

Definition: The n x n matrix that consists of 1's on its main diagonal and 0's elsewhere is called the *identity matrix* of order n and is denoted by I or I_n.

For example;

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad , \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I_2 and I_3 are unit matrices.

NOTE

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if they have the same order (mxn) and $a_{ij} = b_{ij}$ for $1 \le i \le m$ and $1 \le j \le n$. In other words, two matrices are equal if their corresponding entries are equal.

EQUALITY of MATRICES

$$\begin{bmatrix} x & y & z \\ m & n & t \\ a & b & c \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow \begin{cases} x = a_{11}, & y = a_{12}, & z = a_{13} \\ m = a_{21}, & n = a_{22}, & t = a_{23} \\ a = a_{31}, & b = a_{32}, & c = a_{33} \end{cases}$$

EXAMPLE 1

$$K = \begin{bmatrix} x+1 & 2 \\ y-4 & z+5 \end{bmatrix} \quad , \quad L = \begin{bmatrix} 4 & t-5 \\ 6 & 2 \end{bmatrix}$$

$K = L \Rightarrow x + y + z + t = ?$

A) 8 B) 10 C) 13 D) 15 E) 17

SOLUTION

The answer is E

$$\begin{bmatrix} x+1 & 2 \\ y-4 & z+5 \end{bmatrix} = \begin{bmatrix} 4 & t-5 \\ 6 & 2 \end{bmatrix}$$

$K = L \Rightarrow$ $x + 1 = 4$ $\quad$ $2 = t - 5$

$x = 3$ $\quad$ $t = 7$

$y - 4 = 6$ $\quad$ $z + 5 = 2$

$y = 10$ $\quad$ $z = -3$

$x + y + z + t = 3 + 10 + (-3) + 7 = 17$

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MATRİSLERİN TOPLAMI ve FARKI

$A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n} \Rightarrow A + B = [a_{ij} + b_{ij}]_{m\times n}$, $A - B = [a_{ij} - b_{ij}]_{m\times n}$

NOT

$A + B = B + A$ *[Değişme özelliği]*

ÖRNEK 2

$A = \begin{bmatrix} 2 & 4 \\ -5 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} \Rightarrow A+B = ?$

A) $\begin{bmatrix} 7 & 3 \\ -2 & 11 \end{bmatrix}$ B) $\begin{bmatrix} -3 & 5 \\ -8 & 7 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 4 \\ 5 & 9 \end{bmatrix}$

D) $\begin{bmatrix} 7 & 4 \\ 5 & 9 \end{bmatrix}$ E) $\begin{bmatrix} 7 & 5 \\ 2 & 11 \end{bmatrix}$

ÇÖZÜM

Yanıt **A**

$$A+B = \begin{bmatrix} 2 & 4 \\ -5 & 9 \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+5 & 4-1 \\ -5+3 & 9+2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ -2 & 11 \end{bmatrix}$$

BİR MATRİSİN bir SKALERLE ÇARPIMI

$\alpha \in R$

$$\alpha \cdot \begin{bmatrix} a & b & c \\ x & y & z \\ m & n & t \end{bmatrix} = \begin{bmatrix} \alpha\cdot a & \alpha\cdot b & \alpha\cdot c \\ \alpha\cdot x & \alpha\cdot y & \alpha\cdot z \\ \alpha\cdot m & \alpha\cdot n & \alpha\cdot t \end{bmatrix}$$

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NOTE

$A + B = B + A$ *[Commutative]*

SUM and DIFFERENCE of MATRICES

$A = [a_{ij}]_{m\times n}, B = [b_{ij}]_{m\times n} \Rightarrow A + B = [a_{ij} + b_{ij}]_{m\times n}$, $A - B = [a_{ij} - b_{ij}]_{m\times n}$

EXAMPLE 2

$A = \begin{bmatrix} 2 & 4 \\ -5 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} \Rightarrow A+B = ?$

A) $\begin{bmatrix} 7 & 3 \\ -2 & 11 \end{bmatrix}$ B) $\begin{bmatrix} -3 & 5 \\ -8 & 7 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 4 \\ 5 & 9 \end{bmatrix}$

D) $\begin{bmatrix} 7 & 4 \\ 5 & 9 \end{bmatrix}$ E) $\begin{bmatrix} 7 & 5 \\ 2 & 11 \end{bmatrix}$

SOLUTION

The answer is **A**

$$A+B = \begin{bmatrix} 2 & 4 \\ -5 & 9 \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+5 & 4-1 \\ -5+3 & 9+2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ -2 & 11 \end{bmatrix}$$

PRODUCT of SCALAR and MATRIX

$\alpha \in R$

$$\alpha \cdot \begin{bmatrix} a & b & c \\ x & y & z \\ m & n & t \end{bmatrix} = \begin{bmatrix} \alpha\cdot a & \alpha\cdot b & \alpha\cdot c \\ \alpha\cdot x & \alpha\cdot y & \alpha\cdot z \\ \alpha\cdot m & \alpha\cdot n & \alpha\cdot t \end{bmatrix}$$

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ÖRNEK 3

$A=\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}$, $B=\begin{bmatrix}-1 & 0\\ 1 & 4\end{bmatrix} \Rightarrow 2A+B=?$

A) $\begin{bmatrix}3 & 6\\ 1 & 0\end{bmatrix}$ B) $\begin{bmatrix}2 & 6\\ -1 & 3\end{bmatrix}$ C) $\begin{bmatrix}3 & 6\\ 3 & 2\end{bmatrix}$

D) $\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$ E) $\begin{bmatrix}2 & 1\\ 4 & 6\end{bmatrix}$

ÇÖZÜM

Yanıt **C**

$A=\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix} \Rightarrow 2A=\begin{bmatrix}4 & 6\\ 2 & -2\end{bmatrix}$

$2A+B=\begin{bmatrix}4 & 6\\ 2 & -2\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 1 & 4\end{bmatrix}$

$=\begin{bmatrix}4-1 & 6+0\\ 2+1 & -2+4\end{bmatrix}=\begin{bmatrix}3 & 6\\ 3 & 2\end{bmatrix}$

MATRİSLERİN ÇARPIMI

$A=[a_{ij}]_{m\times n}$ ve $B=[b_{ij}]_{n\times p}$

A ve B matrislerinin çarpımı, $A\cdot B = AB$ biçiminde gösterilir.

Örneğin:

$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$, $B=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}$

$\Rightarrow A\cdot B=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\cdot\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22}\\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22}\end{bmatrix}$

NOTLAR

- ✓ *İki matrisin çarpılabilmesi için; birinci matrisin sütun sayısı, ikinci matrisin satır sayısına eşit olmalıdır.*
- ✓ $AB \neq BA$
- ✓ $(AB)\cdot C = A\cdot(BC)$
- ✓ $(B+C)\cdot A = BA + CA$
- ✓ $k\cdot(AB) = (kA)\cdot B = A\cdot(kB)$ $(k \in R)$
- ✓ $IA = AI = A$

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EXAMPLE 3

$A=\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}$, $B=\begin{bmatrix}-1 & 0\\ 1 & 4\end{bmatrix} \Rightarrow 2A+B=?$

A) $\begin{bmatrix}3 & 6\\ 1 & 0\end{bmatrix}$ B) $\begin{bmatrix}2 & 6\\ -1 & 3\end{bmatrix}$ C) $\begin{bmatrix}3 & 6\\ 3 & 2\end{bmatrix}$

D) $\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$ E) $\begin{bmatrix}2 & 1\\ 4 & 6\end{bmatrix}$

SOLUTION

The answer is **C**

$A=\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix} \Rightarrow 2A=\begin{bmatrix}4 & 6\\ 2 & -2\end{bmatrix}$

$2A+B=\begin{bmatrix}4 & 6\\ 2 & -2\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 1 & 4\end{bmatrix}$

$=\begin{bmatrix}4-1 & 6+0\\ 2+1 & -2+4\end{bmatrix}=\begin{bmatrix}3 & 6\\ 3 & 2\end{bmatrix}$

NOTES

- ✓ *For the product of two matrices, the number of columns of the first matrix must equal the number of rows of the second matrix.*
- ✓ $AB \neq BA$
- ✓ $(AB)\cdot C = A\cdot(BC)$
- ✓ $(B + C)\cdot A = BA + CA$
- ✓ $k\cdot(AB) = (kA)\cdot B = A\cdot(kB)$ $(k \in R)$
- ✓ $IA = AI = A$

PRODUCT of MATRICES

$A=[a_{ij}]_{m\times n}$ ve $B=[b_{ij}]_{n\times p}$

Multiplication of matrices A and B is denoted by $A\cdot B = AB$.

For example:

$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$, $B=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}$

$\Rightarrow A\cdot B=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\cdot\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22}\\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22}\end{bmatrix}$

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ÖRNEK 4

$A=[2 \quad -1 \quad 3 \quad 4]$, $B=\begin{bmatrix}3 & -1\\ -2 & 4\\ 0 & 2\\ -3 & 4\end{bmatrix} \Rightarrow A\cdot B=?$

A) [16 –4] B) [–16 4] C) [–4 –16]

D) [4 –16] E) [–4 16]

ÇÖZÜM

Yanıt **E**

$$A\cdot B=[2 \quad -1 \quad 3 \quad 4]\cdot\begin{bmatrix}3 & -1\\ -2 & 4\\ 0 & 2\\ -3 & 4\end{bmatrix}=[6+2+0-12 \quad -2-4+6+16]=[-4 \quad 16]$$

PEKİŞTİRME

$x=[3 \quad 3 \quad 1]$, $y=\begin{bmatrix}3\\ 1\\ -1\end{bmatrix} \Rightarrow x\cdot y=?$

YANIT : 11

ÖRNEK 5

$A=\begin{bmatrix}2 & -1 & 1\\ 3 & -2 & 5\end{bmatrix}$, $B=\begin{bmatrix}0 & 2 & 3\\ 4 & 1 & 0\\ 2 & 1 & -1\end{bmatrix} \Rightarrow A\cdot B=?$

A) $\begin{bmatrix}2 & 4 & 5\\ 2 & 9 & 4\end{bmatrix}$ B) $\begin{bmatrix}-2 & 4 & 5\\ -2 & 9 & 4\end{bmatrix}$ C) $\begin{bmatrix}-2 & 4 & 5\\ 2 & 9 & 4\end{bmatrix}$ D) $\begin{bmatrix}2 & 4 & 5\\ 2 & 9 & -4\end{bmatrix}$ E) $\begin{bmatrix}-2 & 4 & 5\\ 2 & 9 & -4\end{bmatrix}$

ÇÖZÜM

Yanıt **C**

$$A\cdot B=\begin{bmatrix}2 & -1 & 1\\ 3 & -2 & 5\end{bmatrix}\cdot\begin{bmatrix}0 & 2 & 3\\ 4 & 1 & 0\\ 2 & 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}0-4+2 & 4-1+1 & 6+0-1\\ 0-8+10 & 6-2+5 & 9+0-5\end{bmatrix}=\begin{bmatrix}-2 & 4 & 5\\ 2 & 9 & 4\end{bmatrix}$$

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REINFORCE

$x=[3 \quad 3 \quad 1]$, $y=\begin{bmatrix}3\\1\\-1\end{bmatrix} \Rightarrow x\cdot y=?$

ANSWER : 11

EXAMPLE 4

$A=[2 \quad -1 \quad 3 \quad 4]$, $B=\begin{bmatrix}3 & -1\\-2 & 4\\0 & 2\\-3 & 4\end{bmatrix} \Rightarrow A\cdot B=?$

A) $[16 \ -4]$ B) $[-16 \ 4]$ C) $[-4 \ -16]$

D) $[4 \ -16]$ E) $[-4 \ 16]$

SOLUTION

The answer is E

$$A\cdot B=[2 \quad -1 \quad 3 \quad 4]\cdot\begin{bmatrix}3 & -1\\-2 & 4\\0 & 2\\-3 & 4\end{bmatrix}=[6+2+0-12 \quad -2-4+6+16]=[-4 \ 16]$$

EXAMPLE 5

$A=\begin{bmatrix}2 & -1 & 1\\3 & -2 & 5\end{bmatrix}$, $B=\begin{bmatrix}0 & 2 & 3\\4 & 1 & 0\\2 & 1 & -1\end{bmatrix} \Rightarrow A\cdot B=?$

A) $\begin{bmatrix}2 & 4 & 5\\2 & 9 & 4\end{bmatrix}$ B) $\begin{bmatrix}-2 & 4 & 5\\-2 & 9 & 4\end{bmatrix}$ C) $\begin{bmatrix}-2 & 4 & 5\\2 & 9 & 4\end{bmatrix}$ D) $\begin{bmatrix}2 & 4 & 5\\2 & 9 & -4\end{bmatrix}$ E) $\begin{bmatrix}-2 & 4 & 5\\2 & 9 & -4\end{bmatrix}$

SOLUTION

The answer is C

$$A\cdot B=\begin{bmatrix}2 & -1 & 1\\3 & -2 & 5\end{bmatrix}\cdot\begin{bmatrix}0 & 2 & 3\\4 & 1 & 0\\2 & 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}0-4+2 & 4-1+1 & 6+0-1\\0-8+10 & 6-2+5 & 9+0-5\end{bmatrix}=\begin{bmatrix}-2 & 4 & 5\\2 & 9 & 4\end{bmatrix}$$

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MATRİSİN KUVVETİ

$A^n = \underbrace{A \cdot A \cdot A \cdot \ldots \cdot A}_{n \text{ tane}}$

NOT

✓ $I_n^k = I_n$

ÖRNEK 6

$A = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \Rightarrow A^2 B = ?$

A) $\begin{bmatrix} 9 & -18 \\ 14 & 76 \end{bmatrix}$ B) $\begin{bmatrix} 9 & -18 \\ -14 & 76 \end{bmatrix}$ C) $\begin{bmatrix} 9 & 14 \\ -18 & 76 \end{bmatrix}$ D) $\begin{bmatrix} 9 & 76 \\ -18 & 14 \end{bmatrix}$ E) $\begin{bmatrix} 14 & 76 \\ 9 & -18 \end{bmatrix}$

ÇÖZÜM

Yanıt **B**

$$A = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3 + 0 \cdot (-2) & 3 \cdot 0 + 0 \cdot 4 \\ (-2) \cdot 3 + 4 \cdot (-2) & (-2) \cdot 0 + 4 \cdot 4 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ -14 & 16 \end{bmatrix}$$

$$A^2 \cdot B = \begin{bmatrix} 9 & 0 \\ -14 & 16 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 \cdot 1 + 0 \cdot 0 & 9 \cdot (-2) + 0 \cdot 3 \\ -14 \cdot 1 + 16 \cdot 0 & (-14) \cdot (-2) + 16 \cdot 3 \end{bmatrix} = \begin{bmatrix} 9 & -18 \\ -14 & 76 \end{bmatrix}$$

ÖRNEK 7

$A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{18} = ?$

A) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & -9 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -32 \\ 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -54 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & -72 \\ 0 & 1 \end{bmatrix}$

ÇÖZÜM

Yanıt **D**

$$A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 1 + (-3) \cdot 0 & 1 \cdot (-3) - 3 \cdot 1 \\ 0 \cdot 1 + 1 \cdot 0 & 0 \cdot (-3) + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & -6 \\ 0 & 1 \end{bmatrix}$$

$$A^{18} = \begin{bmatrix} 1 & -3 \cdot 18 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -54 \\ 0 & 1 \end{bmatrix}$$

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NOTE

✓ $I_n^k = I_n$

POWER of a MATRIX

$$A^n = \underbrace{A \cdot A \cdot A \cdot \ldots \cdot A}_{n \text{ times}}$$

EXAMPLE 6

$$A = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \Rightarrow A^2 B = ?$$

A) $\begin{bmatrix} 9 & -18 \\ 14 & 76 \end{bmatrix}$ B) $\begin{bmatrix} 9 & -18 \\ -14 & 76 \end{bmatrix}$ C) $\begin{bmatrix} 9 & 14 \\ -18 & 76 \end{bmatrix}$ D) $\begin{bmatrix} 9 & 76 \\ -18 & 14 \end{bmatrix}$ E) $\begin{bmatrix} 14 & 76 \\ 9 & -18 \end{bmatrix}$

SOLUTION

The answer is **B**

$$A = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3 + 0 \cdot (-2) & 3 \cdot 0 + 0 \cdot 4 \\ (-2) \cdot 3 + 4 \cdot (-2) & (-2) \cdot 0 + 4 \cdot 4 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ -14 & 16 \end{bmatrix}$$

$$A^2 \cdot B = \begin{bmatrix} 9 & 0 \\ -14 & 16 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 \cdot 1 + 0 \cdot 0 & 9 \cdot (-2) + 0 \cdot 3 \\ -14 \cdot 1 + 16 \cdot 0 & (-14) \cdot (-2) + 16 \cdot 3 \end{bmatrix} = \begin{bmatrix} 9 & -18 \\ -14 & 76 \end{bmatrix}$$

EXAMPLE 7

$$A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{18} = ?$$

A) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & -9 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -32 \\ 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -54 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & -72 \\ 0 & 1 \end{bmatrix}$

SOLUTION

The answer is **D**

$$A = \begin{bmatrix} 1 & \mathbf{-3} \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 1 + (-3) \cdot 0 & 1 \cdot (-3) - 3 \cdot 1 \\ 0 \cdot 1 + 1 \cdot 0 & 0 \cdot (-3) + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{-6} \\ 0 & 1 \end{bmatrix}$$

$$A^{18} = \begin{bmatrix} 1 & -3 \cdot 18 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -54 \\ 0 & 1 \end{bmatrix}$$

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ÖRNEK 8

$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $A^2-5A+7I=X \Rightarrow X=?$

A) $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ B) $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ C) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

D) $\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$ E) $\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

ÇÖZÜM

Yanıt C

$$X=A^2-5A+7I=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\cdot\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}+\begin{bmatrix}-15 & -5\\ 5 & -10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

ÖRNEK 9

$\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}^{1998}=?$

A) $3^{999}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ B) $9^{999}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ C) $3^{999}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

D) $9^{1998}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ E) $3^{999}\begin{bmatrix}1998 & 0\\ 0 & 1998\end{bmatrix}$

ÇÖZÜM

Yanıt B

$$\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}\cdot\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}=\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}=9I \Rightarrow \begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}^{1998}=\left(\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}^2\right)^{999}$$

$$=(9I)^{999}$$

$$=9^{999}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

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EXAMPLE 8

$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $A^2-5A+7I=x \Rightarrow x=?$

A) $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ B) $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ C) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

D) $\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$ E) $\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

SOLUTION

The answer is **C**

$$x=A^2-5A+7I=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\cdot\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}+\begin{bmatrix}-15 & -5\\ 5 & -10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

EXAMPLE 9

$\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}^{1998}=?$

A) $3^{999}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ B) $9^{999}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ C) $3^{999}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

D) $9^{1998}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ E) $3^{999}\begin{bmatrix}1998 & 0\\ 0 & 1998\end{bmatrix}$

SOLUTION

The answer is **B**

$$\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}\cdot\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}=\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}=9I \Rightarrow \begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}^{1998}=\left(\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}^2\right)^{999}$$

$$=(9I)^{999}$$

$$=9^{999}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

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MATRİSİN TRANSPOZU

Bir A matrisinin transpozu A^t olarak gösterilir ve satırların yerine sütunların, sütunların yerine satırların yazılması ile elde edilir.

$$A=\begin{bmatrix}2 & 1 & 3 & 4\\ -2 & 3 & -1 & 2\\ 6 & -4 & -3 & 5\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}2 & -2 & 6\\ 1 & 3 & -4\\ 3 & -1 & -3\\ 4 & 2 & 5\end{bmatrix}$$

NOTLAR

- ✓ *Matrislerin toplamının transpozu, transpozlarının toplamıdır.*
$$(A+B)^T=A^T+B^T$$
- ✓ *Matrislerin çarpımının transpozu, transpozlarının yer değiştirerek çarpımıdır.*
$$(A\cdot B)^T=B^T\cdot A^T$$
- ✓ *Bir matrisin transpozunun transpozu kendisidir.*
$$(A^T)^T=A$$
- ✓ *Bir matrisin bir sabitle çarpımının transpozu, transpozunun sabitle çarpımına eşittir.*
$$(k\cdot A)^T=k\cdot A^T$$

ÖRNEK 10

$A=\begin{bmatrix}2 & -3 & 1\\ 3 & 0 & 4\end{bmatrix}$, $B=\begin{bmatrix}0 & 1 & -2\\ 2 & 5 & 3\end{bmatrix} \Rightarrow A^T\cdot B=?$

A) $\begin{bmatrix}6 & 17 & 5\\ 0 & -3 & 6\\ 8 & 21 & 10\end{bmatrix}$ B) $\begin{bmatrix}5 & 4 & 3\\ 0 & -3 & 6\\ 8 & 21 & 10\end{bmatrix}$ C) $\begin{bmatrix}6 & 12 & 5\\ 0 & -3 & 6\\ 8 & 21 & 10\end{bmatrix}$

D) $\begin{bmatrix}5 & 17 & 5\\ 0 & -3 & 6\\ 8 & 21 & 10\end{bmatrix}$ E) $\begin{bmatrix}4 & 21 & 4\\ 0 & -3 & 6\\ 8 & 21 & 10\end{bmatrix}$

ÇÖZÜM

Yanıt **A**

$$A=\begin{bmatrix}2 & -3 & 1\\ 3 & 0 & 4\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}2 & 3\\ -3 & 0\\ 1 & 4\end{bmatrix}$$

$$A^T\cdot B=\begin{bmatrix}2 & 3\\ -3 & 0\\ 1 & 4\end{bmatrix}\cdot\begin{bmatrix}0 & 1 & -2\\ 2 & 5 & 3\end{bmatrix}=\begin{bmatrix}2\cdot0+3\cdot2 & 2\cdot1+3\cdot5 & 2\cdot(-2)+3\cdot3\\ (-3)\cdot0+0\cdot2 & (-3)\cdot1+0\cdot5 & (-3)\cdot(-2)+0\cdot3\\ 1\cdot0+4\cdot2 & 1\cdot1+4\cdot5 & 1\cdot(-2)+4\cdot3\end{bmatrix}$$

$$=\begin{bmatrix}6 & 17 & 5\\ 0 & -3 & 6\\ 8 & 21 & 10\end{bmatrix}$$

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NOTES

- ✓ *The transpose of a sum is the sum of transposes.*

$$(A + B)^T = A^T + B^T$$

- ✓ *The transpose of a product is the product of transposes in opposite order.*

$$(A \cdot B)^T = B^T \cdot A^T$$

- ✓ *The transpose of the transpose of a matrix is itself.*

$$(A^T)^T = A$$

- ✓ *The transpose of product of a matrix and a scalar is product scalar and of the transpose of matrix.*

$$(k \cdot A)^T = k \cdot A^T$$

TRANSPOSE of a MATRIX

The transpoze of a matrix A is denoted by A^t. To find the transpose of a matrix, rows of matrix are written instead of columns and columns of matrix are written instead of rows.

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ -2 & 3 & -1 & 2 \\ 6 & -4 & -3 & 5 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 2 & -2 & 6 \\ 1 & 3 & -4 \\ 3 & -1 & -3 \\ 4 & 2 & 5 \end{bmatrix}$$

EXAMPLE 10

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & -2 \\ 2 & 5 & 3 \end{bmatrix} \Rightarrow A^T \cdot B = ?$$

A) $\begin{bmatrix} 6 & 17 & 5 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 4 & 3 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$ C) $\begin{bmatrix} 6 & 12 & 5 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 17 & 5 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$ E) $\begin{bmatrix} 4 & 21 & 4 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$

SOLUTION

The answer is **A**

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 0 & 4 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^T \cdot B = \begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -2 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 0 + 3 \cdot 2 & 2 \cdot 1 + 3 \cdot 5 & 2 \cdot (-2) + 3 \cdot 3 \\ (-3) \cdot 0 + 0 \cdot 2 & (-3) \cdot 1 + 0 \cdot 5 & (-3) \cdot (-2) + 0 \cdot 3 \\ 1 \cdot 0 + 4 \cdot 2 & 1 \cdot 1 + 4 \cdot 5 & 1 \cdot (-2) + 4 \cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 17 & 5 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$$

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MATRİSİN TERSİ

Bir matrisin tersi A^{-1} ile gösterilir.

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

NOTLAR

- ✓ $A \cdot A^{-1} = I_n$
- ✓ $(A^T)^{-1} = (A^{-1})^T$
- ✓ $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$
- ✓ $(A^{-1})^{-1} = A$

ÖRNEK 11

$A=\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} \Rightarrow A^{-1}=?$

A) $\begin{bmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ B) $\begin{bmatrix} -\frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ C) $\begin{bmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$

D) $\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$ E) $\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$

ÇÖZÜM

Yanıt **B**

$$A^{-1}=-\frac{1}{8}\begin{bmatrix} 6 & -5 \\ -4 & 2 \end{bmatrix}=\begin{bmatrix} -\frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

ÖRNEK 12

$\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=I_{2x2} \Rightarrow a+b+c+d=?$

A) 10 B) 13 C) 16 D) 19 E) 22

ÇÖZÜM

Yanıt **D**

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \Rightarrow a+b+c+d=7+5+4+3=19$

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NOTES

- ✓ $A \cdot A^{-1} = I_n$
- ✓ $(A^T)^{-1} = (A^{-1})^T$
- ✓ $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$
- ✓ $(A^{-1})^{-1} = A$

THE INVERSE of a MATRIX

Inverse of a matrix is denoted by A^{-1}.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

EXAMPLE 11

$$A = \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} \Rightarrow A^{-1} = ?$$

A) $\begin{bmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ B) $\begin{bmatrix} -\frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ C) $\begin{bmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$

D) $\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$ E) $\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$

SOLUTION

The answer is **B**

$$A^{-1} = -\frac{1}{8}\begin{bmatrix} 6 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

EXAMPLE 12

$$\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I_{2x2} \Rightarrow a+b+c+d = ?$$

A) 10 B) 13 C) 16 D) 19 E) 22

SOLUTION

The answer is **D**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \Rightarrow a+b+c+d = 7+5+4+3 = 19$$

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BİR MATRİSİN DETERMİNANTI

Tanım: Bir kare matrisinin ilişkilendirildiği reel sayıya o matrisin *determinantı* denir ve detA veya |A| ile gösterilir.

a. 2 x 2'lik Matrisin Determinantı

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A| = \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Etkinlikler

a. $A=\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \Rightarrow |A| = 1\cdot 2 - 3\cdot(-1) = 2+3 = 5$

b. $x=\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \Rightarrow \det(x) = 1\cdot 1 - 0\cdot(-1) = 1$

ÖRNEK 13

$$\begin{vmatrix} 2004 & 2003 \\ 2001 & 2002 \end{vmatrix} = ?$$

A) 2225 B) 3205 C) 3605 D) 3999 E) 4005

ÇÖZÜM

Yanıt **E**

$$\begin{vmatrix} 2004 & 2003 \\ 2001 & 2002 \end{vmatrix} = 2004\cdot 2002 - 2001\cdot 2003$$

$t = 2004 \Rightarrow 2002 = t-2,\ 2001 = t-3,\ 2003 = t-1$

$$\Rightarrow 2004\cdot 2002 - 2001\cdot 2003 = t\cdot(t-2) - (t-3)\cdot(t-1)$$

$$= t^2 - 2t - t^2 + 4t - 3$$

$$= 2t - 3$$

$$= 2\cdot 2004 - 3$$

$$= 4008 - 3$$

$$= 4005$$

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THE DETERMINANT of a MATRIX

Definition: Every square matrix can be associated with a real number called its *determinant* and determinant of a matrix is denoted by **detA** or **|A|.**

a. The Determinant of a 2 x 2 Matrix

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A| = \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Activities

a. $A=\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \Rightarrow |A| = 1\cdot 2 - 3\cdot(-1) = 2+3 = 5$

b. $x=\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \Rightarrow \det(x) = 1\cdot 1 - 0\cdot(-1) = 1$

EXAMPLE 13

$$\begin{vmatrix} 2004 & 2003 \\ 2001 & 2002 \end{vmatrix} = ?$$

A) 2225 B) 3205 C) 3605 D) 3999 E) 4005

SOLUTION

The answer is **E**

$$\begin{vmatrix} 2004 & 2003 \\ 2001 & 2002 \end{vmatrix} = 2004\cdot 2002 - 2001\cdot 2003$$

$t = 2004 \Rightarrow 2002 = t-2,\ 2001 = t-3,\ 2003 = t-1$

$$\Rightarrow 2004\cdot 2002 - 2001\cdot 2003 = t\cdot(t-2) - (t-3)\cdot(t-1)$$

$$= t^2 - 2t - t^2 + 4t - 3$$

$$= 2t - 3$$

$$= 2\cdot 2004 - 3$$

$$= 4008 - 3$$

$$= 4005$$

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b. 3 x 3'lük Matrisin Determinantı (Sarrus Kuralı)

$$A=\begin{bmatrix} a & b & c \\ x & y & z \\ m & n & p \end{bmatrix} \Rightarrow \det A = |A| = \begin{vmatrix} a & b & c \\ x & y & z \\ m & n & p \end{vmatrix} \begin{matrix} \\ \\ \\ a & b \\ x & y \end{matrix} = (a\cdot y\cdot p + x\cdot n\cdot c + m\cdot b\cdot z) - (m\cdot y\cdot c + a\cdot n\cdot z + x\cdot b\cdot p)$$

ÖRNEK 14

$$A=\begin{bmatrix} 3 & 2 & 3 \\ 1 & -2 & -1 \\ -2 & 4 & 2 \end{bmatrix} \Rightarrow \det A = ?$$

A) −1 B) 0 C) 1 D) 2 E) 4

ÇÖZÜM

Yanıt **B**

$$\begin{vmatrix} 3 & 2 & 3 \\ 1 & -2 & -1 \\ -2 & 4 & 2 \\ 3 & 2 & 3 \\ 1 & -2 & -1 \end{vmatrix} \begin{matrix} \\ \\ 12 \\ -12 \\ 4 \\ 4 \end{matrix} \begin{matrix} \\ \\ -12 \\ 12 \\ 4 \\ 4 \end{matrix} \Rightarrow |A| = 4 - 4 = 0$$

c. n x n lik Matrisin Determinantı

Minör: A kare matrisinin i. satır ve j. sütununun silinmesiyle elde edilen matrisin determinantına bu matrisin *küçüğü (minörü)* denir. M_{ij} ile gösterilir.

Etkinlikler

a. $A=\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} \Rightarrow M_{11} = [1]$

b. $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 1 & 1 \end{bmatrix} \Rightarrow M_{21} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

Kofaktör: Bir matrisin i. satır, j. sütununun kofaktörü $A_{ij} = (-1)^{i+j} M_{ij}$ ile gösterilir.

Etkinlik

$$A=\begin{bmatrix} -1 & 1 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \Rightarrow A_{11} = (-1)^2 \cdot M_{11} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

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b. The Determinant of a 3 x 3 Matrix (Rule of Sarrus)

$$A=\begin{bmatrix} a & b & c \\ x & y & z \\ m & n & p \end{bmatrix} \Rightarrow \det A = |A| = \begin{matrix} a & b & c \\ x & y & z \\ m & n & p \\ a & b & c \\ x & y & z \end{matrix} = (a\cdot y\cdot p + x\cdot n\cdot c + m\cdot b\cdot z)-(m\cdot y\cdot c + a\cdot n\cdot z + x\cdot b\cdot p)$$

EXAMPLE 14

$$A=\begin{bmatrix} 3 & 2 & 3 \\ 1 & -2 & -1 \\ -2 & 4 & 2 \end{bmatrix} \Rightarrow \det A = ?$$

A) -1 B) 0 C) 1 D) 2 E) 4

SOLUTION

The answer is **B**

$$\begin{matrix} & 3 & 2 & 3 & \\ & 1 & -2 & -1 & \\ 12 & -2 & 4 & 2 & -12 \\ -12 & 3 & 2 & 3 & 12 \\ 4 & 1 & -2 & -1 & 4 \\ 4 & & & & 4 \end{matrix} \Rightarrow |A| = 4-4=0$$

c. The Determinant of a n x n Matrix

Minor: If A is a square matrix, the minor M_{ij} of the entry aij is the determinant of the matrix obtained by deleting the i^{th} row and j^{th} column of A.

Activities

a. $A=\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} \Rightarrow M_{11}=[1]$

b. $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 1 & 1 \end{bmatrix} \Rightarrow M_{21}=\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

Cofactor: The cofactor A_{ij} of the entry a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$.

Activity

$$A=\begin{bmatrix} -1 & 1 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \Rightarrow A_{11} = (-1)^2 \cdot M_{11} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

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▼ $|A| = (-1)^{i+j} M_{ij} \cdot a_{ij}$

ÖRNEK 15

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & -2 & 3 \\ -4 & 3 & 0 \end{bmatrix} \Rightarrow |A| = ?$$

A) –29 B) –32 C) 0 D) 29 E) 32

ÇÖZÜM

Yanıt A

$|A| = a_{31} \cdot A_{31} + a_{32} \cdot A_{32} + a_{33} \cdot A_{33}$

$$|A| = (-1)^{3+1} \cdot \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} + (-1)^{3+2} \cdot \begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} + (-1)^{3+3} \cdot \begin{vmatrix} -2 & 3 \\ -1 & -2 \end{vmatrix}$$

$|A| = -4 \cdot 11 + 3 \cdot 5 + 0 \cdot 7 = -29$

Determinantın Özellikleri

1. $|A| = |A^T|$
2. Bir çarpımın determinantı; determinantların ayrı ayrı çarpımına eşittir.
   $|A \cdot B| = |A| \cdot |B|$
3. $|A^n| = |A|^n$
4. $|A^{-1}| = \dfrac{1}{|A|}$
5. Bir kare matrisin herhangi satır veya sütunu diğeri ile yer değiştirirse, determinantın değeri işaret değiştirir.
6. A kare matrisinin herhangi satır veya sütununun tamamı 0 ise, $|A| = 0$ dır.
7. A kare matrisinin herhangi iki satır veya sütunu aynı ise, determinantı 0 dır.

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▼ $|A| = (-1)^{i+j} M_{ij} \cdot a_{ij}$

EXAMPLE 15

$$A = \begin{bmatrix} 5 & 0 & 2 & 1 \\ 0 & 4 & -1 & 1 \\ 0 & 0 & 0 & 3 \\ 1 & 2 & 1 & 3 \end{bmatrix} \Rightarrow |A| = ?$$

A) –29 B) –32 C) 0 D) 29 E) 32

SOLUTION

The answer is A

$|A| = a_{31} \cdot A_{31} + a_{32} \cdot A_{32} + a_{33} \cdot A_{33}$

$$|A| = (-1)^{3+1} \cdot \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} + (-1)^{3+2} \cdot \begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} + (-1)^{3+3} \cdot \begin{vmatrix} -2 & 3 \\ -1 & -2 \end{vmatrix}$$

$|A| = -4 \cdot 11 + 3 \cdot 5 + 0 \cdot 7 = -29$

Properties of Determinant

1. $|A| = |A^T|$
2. The determinant of a product is the product of determinants.
   $|A \cdot B| = |A| \cdot |B|$
3. $|A^n| = |A|^n$
4. $|A^{-1}| = \dfrac{1}{|A|}$
5. If any two rows or columns of a square matrix is interchanged with eachother, the sign of its determinant changes.
6. If the whole entries in any row or column of a square matrix A is 0, then $|A| = 0$.
7. If any two rows or columns of a square matrix A are the same, determinant will be 0.

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8. Bir determinantın herhangi iki satır veya sütununun tamamı orantılı ise, determinantı sıfırdır.

9. A kare matrisinin herhangi satır veya sütununun tamamı k reel sayısı ile çarpılırsa, matrisin determinantı k·IAI olur.

10. 2 x 2'lik A matrisi ve $k \in R$ için, $|kA| = k^2|A|$ dir.

11. Bir matrisin herhangi satır veya sütunu k reel sayısı ile çarpılır ve başka sütun veya satıra eklenirse determinant değişmez.

12. Bir matrisin köşegeninin altındaki veya üstündeki tüm elemanlar sıfır ise, determinant değeri, köşegendeki elemanların çarpımına veya negatif işaretlisine eşittir.

ÖRNEK 16

$$A=\begin{bmatrix}1&2&-1\\3&1&1\\-2&5&5\end{bmatrix},\ B=\begin{bmatrix}3&-1&2\\4&0&1\\-2&1&5\end{bmatrix} \Rightarrow \det(A\cdot B)=?$$

A) –837 B) –927 C) –1096 D) –1279 E) –1377

ÇÖZÜM

Yanıt E

$$A=\begin{bmatrix}1&2&-1\\3&1&1\\-2&5&5\end{bmatrix} \Rightarrow |A| = \begin{array}{|ccc|c} 1&2&-1& \\ 3&1&1& \\ -2&5&5& \\ \hline 2&5&30& \\ \hline 37 & & & \end{array} \begin{array}{c} \\ \\ 5 \\ -15 \\ \hline -4 \\ \hline -14 \end{array} = -14-37=-51$$

$$B=\begin{bmatrix}3&-1&2\\4&0&1\\-2&1&5\end{bmatrix} \Rightarrow |B| = \begin{array}{|ccc|c} 3&-1&2& \\ 4&0&1& \\ -2&1&5& \\ \hline 0&3&-20& \\ \hline -17 & & & \end{array} \begin{array}{c} \\ \\ 0 \\ 8 \\ \hline 2 \\ \hline 10 \end{array} = 10-(-17)=27$$

$|A\cdot B| = |A|\cdot|B| = (-51)\cdot(27) = -1377$

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8. If the entries of any two rows or columns of a determinant are proportional, the determinant will be zero.

9. When the entries of any row or column of a square matrix A are multiplied by a real number k, the new determinant becomes $k \cdot |A|$.

10. For a 2 x 2 matrix A and $k \in R$, $|kA| = k^2|A|$.

11. The value of a determinant does not change if its any row or column is multiplied by a real number k and then added to another row or column, respectively.

12. If the entries above or below a diagonal of a determinant are all 0, then the determinant is equal to either the product of entries on this diagonal or its minus sign.

EXAMPLE 16

$$A=\begin{bmatrix}1 & 2 & -1\\ 3 & 1 & 1\\ -2 & 5 & 5\end{bmatrix}, B=\begin{bmatrix}3 & -1 & 2\\ 4 & 0 & 1\\ -2 & 1 & 5\end{bmatrix} \Rightarrow \textbf{det}(A \cdot B) = ?$$

A) –837 B) –927 C) –1096 D) –1279 E) –1377

SOLUTION

The answer is E

$$A=\begin{bmatrix}1 & 2 & -1\\ 3 & 1 & 1\\ -2 & 5 & 5\end{bmatrix} \Rightarrow |A| = \begin{array}{c|ccc} & 1 & 2 & -1 \\ 2 & 3 & 1 & 1 \\ 5 & -2 & 5 & 5 \\ 30 & 1 & 2 & -1 \\ \hline 37 & 3 & 1 & 1 \end{array} \quad \begin{array}{c} \\ 5 \\ -15 \\ -4 \\ \hline -14 \end{array} = -14 - 37 = -51$$

$$B=\begin{bmatrix}3 & -1 & 2\\ 4 & 0 & 1\\ -2 & 1 & 5\end{bmatrix} \Rightarrow |B| = \begin{array}{c|ccc} & 3 & -1 & 2 \\ 0 & 4 & 0 & 1 \\ 3 & -2 & 1 & 5 \\ -20 & 3 & -1 & 2 \\ \hline -17 & 4 & 0 & 1 \end{array} \quad \begin{array}{c} \\ 0 \\ 8 \\ 2 \\ \hline 10 \end{array} = 10 - (-17) = 27$$

$|A \cdot B| = |A| \cdot |B| = (-51) \cdot (27) = -1377$

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ÖRNEK 17

$A=\begin{bmatrix}13 & 6\\ 4 & 2\end{bmatrix}$, $B=\begin{bmatrix}5 & 3\\ -2 & 4\end{bmatrix}$ $\Rightarrow$ $\det(A^{-1}\cdot B)=?$

A) 1 B) 13 C) 14 D) 26 E) 52

ÇÖZÜM

Yanıt **B**

$|A^{-1}\cdot B|=|A^{-1}|\cdot|B|=|A|^{-1}\cdot|B|$

$$=\frac{1}{|A|}\cdot|B|$$

$$=\frac{1}{13\cdot 2-4\cdot 6}\cdot(5\cdot 4-3\cdot(-2))=\frac{1}{2}\cdot 26=13$$

ÖRNEK 18

$$x=\begin{vmatrix}1 & 3 & a\\ 2 & 9 & b\\ 3 & 5 & c\end{vmatrix}\Rightarrow\begin{vmatrix}\mathbf{1} & \mathbf{3} & \mathbf{a+3}\\ \mathbf{2} & \mathbf{9} & \mathbf{b+9}\\ \mathbf{3} & \mathbf{5} & \mathbf{c+5}\end{vmatrix}=?$$

A) x – 3 B) x – 2 C) x –1 D) x E) x +1

ÇÖZÜM

Yanıt **D**

$$x=\begin{vmatrix}1 & 3\ (a+3) & a\\ 2 & 9\ (b+9) & b\\ 3 & 5\ (c+5) & c\end{vmatrix}\Rightarrow\begin{vmatrix}1 & 3 & a+3\\ 2 & 9 & b+9\\ 3 & 5 & c+5\end{vmatrix}=x$$

Bir matrisin herhangi sütunu diğer sütune eklenirse, determinant değişmez.

ÖRNEK 19

$A=\begin{bmatrix}3 & 0 & 0\\ 1 & 2 & 0\\ 4 & 5 & 6\end{bmatrix}$ $\Rightarrow$ $|A|=?$

A) 30 B) 32 C) 36 D) 40 E) 52

ÇÖZÜM

Yanıt **C**

$|A| = 3(12) + 0(6 - 0) - 0(5 - 8)$

$|A| = 3 \cdot 12 = 36$

$|A| = 3 \cdot 2 \cdot 6 = 36$

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EXAMPLE 17

$A=\begin{bmatrix}13 & 6\\ 4 & 2\end{bmatrix}$, $B=\begin{bmatrix}5 & 3\\ -2 & 4\end{bmatrix} \Rightarrow \det(A^{-1}\cdot B)=?$

A) 1 B) 13 C) 14 D) 26 E) 52

SOLUTION

The answer is **B**

$$|A^{-1}\cdot B|=|A^{-1}|\cdot|B|=|A|^{-1}\cdot|B|$$

$$=\frac{1}{|A|}\cdot|B|$$

$$=\frac{1}{13\cdot 2-4\cdot 6}\cdot(5\cdot 4-3\cdot(-2))=\frac{1}{2}\cdot 26=13$$

EXAMPLE 18

$$x=\begin{vmatrix}1 & 3 & a\\ 2 & 9 & b\\ 3 & 5 & c\end{vmatrix} \Rightarrow \begin{vmatrix}\mathbf{1} & \mathbf{3} & \mathbf{a+3}\\ \mathbf{2} & \mathbf{9} & \mathbf{b+9}\\ \mathbf{3} & \mathbf{5} & \mathbf{c+5}\end{vmatrix}=?$$

A) x – 3 B) x – 2 C) x –1 D) x E) x +1

SOLUTION

The answer is **D**

$$x=\begin{vmatrix}1 & 3 \overset{a+3}{\curvearrowright} & a\\ 2 & 9 \overset{b+9}{\curvearrowright} & b\\ 3 & 5 \overset{c+5}{\curvearrowright} & c\end{vmatrix} \Rightarrow \begin{vmatrix}1 & 3 & a+3\\ 2 & 9 & b+9\\ 3 & 5 & c+5\end{vmatrix}=x$$

The value of a determinant does not change if its column added to another column.

EXAMPLE 19

$$A=\begin{bmatrix}3 & 0 & 0\\ 1 & 2 & 0\\ 4 & 5 & 6\end{bmatrix} \Rightarrow |A|=?$$

A) 30 B) 32 C) 36 D) 40 E) 52

SOLUTION

The answer is **C**

$|A| = 3(12) + 0(6 - 0) - 0(5 - 8)$

$|A| = 3 \cdot 12 = 36$

$|A| = 3 \cdot 2 \cdot 6 = 36$

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LİNEER DENKLEM SİSTEMLERİ

$Ax = B \Rightarrow x = A^{-1} \cdot B$

ÖRNEK 20

$$\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \Rightarrow x \cdot y = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt **B**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \frac{1}{-3-2} \begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$= \frac{-1}{5} \begin{bmatrix} -10 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$x = 2, y = 1 \Rightarrow x \cdot y = 2 \cdot 1 = 2$

ÖRNEK 21

$$x + y = 9 \text{ ve } \begin{bmatrix} 2 & 3 & -4 \\ 3 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \Rightarrow z = ?$$

A) $\frac{11}{2}$ B) $\frac{-9}{2}$ C) -4 D) $\frac{5}{2}$ E) 5

ÇÖZÜM

Yanıt **A**

$$\left.\begin{array}{r} 2x + 3y - 4z = 5 \\ 2 \,/\, 3x - y + 2z = 2 \end{array}\right\} \Rightarrow \begin{array}{r} 2x + 3y - 4z = 5 \\ 6x - 2y + 4z = 4 \\ \hline 8x + y = 9 \\ + \quad x + y = 9 \\ \hline 7x = 0 \\ x = 0 \Rightarrow y = 9 \end{array}$$

$$\begin{aligned} 2x + 3y - 4z &= 5 \\ 2 \cdot 0 + 3 \cdot 9 - 4z &= 5 \\ 4z &= 22 \\ z &= \frac{11}{2} \end{aligned}$$

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SYSTEMS of LINEAR EQUATION

$Ax = B \Rightarrow x = A^{-1} \cdot B$

EXAMPLE 20

$$\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \Rightarrow x \cdot y = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION

The answer is B

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \frac{1}{-3-2} \begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$= \frac{-1}{5} \begin{bmatrix} -10 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$x = 2, y = 1 \Rightarrow x \cdot y = 2 \cdot 1 = 2$

EXAMPLE 21

$$x + y = 9 \text{ and } \begin{bmatrix} 2 & 3 & -4 \\ 3 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \Rightarrow z = ?$$

A) $\frac{11}{2}$ B) $\frac{-9}{2}$ C) -4 D) $\frac{5}{2}$ E) 5

SOLUTION

The answer is A

$$\left.\begin{array}{l} 2x + 3y - 4z = 5 \\ 2 \,/\, 3x - y + 2z = 2 \end{array}\right\} \Rightarrow \begin{array}{r} 2x + 3y - 4z = 5 \\ 6x - 2y + 4z = 4 \\ \hline 8x + y = 9 \\ + \quad x + y = 9 \\ \hline 7x = 0 \\ x = 0 \Rightarrow y = 9 \end{array}$$

$$2x + 3y - 4z = 5$$
$$2 \cdot 0 + 3 \cdot 9 - 4z = 5$$
$$4z = 22$$
$$z = \frac{11}{2}$$

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1. $A=\begin{bmatrix}1&2&3\\3&4&1\end{bmatrix}$, $B=\begin{bmatrix}-1&1&4\\2&-4&3\end{bmatrix} \Rightarrow A+B=?$

A) $\begin{bmatrix}0&3&7\\5&0&4\end{bmatrix}$ B) $\begin{bmatrix}0&3&7\\3&4&1\end{bmatrix}$ C) $\begin{bmatrix}0&3&7\\2&-4&3\end{bmatrix}$

D) $\begin{bmatrix}0&3&3\\2&-4&1\end{bmatrix}$ E) $\begin{bmatrix}0&3&4\\5&0&7\end{bmatrix}$

2. $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}+\begin{bmatrix}5&6\\7&8\end{bmatrix}=\begin{bmatrix}6&8\\10&12\end{bmatrix} \Rightarrow 2a+b+c-2d=?$

A) –3
B) –2
C) –1
D) 0
E) 1

3. $A=\begin{bmatrix}3&-1\\4&4\\2&3\end{bmatrix}$, $B=\begin{bmatrix}1&3\\2&4\end{bmatrix} \Rightarrow A\cdot B=?$

A) $\begin{bmatrix}1&15&6\\5&28&16\end{bmatrix}$ B) $\begin{bmatrix}1&5\\12&28\\8&18\end{bmatrix}$ C) $\begin{bmatrix}1&5\\12&28\\6&18\end{bmatrix}$

D) $\begin{bmatrix}1&12&8\\5&28&18\end{bmatrix}$ E) $\begin{bmatrix}1&5\\12&28\\18&8\end{bmatrix}$

4. $A=\begin{bmatrix}-1&1\\3&1\end{bmatrix} \Rightarrow A^{16}=?$

A) $2^{18}\begin{bmatrix}1&0\\0&1\end{bmatrix}$ B) $2^{16}\begin{bmatrix}0&1\\1&0\end{bmatrix}$ C) $2^{32}\begin{bmatrix}0&1\\1&0\end{bmatrix}$

D) $2^{16}\begin{bmatrix}1&0\\0&1\end{bmatrix}$ E) $2^{32}\begin{bmatrix}1&0\\0&1\end{bmatrix}$

5. $A=\begin{bmatrix}9&-9\\6&-6\end{bmatrix} \Rightarrow A^5=?$

A) 2^3A
B) 2^4A
C) 2^5A
D) 3^2A
E) 3^4A

6. $\begin{bmatrix}1&3\\2&4\end{bmatrix}\cdot\begin{bmatrix}x&\frac{3}{2}\\y&-\frac{1}{2}\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix} \Rightarrow x+y=?$

A) 0
B) –1
C) –2
D) –3
E) –4

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7. $\begin{bmatrix} a & x \\ -x & 3 \end{bmatrix}^T \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix} \Rightarrow \mathbf{a = ?}$

A) –4
B) –3
C) –1
D) 2
E) 3

8. $A = \begin{bmatrix} 1 & x \\ y & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 22 & a & -8 \\ -2 & b & -7 \end{bmatrix}$

$A \cdot B = C \Rightarrow \mathbf{a + b = ?}$

A) 8
B) 9
C) 10
D) 11
E) 12

9. $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$A \cdot B = A - B \Rightarrow \mathbf{a + b + c + d = ?}$

A) 2
B) $\frac{5}{2}$
C) 3
D) $\frac{7}{2}$
E) 4

10. $A = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix}$, $f(x) = x^8 + x^2 - 2 \Rightarrow \mathbf{f(A) = ?}$

A) $\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ B) $\begin{bmatrix} -1 & 3 \\ 1 & -4 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}$

D) $\begin{bmatrix} -1 & -3 \\ -1 & 4 \end{bmatrix}$ E) $\begin{bmatrix} -6 & 6 \\ -2 & 0 \end{bmatrix}$

11. $A = \begin{vmatrix} x+1 & 3 & -2 \\ 4 & 1 & 0 \\ 3 & x & 1 \end{vmatrix} = 9 \Rightarrow \mathbf{x = ?}$

A) –2
B) –1
C) 0
D) 1
E) 2

12. $\begin{vmatrix} 0 & 1 & x-2 \\ 0 & -1 & 3 \\ x+2 & 0 & 1 \end{vmatrix} = 0$

Denkleminin çözüm kümesi hangisidir?

What is the solution set of the equation above?

A) {–1} B) {1} C) {1, 2}
D) {–2} E) {–2, –1}

YANITLAR / ANSWERS							
1. A	2. C	3. B	4. D	5. E	6. B	7. A	8. C
9. B	10. E	11. A	12. E				

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1.
$$\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1 & 4\\ 2 & -4 & 3\end{bmatrix}=\begin{bmatrix}0 & 3 & 7\\ 5 & 0 & 4\end{bmatrix}$$

Yanıt / The Answer is A

2.
$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}+\begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}=\begin{bmatrix}a+5 & b+6\\ c+7 & d+8\end{bmatrix}=\begin{bmatrix}6 & 8\\ 10 & 12\end{bmatrix}$$

$a+5=6\Rightarrow a=1$

$b+6=8\Rightarrow b=2$

$c+7=10\Rightarrow c=3$

$d+8=12\Rightarrow d=4$

$2a+b+c-2d=2\cdot1+2+3-2\cdot4=-1$

Yanıt / The Answer is C

3.
$$\begin{bmatrix}3 & -1\\ 4 & 4\\ 2 & 3\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}=\begin{bmatrix}3\cdot1+(-1)\cdot2 & 3\cdot3+(-1)\cdot4\\ 4\cdot1+4\cdot2 & 4\cdot3+4\cdot4\\ 2\cdot1+3\cdot2 & 2\cdot3+3\cdot4\end{bmatrix}$$
$$=\begin{bmatrix}1 & 5\\ 12 & 28\\ 8 & 18\end{bmatrix}$$

Yanıt / The Answer is B

4.
$$A^2=A\cdot A=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}=\begin{bmatrix}2^2 & 0\\ 0 & 2^2\end{bmatrix}$$
$$A^4=A^2\cdot A^2=\begin{bmatrix}16 & 0\\ 0 & 16\end{bmatrix}=\begin{bmatrix}2^4 & 0\\ 0 & 2^4\end{bmatrix}$$
$$A^8=A^4\cdot A^4=\begin{bmatrix}256 & 0\\ 0 & 256\end{bmatrix}=\begin{bmatrix}2^8 & 0\\ 0 & 2^8\end{bmatrix}$$
$$A^{16}=A^8\cdot A^8=\begin{bmatrix}2^{16} & 0\\ 0 & 2^{16}\end{bmatrix}=2^{16}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Yanıt / The Answer is D

5.
$$A^2=\begin{bmatrix}9 & -9\\ 6 & -6\end{bmatrix}\cdot\begin{bmatrix}9 & -9\\ 6 & -6\end{bmatrix}=\begin{bmatrix}27 & -27\\ 18 & -18\end{bmatrix}$$
$$A^3=\begin{bmatrix}27 & -27\\ 18 & -18\end{bmatrix}\cdot\begin{bmatrix}9 & -9\\ 6 & -6\end{bmatrix}=\begin{bmatrix}81 & -81\\ 54 & -54\end{bmatrix}$$
$$A^5=\begin{bmatrix}3^6 & -3^6\\ 3^5\cdot2 & -3^5\cdot2\end{bmatrix}=3^4\begin{bmatrix}9 & -9\\ 6 & -6\end{bmatrix}=3^4A$$

Yanıt / The Answer is E

6.
$$\begin{bmatrix}x & \frac{3}{2}\\ y & -\frac{1}{2}\end{bmatrix}=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}^{-1}=-\frac{1}{2}\begin{bmatrix}4 & -3\\ -2 & 1\end{bmatrix}$$

$\Rightarrow x=-2\ ,\ y=1$

$x+y=(-2)+1=-1$

Yanıt / The Answer is B

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7. $\begin{bmatrix} a & -x \\ x & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2a-x \\ -2x+3 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$

$-2x + 3 = 7 \Rightarrow x = -2$

$-2a - x = 10 \Rightarrow 2 - 2a = 10$

$a = -4$

Yanıt / The Answer is A

8. $A \cdot B = \begin{bmatrix} 2+4x & 1+2x & -3-x \\ 2y-8 & y-4 & 2-3y \end{bmatrix} = \begin{bmatrix} 22 & a & -8 \\ -2 & b & -7 \end{bmatrix}$

$2 + 4x = 22 \Rightarrow x = 5$

$2y - 8 = -2 \Rightarrow y = 3$

$a = 1 + 2x = 1 + 2 \cdot 5 = 11$

$b = y - 4 = 3 - 4 = -1$

$\Rightarrow a + b = 10$

Yanıt / The Answer is C

9. $A \cdot B = A - B$

$\begin{bmatrix} a & b \\ -2c & -2d \end{bmatrix} = \begin{bmatrix} 1-a & -b \\ -c & -2-d \end{bmatrix}$

$a = 1 - a \Rightarrow a = \frac{1}{2}$

$b = -b \Rightarrow b = 0$

$-2c = -c \Rightarrow c = 0$

$-2d = -2 - d \Rightarrow d = 2$

$\Rightarrow a+b+c+d = \frac{1}{2} + 0 + 0 + 2 = \frac{5}{2}$

Yanıt / The Answer is B

10. $A^2 = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}$

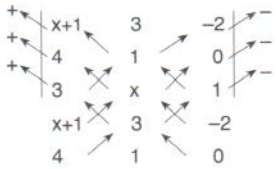
$A^3 = \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^3 = -I \Rightarrow A^8 = (A^3)^2 \cdot A^2 = A^2 = \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}$

$f(A) = \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} - 2 = \begin{bmatrix} -6 & 6 \\ -2 & 0 \end{bmatrix}$

Yanıt / The Answer is E

11.



$[(x+1)\cdot 1\cdot 1 + (-2)\cdot x\cdot 4 + 0\cdot 3\cdot 3] - [3\cdot 1\cdot(-2) + (x+1)\cdot x\cdot 0 + 4\cdot 3\cdot 1] = 9$

$(x + 1 - 8x) - (-6 + 12) = 9$

$-7x + 1 - 6 = 9$

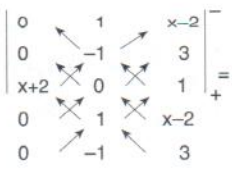
$-7x - 5 = 9$

$7x = -14$

$x = -2$

Yanıt / The Answer is A

12.



$= 3(x+2) + (x+2)\cdot(x-2) = 0$

$(x + 2)\cdot(3 + x - 2) = 0$

$(x + 2)(1 + x) = 0$

$x_1 = -2, \; x_2 = -1$

ÇK(SS) = {−2, −1}

Yanıt / The Answer is E

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1. $\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = I_{2x2} \Rightarrow a+b+c+d=?$

A) 17
B) 18
C) 19
D) 20
E) 21

2. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$

$x+2A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow x=?$

A) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$ C) $\begin{bmatrix} -1 & -2 \\ -1 & -2 \end{bmatrix}$

D) $\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$ E) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

3. $A = \begin{bmatrix} \sqrt{8} & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2^y & \log_4 x \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} \sqrt{32} & \log_9 27 \\ 1 & 2 \end{bmatrix}$

$A+B=C \Rightarrow x \cdot y = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

4. $A = \begin{bmatrix} a & 3 \\ b & c \end{bmatrix}, A = A^{-1} \Rightarrow a+c=?$

A) 0
B) 1
C) 2
D) 3
E) 4

5. $A = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}, A \cdot B = \begin{bmatrix} 4 & -5 \\ -10 & 9 \end{bmatrix} \Rightarrow B=?$

A) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$ B) $\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$ C) $\begin{bmatrix} -3 & 1 \\ 2 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ E) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

6. $\begin{vmatrix} a & 2 & -1 \\ 3 & -2 & 4 \\ 5 & 0 & 1 \end{vmatrix} = 46 \Rightarrow a=?$

A) −11
B) −1
C) 1
D) 2
E) 12

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7. $\begin{bmatrix} -1 & 0 \\ 2 & -2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} a & b & x \\ c & y & d \\ z & e & f \end{bmatrix} \Rightarrow x \cdot y \cdot z = ?$

A) –12
B) –10
C) 6
D) 10
E) 12

8. $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 7 & 9 \\ 19 & 23 \end{bmatrix}$, $A \cdot B = C \Rightarrow B = ?$

A) $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ B) $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ C) $\begin{bmatrix} -3 & 1 \\ 2 & -4 \end{bmatrix}$
D) $\begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$ E) $\begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}$

9. $A = \begin{bmatrix} a & \frac{1}{4} \\ \frac{1}{3} & b \end{bmatrix}$, $A = A^{-1} \Rightarrow a \cdot b = ?$

A) $-\frac{1}{12}$
B) $-\frac{11}{12}$
C) -1
D) $\frac{11}{12}$
E) 1

10. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{80} = ?$

A) $\begin{bmatrix} 40 & 1 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 40 \\ 1 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 80 \\ 1 & 1 \end{bmatrix}$
D) $\begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 0 & 80 \\ 0 & 1 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x & 4 & y \\ 6 & z & -3 \end{bmatrix} = \begin{bmatrix} a & b & 1 \\ c & 2 & d \\ 3 & e & f \end{bmatrix} \Rightarrow z + y - x = ?$

A) 6
B) 5
C) 4
D) 3
E) 2

12. $\begin{vmatrix} 2-x & x-2 \\ x & -1 \end{vmatrix} = -12 \Rightarrow ÇK(SS) = ?$

A) {–2, 3}
B) {–2, 5}
C) {2, 3}
D) {2, 0}
E) {2, 5}

YANITLAR / ANSWERS

1. C	2. D	3. C	4. A	5. E	6. A	7. E	8. B
9. B	10. D	11. A	12. B				

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1. $|A| = 21 - 20 = 1$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$$

$a + b + c + d = 19$

Yanıt / The Answer is C

2. $$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} + 2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a + 1 = 0 \Rightarrow a = -1$

$b + 2 = 0 \Rightarrow b = -2$

$c + 2 = 0 \Rightarrow c = -2$

$d + 1 = 0 \Rightarrow d = -1$

$$\Rightarrow X = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$$

Yanıt / The Answer is D

3. $A + B = C$

$$\begin{bmatrix} \sqrt{8} & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2^y & \log_4 x \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{32} & \log_9 27 \\ 1 & 2 \end{bmatrix}$$

$\sqrt{8} + 2^y = \sqrt{32} \Rightarrow 2^y = 4\sqrt{2} - 2\sqrt{2}$

$2^y = 2\sqrt{2}$

$2^y = 2^{\frac{3}{2}} \Rightarrow y = \frac{3}{2}$

$1 + \log_4 x = \log_9 27 = \log_{3^2} 3^3$

$1 + \log_4 x = \frac{3}{2}$

$\log_4 x = \frac{3}{2} - 1 = \frac{1}{2} \Rightarrow x = 4^{\frac{1}{2}} = 2 \Rightarrow x = 2$

$x \cdot y = 2 \cdot \frac{3}{2} = 3$

Yanıt / The Answer is C

4. $|A| = c \cdot a - 3 \cdot b$

$A = A^{-1} \Rightarrow |A| = -1, \quad \frac{a_{11}}{|A|} = a_{22}$

$c \cdot a - 3b = -1$

$\frac{a}{-1} = c \Rightarrow a + c = 0$

Yanıt / The Answer is A

5. $B = A^{-1}(A \cdot B)$

$$= -\frac{1}{2}\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -5 \\ -10 & 9 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} -4 & -2 \\ 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

Yanıt / The Answer is E

6.

	a	2	−1	
	3	−2	4	
10	5	0	1	−2a
0	a	2	−1	0
6	3	−2	4	40
16				40 − 2a

$\Rightarrow 40 - 2a - 16 = 46$

$24 - 2a = 46$

$2a = -22$

$a = -11$

Yanıt / The Answer is A

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7. $x = [-1 \quad 0] \cdot \begin{bmatrix} 3 \\ 0 \end{bmatrix} = -3$

$y = [2 \quad -2] \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2$

$z = [0 \quad 1] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2$

$\Rightarrow x \cdot y \cdot z = (-3) \cdot 2(-2) = 12$

Yanıt / The Answer is E

8. $A \cdot B = C \Rightarrow B = A^{-1} \cdot C$

$$= \frac{1}{-1} \cdot \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 9 \\ 19 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

Yanıt / The Answer is B

9. $A = A^{-1} \Rightarrow |A| = -1$

$a \cdot b - \frac{1}{12} = -1$

$a \cdot b = -\frac{11}{12}$

Yanıt / The Answer is B

10. $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{2} \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^{80} = \begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix}$

Yanıt / The Answer is D

11. $\begin{bmatrix} 2x+6 & 8+z & 2y-3 \\ 12-x & 2z-4 & -y-6 \\ 3x+6 & 12+z & 3y-3 \end{bmatrix} = \begin{bmatrix} a & b & 1 \\ c & 2 & d \\ 3 & e & f \end{bmatrix}$

$2y - 3 = 1 \Rightarrow y = 2$

$2z - 4 = 2 \Rightarrow z = 3$

$3x + 6 = 3 \Rightarrow x = -1$

$z + y - x = 3 + 2 + 1 = 6$

Yanıt / The Answer is A

12. $\begin{vmatrix} 2-x & x-2 \\ x & -1 \end{vmatrix} = (x-2) - x \cdot (x-2) = -12$

$(x - 2)(1 - x) = -12$

$-x^2 + 3x - 2 = -12$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x_1 = 5 \ , \ x_2 = -2$

Yanıt / The Answer is B

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ZTC زنگنه مجموعه موسسات

YÖS Soruları / YÖS Questions

1. $\begin{bmatrix} i & 0 & 0 \\ 2013 & i & 0 \\ 2014 & 2015 & i \end{bmatrix} \Rightarrow \textbf{detA} = ?$

A) $-i$ B) 0 C) i
D) $2012i$ E) 2014

[MARMARA ÜNİV. YÖS 2013]

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2. $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix} \Rightarrow A^T \cdot A = ?$

A) $\begin{bmatrix} 5 & -4 & -1 \\ -4 & 4 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 5 & -4 & -1 \\ 4 & -4 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

A) $\begin{bmatrix} 5 & 4 & -1 \\ -4 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ D) $\begin{bmatrix} 5 & -4 & 1 \\ 4 & -4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

E) $\begin{bmatrix} 5 & 4 & 1 \\ 4 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

[ANKARA ÜNİV. YÖS 2013]

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3. $A = \begin{bmatrix} \sin x & 1 \\ 1 & \sin x \end{bmatrix} \Rightarrow |A| = \det(A) = ?$

A) $\sin^2 x$ B) -1 C) 1
D) $-\cos^2 x$ E) $\cos^2 x$

[ANKARA ÜNİV. YÖS 2013]

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4. $\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix}^2 = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix} \Rightarrow (x,y) = ?$

A) (1, 2) B) (1, 0) C) (2, 0)
D) (2, 1) E) (2, 3)

[İSTANBUL ÜNİV. YÖS 2013]

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5. $A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} \Rightarrow A = ?$

A) $\begin{bmatrix} -3/2 & 3/4 \\ 1/2 & 1/2 \end{bmatrix}$ B) $\begin{bmatrix} 1/3 & 1/4 \\ 2/3 & 1 \end{bmatrix}$ C) $\begin{bmatrix} -1/4 & 1/4 \\ 0 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 1/2 & -3/4 \\ -1/2 & 3/4 \end{bmatrix}$ E) $\begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

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6. $A = \begin{bmatrix} 1 & x \\ 1 & y \end{bmatrix}$, $B = \begin{bmatrix} y & 1 \\ x & 1 \end{bmatrix} \Rightarrow |AB| = ?$

A) $x^2 - y^2$ B) $(x - y)^2$ C) $x - 2y$
D) $2(x + y)^2$ E) $2x + y$

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

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7. $A-3\begin{bmatrix}1 & 5\\ 2 & 4\end{bmatrix}+6=0 \Rightarrow A=?$

A) $\begin{bmatrix}-3 & 15\\ 6 & 6\end{bmatrix}$ B) $\begin{bmatrix}1/3 & 9/4\\ 2/3 & 1\end{bmatrix}$

C) $\begin{bmatrix}-1/4 & 9/4\\ 0 & -1\end{bmatrix}$ D) $\begin{bmatrix}9 & 15\\ 6 & 30\end{bmatrix}$

E) $\begin{bmatrix}3 & 5\\ 0 & 0\end{bmatrix}$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

8. $A=\begin{bmatrix}x & 0\\ 3 & -2\end{bmatrix}$

$B=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$\det(A+B^{-1})=|A+B^{-1}|=1$

$x=?$

A) –2 B) –1 C) 1 D) 2 E) 4

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

9. $\begin{bmatrix}1 & -2\\ 4 & 1\\ 0 & 3\end{bmatrix}\begin{bmatrix}-3\\ 2\end{bmatrix}=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$

$a+b+c=?$

A) –9 B) –11 C) –19 D) 6 E) 10

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

10. $A=\begin{bmatrix}-1 & 1\\ 0 & 2\end{bmatrix}$

$B=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$

$\Rightarrow A\cdot B^{-1}=?$

A) $\begin{bmatrix}-5 & 3\\ -6 & 4\end{bmatrix}$ B) $\begin{bmatrix}5 & 4\\ 3 & -6\end{bmatrix}$ C) $\begin{bmatrix}-4 & 3\\ 2 & 6\end{bmatrix}$

D) $\begin{bmatrix}3 & 4\\ -2 & -6\end{bmatrix}$ E) $\begin{bmatrix}4 & -6\\ 2 & -3\end{bmatrix}$

[YÖS 2009]

11. $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$

$B=\begin{bmatrix}-1 & 1\\ -2 & 0\end{bmatrix}$

$\det(A+B)=|A+B|=?$

A) –3 B) –2 C) 0 D) 6 E) 8

[YÖS 2009]

12. $\begin{bmatrix}x^2 & 1\\ x^2-9 & 0\end{bmatrix}=\begin{bmatrix}6x-9 & 1\\ 0 & \cos\frac{\pi x}{2}\end{bmatrix}$

$x=?$

A) –1 B) –2 C) 1 D) 2 E) 3

[YÖS 2008]

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13. $A=\begin{bmatrix}3 & 2\\ 9 & 5\end{bmatrix}$

$\det\left(A^{-1}\right)=?$

A) $-\frac{1}{3}$ B) $-\frac{1}{2}$ C) $\frac{3}{2}$ D) 2 E) 3

(YÖS 2008)

14. $A=\begin{bmatrix}2 & 3\\ 1 & 1\\ 4 & 6\end{bmatrix}$, $B=\begin{bmatrix}-3 & 0 & 5\\ 4 & 1 & -1\end{bmatrix} \Rightarrow A\cdot B=?$

A) $\begin{bmatrix}6 & 3 & 7\\ 1 & 1 & 4\\ 12 & 6 & 14\end{bmatrix}$ B) $\begin{bmatrix}-6 & 3 & 7\\ 1 & -11 & 4\\ 12 & 6 & -14\end{bmatrix}$

C) $\begin{bmatrix}-1 & 2 & 3\\ 1 & 1 & 4\\ -2 & 4 & 24\end{bmatrix}$ D) $\begin{bmatrix}-14 & 5\\ 21 & -7\end{bmatrix}$

E) $\begin{bmatrix}14 & 5\\ 21 & 7\end{bmatrix}$

(YÖS 2007)

15. $\begin{vmatrix}108765 & 108766\\ 108763 & 108764\end{vmatrix}=?$

A) 1 B) 2 C) 3 D) 4 E) 6

[YÖS 2007]

16. $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$A^2-5A-2I=?$

A) $\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$ B) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ C) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

D) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ E) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

[YÖS 2006]

17. $A=\begin{bmatrix}-5 & 1\\ 3 & -1\end{bmatrix} \Rightarrow A^{-1}=?$

A) $\begin{bmatrix}\frac{-1}{2} & \frac{-1}{2}\\ \frac{-3}{2} & \frac{-5}{2}\end{bmatrix}$ B) $\begin{bmatrix}-1 & -1\\ \frac{-3}{2} & -5\end{bmatrix}$

C) $\begin{bmatrix}\frac{-1}{2} & \frac{1}{2}\\ -3 & \frac{5}{2}\end{bmatrix}$ D) $\begin{bmatrix}-1 & 2\\ -3 & -5\end{bmatrix}$

E) $\begin{bmatrix}5 & 3\\ 2 & 1\end{bmatrix}$

[YÖS 2006]

18. $\begin{vmatrix}\sin x & \cos x\\ \sin x & \cos x\end{vmatrix}=?$

A) 0 B) 1 C) $\sin 2x$ D) $\sin^2 x$ E) $\sin^2 x-\cos^2 x$

[YÖS 2006]

YANITLAR / ANSWERS							
1. A	2. A	3. D	4. D	5. E	6. B	7. A	8. A
9. B	10. A	11. D	12. E	13. A	14. A	15. B	16. C
17. A	18. A						

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1.
$$\begin{bmatrix} i & 0 & 0 \\ 2013 & i & 0 \\ 2014 & 2015 & i \end{bmatrix}$$
$$\Rightarrow \det A = i(i^2 - 0) - 0(2013i - 0) + 0(2013 \cdot 2015 - 2014i)$$
$$= -i$$

Yanıt / The Answer is A

2.
$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ -1 & 0 \end{bmatrix}$$
$$A^T \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -4 & -1 \\ -4 & 4 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Yanıt / The Answer is A

3.
$$|A| = \sin x \cdot \sin x - 1$$
$$= \sin^2 x - 1$$
$$= -\cos^2 x$$

Yanıt / The Answer is D

4.
$$\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} \cdot \begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$
$$\begin{bmatrix} x^2 + 2y & 2x - 2 \\ xy - y & 2y + 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$
$$2x - 2 = 2 \qquad 2y + 1 = 3$$
$$2x = 4 \qquad 2y = 2$$
$$x = 2 \qquad y = 1$$
$$\Rightarrow (x, y) = (2, 1)$$

Yanıt / The Answer is D

5.
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A \cdot A^{-1} = I$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$$

$3a - 2b = 0$	$3c - 2d = 0$
$+ \; -a + 2b = 0$	$+ \; -c + 2d = 1$
$2a = 1$	$2c = 2$
$a = \frac{1}{2}$, $b \frac{1}{4}$	$c = \frac{1}{2}$, $d = \frac{3}{4}$

$$\Rightarrow A = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$$

Yanıt / The Answer is E

6.
$$A \cdot B = \begin{bmatrix} 1 & x \\ 1 & y \end{bmatrix} \cdot \begin{bmatrix} y & 1 \\ x & 1 \end{bmatrix} = \begin{bmatrix} y + x^2 & 1 + x \\ y + xy & 1 + y \end{bmatrix}$$
$$|A \cdot B| = (y + y^2 + x^2 + x^2 y) - (y + xy + 2y + x^2 y)$$
$$= x^2 + y^2 - 2xy$$
$$= (x - y)^2$$

Yanıt / The Answer is B

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7. $A-3\begin{bmatrix}1 & 5\\ 2 & 4\end{bmatrix}+6=0$

$$A-\begin{bmatrix}3 & 15\\ 6 & 12\end{bmatrix}+\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$A=\begin{bmatrix}-3 & 15\\ 6 & 6\end{bmatrix}$$

Yanıt / The Answer is A

8. $A+B^{-1}=\begin{bmatrix}x & 0\\ 3 & -2\end{bmatrix}+\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}=\begin{bmatrix}x+1 & 0\\ 1 & -1\end{bmatrix}$

$\left|A+B^{-1}\right|=-x-1=1 \Rightarrow x=-2$

Yanıt / The Answer is A

9. $\begin{bmatrix}1 & -2\\ 4 & 1\\ 0 & 3\end{bmatrix}\begin{bmatrix}-3\\ 2\end{bmatrix}=\begin{bmatrix}-7\\ -10\\ 6\end{bmatrix}=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$

$\Rightarrow a=-7,\ b=-10,\ c=6$

$a+b+c=(-7)+(-10)+6=-11$

Yanıt / The Answer is B

10. $B^{-1}=\frac{1}{4-3}\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$

$$A\cdot B^{-1}=\begin{bmatrix}-1 & 1\\ 0 & 2\end{bmatrix}\cdot\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-5 & 3\\ -6 & 4\end{bmatrix}$$

Yanıt / The Answer is A

11. $A+B=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 1\\ -2 & 0\end{bmatrix}=\begin{bmatrix}0 & 3\\ -2 & 3\end{bmatrix}$

$\det(A+B)=0-(-6)=6$

Yanıt / The Answer is D

12. $x^2-9=0 \Rightarrow x^2=9$

$x=\pm 3$

Yanıt / The Answer is E

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13. $\det\left(A^{-1}\right)=\frac{1}{\det(A)}=\frac{1}{3\cdot5-2\cdot9}=\frac{1}{15-18}=-\frac{1}{3}$

Yanıt / The Answer is A

14. $A\cdot B=\begin{bmatrix}2&3\\1&1\\4&6\end{bmatrix}\cdot\begin{bmatrix}-3&0&5\\4&1&-1\end{bmatrix}=\begin{bmatrix}6&3&7\\1&1&4\\12&6&14\end{bmatrix}$

Yanıt / The Answer is A

15. $108765 = x$

$$\begin{vmatrix}x&x+1\\x-2&x-1\end{vmatrix}=x^2-x-(x^2-x-2)$$

$$=x^2-x-x^2+x+2$$

$$=2$$

Yanıt / The Answer is B

16. $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}\Rightarrow A^2=\begin{bmatrix}7&10\\15&22\end{bmatrix}$

$$\begin{bmatrix}7&10\\15&22\end{bmatrix}-5\begin{bmatrix}1&2\\3&4\end{bmatrix}-2\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

Yanıt / The Answer is C

17. $A=\begin{bmatrix}-5&1\\3&-1\end{bmatrix}\Rightarrow \det(A)=(-5)(-1)-(3\cdot1)=2$

$$\Rightarrow A^{-1}=\frac{1}{2}\begin{bmatrix}-1&-1\\-3&-5\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}&-\frac{1}{2}\\-\frac{3}{2}&-\frac{5}{2}\end{bmatrix}$$

Yanıt / The Answer is A

18. $\begin{vmatrix}\sin x&\cos x\\\sin x&\cos x\end{vmatrix}=\sin x\cdot\cos x-\sin x\cdot\cos x=0$

Yanıt / The Answer is A

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1. $A=\begin{bmatrix}3 & 0\\ 1 & 2\end{bmatrix} \Rightarrow A+A^T=?$

Yanıt / The Answer is $\begin{bmatrix}6 & 1\\ 1 & 4\end{bmatrix}$

2. $A=\begin{bmatrix}3 & 4 & 5\\ 1 & 2 & 3\\ -3 & 4 & -1\end{bmatrix} \Rightarrow 2A=?$

Yanıt / The Answer is $\begin{bmatrix}6 & 8 & 10\\ 2 & 4 & 6\\ -6 & 8 & -2\end{bmatrix}$

3. $\begin{bmatrix}2 & -3\\ 4 & 0\end{bmatrix}-x=\begin{bmatrix}-3 & 4\\ 5 & -1\end{bmatrix} \Rightarrow x=?$

Yanıt / The Answer is $\begin{bmatrix}5 & -7\\ -1 & 1\end{bmatrix}$

4. $\begin{bmatrix}-2 & 2\\ 4 & 5\end{bmatrix}-3\cdot A=\begin{bmatrix}4 & 8\\ -5 & -7\end{bmatrix} \Rightarrow A=?$

Yanıt / The Answer is $\begin{bmatrix}-2 & -2\\ 3 & 4\end{bmatrix}$

5. $A=\begin{bmatrix}1 & 2 & 0\\ 3 & -1 & 4\end{bmatrix} \Rightarrow A\cdot A^T=?$

Yanıt / The Answer is $\begin{bmatrix}5 & 1\\ 1 & 26\end{bmatrix}$

6. $A=\begin{bmatrix}1 & -2 & 3\\ 2 & 0 & -1\end{bmatrix}$, $B=\begin{bmatrix}0 & 2\\ -3 & 1\\ 4 & -2\end{bmatrix}$,

$A\cdot B=\begin{bmatrix}3x-6 & 4-2y\\ z-2 & 2t-4\end{bmatrix} \Rightarrow x+y+t-z=?$

Yanıt / The Answer is 20

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7. $A=\begin{bmatrix}1 & 2\\ 2 & 1\\ -3 & 0\end{bmatrix}$, $B=\begin{bmatrix}-1 & 0 & -2\\ 2 & 3 & 4\end{bmatrix} \Rightarrow (A\cdot B)^T = ?$

Yanıt / The Answer is $\begin{bmatrix}3 & 0 & 3\\ 6 & 3 & 0\\ 6 & 0 & 6\end{bmatrix}$

8. $A=\begin{bmatrix}1 & 0 & 2\\ -2 & 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}2 & 1\\ -1 & 1\\ 1 & 2\end{bmatrix} \Rightarrow (A\cdot B)^T = ?$

Yanıt / The Answer is $\begin{bmatrix}4 & -2\\ 5 & 5\end{bmatrix}$

9. $A=\begin{bmatrix}2 & -1 & 3\\ 1 & -2 & -1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1\\ 1 & 2\\ -1 & 1\end{bmatrix}$,

$A\cdot B=\begin{bmatrix}x-3 & y-2\\ z-5 & t-10\end{bmatrix} \Rightarrow x+y+t+z = ?$

Yanıt / The Answer is **17**

10. $A=\begin{bmatrix}2 & -3 & 1\\ 0 & 1 & -1\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 2\\ 2 & 3 & -1\end{bmatrix}$,

$C = 3A - 2B \Rightarrow C = ?$

Yanıt / The Answer is $\begin{bmatrix}4 & -7 & -1\\ -4 & -3 & -1\end{bmatrix}$

11. $A=\begin{bmatrix}2 & 3 & 1\\ -1 & 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\ 0 & 1\\ 1 & 1\end{bmatrix} \Rightarrow (A\cdot B)^T = ?$

Yanıt / The Answer is $\begin{bmatrix}3 & 1\\ 6 & 1\end{bmatrix}$

12. $A=\begin{bmatrix}-2 & 5\\ 2 & 3\end{bmatrix}$, $B = A^T - 2A \Rightarrow B = ?$

Yanıt / The Answer is $\begin{bmatrix}2 & -8\\ 1 & -3\end{bmatrix}$

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13. $\begin{bmatrix} x & 4 \\ y & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 4 \end{bmatrix} \Rightarrow \mathbf{x \cdot y = ?}$

Yanıt / The Answer is **24**

14. $A = \begin{bmatrix} -3 & 0 \\ 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 4 \\ 6 & 0 \end{bmatrix} \Rightarrow \mathbf{2A - 3B = ?}$

Yanıt / The Answer is $\begin{bmatrix} \mathbf{-3} & \mathbf{-12} \\ \mathbf{-16} & \mathbf{12} \end{bmatrix}$

15. $A = \begin{bmatrix} 1 & -3 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$, $A \cdot B = C \Rightarrow \mathbf{C = ?}$

Yanıt / The Answer is $\begin{bmatrix} \mathbf{-9} & \mathbf{2} \\ \mathbf{15} & \mathbf{-8} \end{bmatrix}$

16. $\begin{bmatrix} 1 & x \\ 2 & x+1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ a \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix} \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is **3**

17. $A = \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{A^2 = ?}$

Yanıt / The Answer is $\begin{bmatrix} \mathbf{49} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

18. $A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix} \Rightarrow \mathbf{A^2 = ?}$

Yanıt / The Answer is $\begin{bmatrix} \mathbf{4} & \mathbf{18} \\ \mathbf{0} & \mathbf{16} \end{bmatrix}$

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19. $A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} \Rightarrow A^4 = ?$

Yanıt / The Answer is $\begin{bmatrix}1 & 0\\ 0 & 16\end{bmatrix}$

20. $\begin{bmatrix}1 & -2\\ 3 & -4\end{bmatrix}^2 - 3\cdot\begin{bmatrix}-1 & 2\\ 3 & -4\end{bmatrix} + \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = ?$

Yanıt / The Answer is $\begin{bmatrix}-2 & 1\\ -17 & 22\end{bmatrix}$

21. $A=\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix} \Rightarrow A^{62} = ?$

Yanıt / The Answer is 3^{62}

22. $A=\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix} \Rightarrow A^9 = ?$

Yanıt / The Answer is $\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$

23. $A=\begin{bmatrix}-1 & 1\\ 0 & 1\end{bmatrix} \Rightarrow A^{19} = ?$

Yanıt / The Answer is $\begin{bmatrix}-1 & 1\\ 0 & 1\end{bmatrix}$

24. $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \Rightarrow A^5 = ?$

Yanıt / The Answer is $\begin{bmatrix}1 & 5\\ 0 & 1\end{bmatrix}$

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25. $A=\begin{bmatrix}2 & 2\\ 3 & -1\end{bmatrix} \Rightarrow A^3 = ?$

Yanıt / The Answer is $\begin{bmatrix}26 & 18\\ 27 & -1\end{bmatrix}$

26. $A=\begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix} \Rightarrow A^{-1} = ?$

Yanıt / The Answer is $\begin{bmatrix}4 & -1\\ -11 & 3\end{bmatrix}$

27. $A=\begin{bmatrix}1 & 2\\ 6 & 13\end{bmatrix} \Rightarrow A^{-1} = ?$

Yanıt / The Answer is $\begin{bmatrix}13 & -2\\ -6 & 1\end{bmatrix}$

28. $A=\begin{bmatrix}4 & 3\\ 3 & 2\end{bmatrix} \Rightarrow A^{-1} = ?$

Yanıt / The Answer is $\begin{bmatrix}-2 & 3\\ 3 & -4\end{bmatrix}$

29. $A=\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}$, $A^{-1}=\begin{bmatrix}a & b\\ c & d\end{bmatrix} \Rightarrow d = ?$

Yanıt / The Answer is **–1**

30. $A=\begin{vmatrix}x+2y & x+y\\ x-2y & x-y\end{vmatrix} \Rightarrow \det(A) = ?$

Yanıt / The Answer is **2xy**

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1. $A=\begin{bmatrix}1 & 3 & 5\\ 5 & x & -3x\\ -y & 10 & 14\end{bmatrix}$

$a_{22}+a_{23}=a_{31} \Rightarrow y=?$

A) $-2x$

B) $-x$

C) 0

D) x

E) $2x$

2. $\begin{bmatrix}-1 & 2\\ 1 & 2\end{bmatrix}\cdot\begin{bmatrix}2 & 2 & 2\\ 1 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}3\\ 4\\ 5\end{bmatrix}=?$

A) $\begin{bmatrix}-40\\ -8\end{bmatrix}$ B) $\begin{bmatrix}40\\ 8\end{bmatrix}$ C) $\begin{bmatrix}-8\\ -40\end{bmatrix}$

D) $\begin{bmatrix}-8\\ 40\end{bmatrix}$ E) $\begin{bmatrix}8\\ 40\end{bmatrix}$

3. $\begin{bmatrix}2 & -1\\ 5 & 2\end{bmatrix}\cdot\begin{bmatrix}4 & 0 & 3\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}3 & 1\\ -5 & -4\end{bmatrix}\cdot\begin{bmatrix}4 & 0 & 3\\ 2 & -1 & 3\end{bmatrix}=?$

A) $\begin{bmatrix}20 & 0 & 15\\ -4 & 2 & -6\end{bmatrix}$ B) $\begin{bmatrix}20 & 0 & 15\\ 4 & 2 & -6\end{bmatrix}$

A) $\begin{bmatrix}20 & 0 & 15\\ 4 & 2 & 6\end{bmatrix}$ D) $\begin{bmatrix}20 & 0 & 15\\ 4 & -2 & -6\end{bmatrix}$

E) $\begin{bmatrix}20 & 0 & 15\\ -4 & -2 & -6\end{bmatrix}$

4. $A=\begin{bmatrix}-3 & 4\\ 5 & -2\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}4 & -2\\ 7 & 3\\ 2 & -5\end{bmatrix} \Rightarrow A+B=?$

A) $\begin{bmatrix}1 & 2\\ 12 & 1\\ 2 & -4\end{bmatrix}$ B) $\begin{bmatrix}1 & 2\\ 8 & 3\\ -2 & 5\end{bmatrix}$ C) $\begin{bmatrix}1 & 3\\ 12 & 1\\ -4 & 2\end{bmatrix}$

D) $\begin{bmatrix}2 & 3\\ 8 & 5\\ 2 & 4\end{bmatrix}$ E) $\begin{bmatrix}7 & -2\\ 3 & 1\\ 2 & -4\end{bmatrix}$

5. $A=\begin{bmatrix}x-4 & 6\\ 2 & 4-x\end{bmatrix}$, $B=\begin{bmatrix}x-y & 4\\ 12 & y-2\end{bmatrix}$

$2A=B^T \Rightarrow x\cdot y=?$

A) 14

B) 12

C) 10

D) 6

E) 2

6. $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, $A^{27}=\begin{bmatrix}a & b\\ c & d\end{bmatrix} \Rightarrow a+b+c+d=?$

A) 13

B) 19

C) 26

D) 29

E) 36

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7. $A=\begin{bmatrix} a-b & 1 \\ 3 & b-2 \end{bmatrix}$, $B=\begin{bmatrix} a-6 & 6 \\ 2 & 4-a \end{bmatrix}$

$2A = B^T \Rightarrow \mathbf{a+2b=?}$

A) 8

B) 6

C) 4

D) 2

E) 1

8. $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $A \cdot X = B \Rightarrow \mathbf{X=?}$

A) $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ E) $\begin{bmatrix} -1 & 1 \\ 1 & 3 \end{bmatrix}$

9. $A=\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow \mathbf{A^2 \cdot A^t = ?}$

A) $\begin{bmatrix} 11 & -12 \\ 2 & 29 \end{bmatrix}$ B) $\begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix}$ C) $\begin{bmatrix} -11 & 12 \\ 2 & -29 \end{bmatrix}$

D) $\begin{bmatrix} 12 & 13 \\ 4 & 27 \end{bmatrix}$ E) $\begin{bmatrix} 11 & 2 \\ -12 & 29 \end{bmatrix}$

10. $A=\begin{bmatrix} x & 3 \\ -3 & y+1 \end{bmatrix}$, $B=\begin{bmatrix} -2 & 0 \\ 3 & 5 \end{bmatrix}$, $C=\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$

$A-2B=3C \Rightarrow \mathbf{x \cdot y = ?}$

A) –9

B) –6

C) –2

D) 6

E) 9

11. $\begin{vmatrix} 2 & -\frac{1}{2} & x \\ x & 3 & 0 \\ x & 2 & 4 \end{vmatrix} = 0 \Rightarrow \mathbf{ÇK(SS)=?}$

A) {–2, –4}

B) {–4, –6}

C) {–4,6}

D) {4,6}

E) {2,6}

12. $A=\begin{bmatrix} \tan\alpha & 0 \\ 0 & -6 \end{bmatrix} \Rightarrow \mathbf{A^{-1}=?}$

A) $\begin{bmatrix} \tan\alpha & 0 \\ 0 & \sin\alpha \end{bmatrix}$ B) $\begin{bmatrix} \cot\alpha & 0 \\ 0 & -\frac{1}{6} \end{bmatrix}$

C) $\begin{bmatrix} \cot\alpha & 0 \\ 0 & -6 \end{bmatrix}$ D) $\begin{bmatrix} \sin\alpha & 0 \\ 0 & 6 \end{bmatrix}$

E) $\begin{bmatrix} \tan\alpha & 0 \\ 0 & \sec\alpha \end{bmatrix}$

YANITLAR / ANSWERS

1. E	2. D	3. A	4. A	5. B	6. D	7. A	8. C
9. A	10. D	11. C	12. B				

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1. $\begin{bmatrix} 2 & 1 \\ 1 & 8 \\ 2 & 1 \end{bmatrix}^T \cdot \begin{bmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} = ?$

A) $\begin{bmatrix} 4 & 4 \\ 5 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 4 \\ 5 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 5 & 5 \\ 4 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 4 \\ 9 & 9 \end{bmatrix}$ E) $\begin{bmatrix} 5 & 5 \\ 10 & 10 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & y \\ z & t \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{x} & \mathbf{y} \\ \mathbf{z} & \mathbf{t} \end{bmatrix} = ?$

A) $\begin{bmatrix} 2 & 3 \\ -2 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 3 \\ -2 & -2 \end{bmatrix}$

D) $\begin{bmatrix} -2 & 3 \\ 2 & 2 \end{bmatrix}$ E) $\begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ -1 & 2 & 1 \end{bmatrix}$

$\Rightarrow (A \cdot B)^T = ?$

A) $\begin{bmatrix} 5 & -6 \\ 3 & 11 \\ -4 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 6 & 2 & -3 \\ 4 & 9 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 9 & 2 & -1 \\ -6 & 4 & -3 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 3 & -4 \\ -6 & 11 & -1 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 2 & -4 \\ -5 & 10 & -2 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$\Rightarrow ((A \cdot B) \cdot C)^{-1} = ?$

A) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & -3 \\ 1 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -3 \\ 1 & -1 \end{bmatrix} \Rightarrow A \cdot B^3 = ?$

A) $\begin{bmatrix} 13 & -9 \\ -2 & 3 \end{bmatrix}$ B) $\begin{bmatrix} -9 & 13 \\ 3 & -2 \end{bmatrix}$ C) $\begin{bmatrix} 3 & -9 \\ 13 & -2 \end{bmatrix}$

D) $\begin{bmatrix} 13 & -2 \\ 3 & -9 \end{bmatrix}$ E) $\begin{bmatrix} 13 & -9 \\ 3 & -2 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{17} = ?$

A) $2^{12} \cdot A$

B) $-2^{12} \cdot A$

C) $2^8 \cdot A$

D) $-2^{16} \cdot A$

E) $2^{14} \cdot A$

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7. $A=\begin{bmatrix}-2 & \\ & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$

$A+x=B^{-1}\cdot x \Rightarrow \mathbf{x=?}$

A) $\begin{bmatrix}1\\0\end{bmatrix}$ B) $\begin{bmatrix}-1\\2\end{bmatrix}$ C) $\begin{bmatrix}-1\\0\end{bmatrix}$

D $\begin{bmatrix}-2\\0\end{bmatrix}$ E) $\begin{bmatrix}-1\\1\end{bmatrix}$

8. $A=\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix} \Rightarrow \mathbf{A^4=?}$

A) $\begin{bmatrix}-5 & -24\\ 24 & -5\end{bmatrix}$ B) $\begin{bmatrix}-7 & -24\\ 24 & -7\end{bmatrix}$ C) $\begin{bmatrix}-6 & 18\\ 18 & -6\end{bmatrix}$

D) $32\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ E) $64\begin{bmatrix}-2 & 1\\ -1 & -2\end{bmatrix}$

9. $\begin{bmatrix}-1 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}a & 1\\ -1 & b\end{bmatrix}=\begin{bmatrix}2 & -2\\ x & y\end{bmatrix} \Rightarrow \mathbf{x+y=?}$

A) -8

B) $-\frac{15}{2}$

C) -7

D) $-\frac{13}{2}$

E) -6

10. $A=\begin{bmatrix}3 & 1 & 4\\ 2 & 1 & 0\\ -1 & 2 & 0\end{bmatrix} \Rightarrow \mathbf{\det A=?}$

A) -20

B) 20

C) 12

D) 10

E) 8

11. $\begin{vmatrix}x & y\\ z & t\end{vmatrix}=5,\ \begin{vmatrix}x & y+m\\ z & t+n\end{vmatrix}=7 \Rightarrow \begin{vmatrix}\mathbf{x} & \mathbf{m}\\ \mathbf{z} & \mathbf{n}\end{vmatrix}=\mathbf{?}$

A) 2

B) 5

C) 7

D) 12

E) 35

12. $\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}\cdot\begin{bmatrix}a & 2\\ 3 & b\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \Rightarrow \mathbf{a+b=?}$

A) -1 B) -5 C) -6

D) -7 E) 5

YANITLAR / ANSWERS							
1. E	2. A	3. A	4. A	5. E	6. C	7. E	8. B
9. B	10. B	11. A	12. C				

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1. $A=\begin{bmatrix}-2 & x\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}x & -6\\ 2 & 4\end{bmatrix}$, $C=\begin{bmatrix}y-4 & -8\\ -7 & -2\end{bmatrix}$

$A \cdot B = C \Rightarrow y - x = ?$

A) 9
B) 10
C) 11
D) 13
E) 15

2. $A=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix} \Rightarrow A^6 = ?$

A) $\begin{bmatrix}64 & 0\\ 0 & 729\end{bmatrix}$ B) $\begin{bmatrix}729 & 0\\ 0 & 64\end{bmatrix}$ C) $\begin{bmatrix}0 & 729\\ 64 & 0\end{bmatrix}$

D) $\begin{bmatrix}0 & 64\\ 729 & 0\end{bmatrix}$ E) $\begin{bmatrix}0 & 729\\ 0 & 64\end{bmatrix}$

3. $\begin{bmatrix}1 & -1\\ 2 & 1\\ -1 & 2\end{bmatrix} \cdot \begin{bmatrix}1 & 2 & 4\\ 2 & 1 & 5\end{bmatrix} = \begin{bmatrix}a & \cdot & \cdot\\ \cdot & b & \cdot\\ \cdot & \cdot & c\end{bmatrix}$

$\Rightarrow a+b+c = ?$

A) 6
B) 7
C) 8
D) 9
E) 10

4. $A=\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$, $(A^T)^{-1}=\begin{bmatrix}a & b\\ c & d\end{bmatrix} \Rightarrow a+b = ?$

A) –2
B) –1
C) 4
D) 5
E) 8

5. $\begin{vmatrix}x & 1 & x\\ 2 & 3 & 4\\ x & 5 & x\end{vmatrix} = 24 \Rightarrow x = ?$

A) 0
B) –1
C) –2
D) –3
E) –4

6. $\begin{vmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\\ \mathbf{2} & \mathbf{3} & \mathbf{4}\\ \mathbf{4} & \mathbf{9} & \mathbf{16}\end{vmatrix} = ?$

A) –2
B) –1
C) 0
D) 1
E) 2

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7. $K=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $K\cdot\begin{bmatrix} 3 \\ 2 \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $K\cdot\begin{bmatrix} -1 \\ 0 \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow K\cdot\begin{bmatrix} \mathbf{2} \\ \mathbf{-1} \end{bmatrix}=?$

A) $\begin{bmatrix} -7 \\ -6 \end{bmatrix}$ B) $\begin{bmatrix} -4 \\ -7 \end{bmatrix}$ C) $\begin{bmatrix} -7 \\ -4 \end{bmatrix}$

D) $\begin{bmatrix} -4 \\ 7 \end{bmatrix}$ E) $\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

8. $A=\begin{bmatrix} a & \frac{1}{3} \\ \frac{1}{3} & b \end{bmatrix}$ $A=A^{-1} \Rightarrow \mathbf{a-b=?}$

A) 0

B) 1

C) $\frac{2\sqrt{2}}{3}$

D) $\frac{-2\sqrt{2}}{3}$

E) $\frac{4\sqrt{2}}{3}$

9. $A=\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $f(x)=(x-2)^2 \Rightarrow \mathbf{f(A)=?}$

A) $\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 9 & 0 \\ 1 & 2 \end{bmatrix}$

10. $A=\begin{bmatrix} x-3 & 5 \\ -4 & 2x \end{bmatrix}$, $|A|=4 \Rightarrow \prod \mathbf{x=?}$

A) 4

B) 6

C) 8

D) 10

E) 12

11. $\begin{vmatrix} 2 & 2 & a-1 \\ 4 & 0 & 0 \\ 1 & -1 & 4 \end{vmatrix}=8 \Rightarrow \mathbf{a=?}$

A) 9

B) 11

C) 18

D) 20

E) 22

12. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 1 & 2 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 26 \\ -20 \\ 34 \end{bmatrix} \Rightarrow \mathbf{x+y+z=?}$

A) 12

B) 14

C) 18

D) 20

E) 22

YANITLAR / ANSWERS							
1. A	2. A	3. E	4. D	5. D	6. E	7. C	8. A
9. D	10. C	11. A	12. E				

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ÜNİTE 5

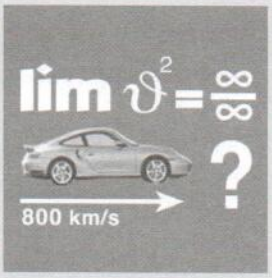
UNIT 5

Bu ünitede, limit ayrıntılı olarak anlatılmıştır. Ilk olarak, limit ve limitin özellikleri, daha sonra limitte belirsizlikler anlatılmıştır.

Bu ünite; 56 çözümlü örnek, 24 alıştırma ve 36 test sorusundan oluşmaktadır.

In this unit, limits is explained in detailed. Firstly, limits and properties of limits are explained. Later, indeterminate forms are explained.

This unit includes; 56 examples, 24 exercises, and 36 test questions.



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BÖLÜM 1
CHAPTER 1

TARİHSEL NOT / HISTORICAL NOTE

Cramer, Gabriel [1704 – 1752]

Isveç matematikçi. 1750 de basılan cebirsel eğriler, Cramer kuralı ile bilinir. Bu kural MacLaurin olarak bilinir.

Swiss mathematician whose introduction to algebraic curves, published in 1750, contains the so-called Cramer's rule. The rule was known earlier by MacLaurin.

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LİMİT / LIMITS

BÖLÜM 1
CHAPTER

$A \subset R$, $a \in A$, $f: A \to R$ ya da $f: A - \{a\} \to R$

$y = f(x)$ şeklindeki fonksiyonlarda x değerleri a ya soldan artan değerlerle yaklaşıyorsa, f fonksiyonunun değeri L_1 e yaklaşır. Buna fonksiyonun *soldan limiti* denir.

$$\lim_{x \to a^-} f(x) = L_1$$

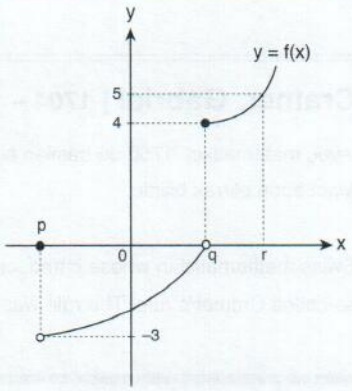
x değerleri a ya sağdan azalan değerlerle yaklaşırsa fonksiyonunun değeri L_2 ye yaklaşır. Buna fonksiyonun *sağdan limiti* denir.

$$\lim_{x \to a^+} f(x) = L_2$$

Fonksiyonun x = a noktasında limiti varsa sağdan ve soldan limitleri eşittir.

$$\left.\begin{array}{l} \lim\limits_{x \to a^-} f(x) = L_1 \\ \lim\limits_{x \to a^+} f(x) = L_2 \end{array}\right\} \quad \begin{array}{l} L_1 = L_2 \Rightarrow \lim\limits_{x \to a} f(x) = L \\ L_1 \neq L_2 \Rightarrow \lim\limits_{x \to a} f(x) \text{ limit yoktur.} \end{array}$$

Düşünün

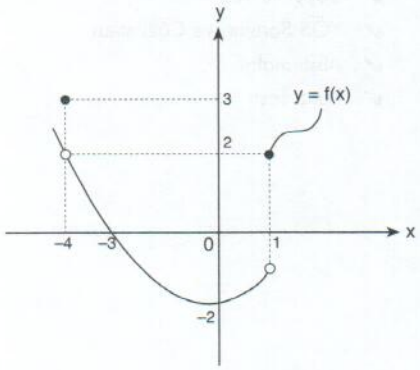


$\lim\limits_{x \to p^+} f(x) + \lim\limits_{x \to q^-} f(x) + \lim\limits_{x \to r} f(x) = ?$

ÖRNEK 1

$\lim\limits_{x \to -4} f(x) + \lim\limits_{x \to -3} f(x) + \lim\limits_{x \to 1^+} f(x) = ?$

A) 1
B) 2
C) 3
D) 4
E) 5



ÇÖZÜM

Yanıt D

$$\left.\begin{array}{l} \lim\limits_{x \to -4^+} f(x) = 2 \\ \lim\limits_{x \to -4^-} f(x) = 2 \end{array}\right\} \Rightarrow \lim_{x \to -4} f(x) = 2 \qquad \left.\begin{array}{l} \lim\limits_{x \to -3^+} f(x) = 0 \\ \lim\limits_{x \to -3^-} f(x) = 0 \end{array}\right\} \Rightarrow \lim_{x \to -3} f(x) = 0$$

$$\lim_{x \to 1^+} f(x) = 2$$

$$\Rightarrow \lim_{x \to -4} f(x) + \lim_{x \to -3} f(x) + \lim_{x \to 1^+} f(x) = 2 + 0 + 2 = 4$$

Yanıt : 2

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Let $A \subset R$, $a \in A$, $f: A \to R$ or $f: A - \{a\} \to R$

If x approaches a from values less than a, the value of function approaches L_1.

That is *limit from the left* of the function.

$$\lim_{x \to a^-} f(x) = L_1$$

If x approaches a from values greater than a, the value of function approaches L_2.

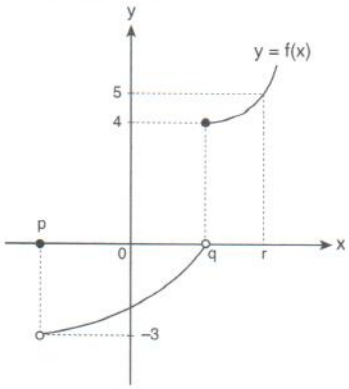
That is *limit from the right* of the function.

$$\lim_{x \to a^+} f(x) = L_2$$

If function has a limit at point x = a, limit from the right and left is equal.

$$\left.\begin{array}{l} \lim\limits_{x \to a^-} f(x) = L_1 \\ \lim\limits_{x \to a^+} f(x) = L_2 \end{array}\right\} \quad \begin{array}{l} L_1 = L_2 \Rightarrow \lim\limits_{x \to a} f(x) = L \\ L_1 \neq L_2 \Rightarrow \lim\limits_{x \to a} f(x) \text{ there are not limit.} \end{array}$$

Think about it

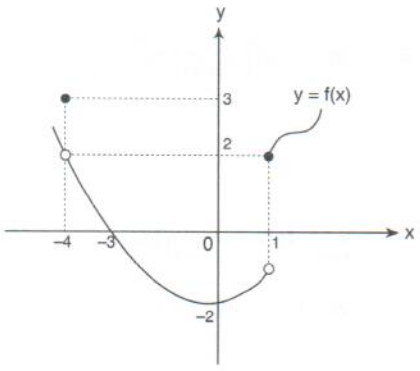


$\lim\limits_{x \to p^+} f(x) + \lim\limits_{x \to q^-} f(x) + \lim\limits_{x \to r} f(x) = ?$

EXAMPLE 1

$\lim\limits_{x \to -4} f(x) + \lim\limits_{x \to -3} f(x) + \lim\limits_{x \to 1^+} f(x) = ?$

A) 1
B) 2
C) 3
D) 4
E) 5



SOLUTION

The answer is **D**

$$\left.\begin{array}{l} \lim\limits_{x \to -4^+} f(x) = 2 \\ \lim\limits_{x \to -4^-} f(x) = 2 \end{array}\right\} \Rightarrow \lim_{x \to -4} f(x) = 2 \qquad \left.\begin{array}{l} \lim\limits_{x \to -3^+} f(x) = 0 \\ \lim\limits_{x \to -3^-} f(x) = 0 \end{array}\right\} \Rightarrow \lim_{x \to -3} f(x) = 0$$

$$\lim_{x \to 1^+} f(x) = 2$$

$$\Rightarrow \lim_{x \to -4} f(x) + \lim_{x \to -3} f(x) + \lim_{x \to 1^+} f(x) = 2 + 0 + 2 = 4$$

Answer: 2

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LİMİTİN ÖZELLİKLERİ

1. $\lim_{x\to a} k = k \ (k \in R)$

2. $\lim_{x\to a} [(f(x) \mp g(x)] = \lim_{x\to a} f(x) \mp \lim_{x\to a} g(x)$

3. $\lim_{x\to a} [f(x)\cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

4. $k \in R, \ \lim_{x\to a} [k\cdot f(x)] = k \cdot \lim_{x\to a} f(x)$

5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \left(\lim_{x\to a} g(x) \neq 0\right)$

6. $\lim_{x\to a} \left[(f(x))^n\right] = \left[\lim_{x\to a} f(x)\right]^n$

7. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

   $f(x) > 0$ ve n tek doğal sayı

8. $\lim_{x\to a} \left[k^{f(x)}\right] = k^{\lim_{x\to a} f(x)}, \ (k \in R)$

9. $\lim_{x\to k} \left[\log_a f(x)\right] = \log_a \left[\lim_{x\to k} f(x)\right]$

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PROPERTIES of LIMITS

1. $\lim\limits_{x \to a} k = k \ (k \in R)$

2. $\lim\limits_{x \to a} [(f\ (x) \mp g(x)] = \lim\limits_{x \to a} f(x) \mp \lim\limits_{x \to a} g(x)$

3. $\lim\limits_{x \to a} [f(x) \cdot g(x)] = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)$

4. $k \in R, \ \lim\limits_{x \to a} [k \cdot f(x)] = k \cdot \lim\limits_{x \to a} f(x)$

5. $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} \left(\lim\limits_{x \to a} g(x) \neq 0 \right)$

6. $\lim\limits_{x \to a} \left[(f(x))^n \right] = \left[\lim\limits_{x \to a} f(x) \right]^n$

7. $\lim\limits_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x \to a} f(x)}$

   $f(x) > 0$ and n is an odd natural number

8. $\lim\limits_{x \to a} \left[k^{f(x)} \right] = k^{\lim\limits_{x \to a} f(x)}, \ (k \in R)$

9. $\lim\limits_{x \to k} \left[\log_a f(x) \right] = \log_a \left[\lim\limits_{x \to k} f(x) \right]$

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▼ $f : R \to R$, $\lim_{x \to a} f(x)$ bulunurken;

1. x = a noktası kritik nokta ise, sağdan ve soldan limit incelenir.
2. x = a noktası kritik nokta değilse, $\lim_{x \to a} f(x) = f(a)$ dır.

ÖRNEK 2

$f(x) = \dfrac{x^3+x+3}{x^2+2} \Rightarrow \lim_{x \to 3} f(x) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM

Yanıt C

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{3^3+3+3}{3^2+2}$$

$$= \lim_{x \to 3} = \frac{33}{11} = 3$$

PEKİŞTİRME

$\lim_{x \to 1} \dfrac{x^2+x+1}{2x-1} = ?$

YANIT : 3

ÖRNEK 3

$\lim_{x \to 3} \dfrac{x^3-6x+1}{x^2-1} = ?$

A) $\frac{5}{4}$ B) $\frac{3}{2}$ C) $\frac{7}{4}$ D) 2 E) $\frac{9}{4}$

ÇÖZÜM

Yanıt A

$$\lim_{x \to 3} \frac{x^3-6x+1}{x^2-1} = \frac{3^3-6\cdot 3+1}{3^2-1}$$

$$= \frac{27-18+1}{9-1}$$

$$= \frac{10}{8} = \frac{5}{4}$$

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▼ To find the limit of a function f(x) at a point x = a

1. Examine the left and the right limits if x = a is a critical number.
2. $\lim_{x \to a} f(x) = f(a)$ if x = a is not a critical number.

REINFORCE

$\lim_{x \to 1} \frac{x^2+x+1}{2x-1} = ?$

ANSWER : 3

EXAMPLE 2

$f(x) = \frac{x^3+x+3}{x^2+2} \Rightarrow \lim_{x \to 3} f(x) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

SOLUTION

The answer is C

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{3^3+3+3}{3^2+2}$$

$$= \lim_{x \to 3} = \frac{33}{11} = 3$$

EXAMPLE 3

$\lim_{x \to 3} \frac{x^3-6x+1}{x^2-1} = ?$

A) $\frac{5}{4}$ B) $\frac{3}{2}$ C) $\frac{7}{4}$ D) 2 E) $\frac{9}{4}$

SOLUTION

The answer is A

$$\lim_{x \to 3} \frac{x^3-6x+1}{x^2-1} = \frac{3^3-6\cdot 3+1}{3^2-1}$$

$$= \frac{27-18+1}{9-1}$$

$$= \frac{10}{8} = \frac{5}{4}$$

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ÖRNEK 4

$$f(x)=\begin{cases} x+3 & x\geq 3 \\ x^2-1 & x<3 \end{cases}$$

$\Rightarrow \lim\limits_{x\to 3^+} f(x)+\lim\limits_{x\to 3^-} f(x)=?$

A) 6 B) 8 C) 10 D) 12 E) 14

ÇÖZÜM

Yanıt **E**

$\lim\limits_{x\to 3^+} f(x)=3+3=6$

$\lim\limits_{x\to 3^-} f(x)=3^2-1=8$

$\Rightarrow \lim\limits_{x\to 3^+} f(x)+\lim\limits_{x\to 3^-} f(x)=6+8=14$

ÖRNEK 5

$$f(x)=\begin{cases} \dfrac{x-1}{x-3}+2 & x\geq 0 \\ \dfrac{x-2}{x-5}+1 & x<0 \end{cases} \Rightarrow \lim_{x\to 0} f(x)=?$$

A) $\frac{7}{3}$ B) $\frac{7}{5}$ C) 1 D) 2 E) Limit yoktur.

ÇÖZÜM

Yanıt **E**

$\lim\limits_{x\to 0^+} f(x)=\dfrac{0-1}{0-3}+2=\dfrac{7}{3}$

$\lim\limits_{x\to 0^-} f(x)=\dfrac{0-2}{0-5}+1=\dfrac{7}{5}$

$\Rightarrow \lim\limits_{x\to 0^+} f(x)\neq \lim\limits_{x\to 0^-} f(x)$

$\lim\limits_{x\to 0^+} f(x)$ yoktur.

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EXAMPLE 4

$$f(x)=\begin{cases} x+3 & x\geq 3 \\ x^2-1 & x<3 \end{cases}$$

$$\Rightarrow \lim_{x\to 3^+} f(x)+\lim_{x\to 3^-} f(x)=?$$

A) 6 B) 8 C) 10 D) 12 E) 14

SOLUTION

The answer is E

$$\lim_{x\to 3^+} f(x)=3+3=6$$

$$\lim_{x\to 3^-} f(x)=3^2-1=8$$

$$\Rightarrow \lim_{x\to 3^+} f(x)+\lim_{x\to 3^-} f(x)=6+8=14$$

EXAMPLE 5

$$f(x)=\begin{cases} \dfrac{x-1}{x-3}+2 & x\geq 0 \\ \dfrac{x-2}{x-5}+1 & x<0 \end{cases} \Rightarrow \lim_{x\to 0} f(x)=?$$

A) $\frac{7}{3}$ B) $\frac{7}{5}$ C) 1 D) 2 E) It doesn't exist.

SOLUTION

The answer is E

$$\lim_{x\to 0^+} f(x)=\frac{0-1}{0-3}+2=\frac{7}{3}$$

$$\lim_{x\to 0^-} f(x)=\frac{0-2}{0-5}+1=\frac{7}{5}$$

$$\Rightarrow \lim_{x\to 0^+} f(x)\neq \lim_{x\to 0^-} f(x)$$

$\lim_{x\to 0^+} f(x)$ does not exist.

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GENİŞLETİLMİŞ REEL SAYI ARALIKLARI

Tanım: Her $x \in R$ için reel sayılar kümesi $-\infty < x < \infty$ veya $R = (-\infty , \infty)$ ile gösterilir. Burada $-\infty$ ve $+\infty$, reel sayılar kümesine dahil değildir. R gerçel sayılar kümesi ile $-\infty$ ve $+\infty$ un birleşmesiyle elde edilen kümeye *genişletilmiş reel sayı aralıkları* denir.

Sonuçlar

1. $a \in R$

$a \mp \infty = \mp \infty$, $a \mp (-\infty) = \pm \infty$

2. $a \in R^+ \Rightarrow a \cdot (+\infty) = \infty$, $a \cdot (-\infty) = -\infty$

$a \in R^- \Rightarrow a \cdot (+\infty) = -\infty$, $a \cdot (-\infty) = +\infty$

3. $a \in R^+ \Rightarrow \frac{+\infty}{a} = +\infty$, $\frac{-\infty}{a} = -\infty$

$a \in R^- \Rightarrow \frac{+\infty}{a} = -\infty$, $\frac{-\infty}{a} = +\infty$

4. $a \in R \Rightarrow \frac{a}{\pm\infty} = 0$, $a \neq 0$

5. $a \in R^+ \Rightarrow \frac{a}{0^+} = +\infty$, $\frac{a}{0^-} = -\infty$

$a \in R^- \Rightarrow \frac{a}{0^+} = -\infty$, $\frac{a}{0^-} = +\infty$

6. $\infty + \infty = \infty$, $(-\infty) + (-\infty) = -\infty$

7. $(\mp\infty) \cdot (\mp\infty) = \infty$, $(\pm\infty) \cdot (\mp\infty) = -\infty$

8. $n \in Z$

$\sqrt[2n-1]{\pm\infty} = \pm\infty$

$\sqrt[2n]{+\infty} = \infty$, $\sqrt[2n]{-\infty}$ = tanımsız

9. $n \in Z$

$(+\infty)^n = +\infty$

$(-\infty)^{2n} = +\infty$

$(-\infty)^{2n-1} = -\infty$

10. $a > 1 \Rightarrow a^{+\infty} = \infty$

$a^{-\infty} = 0$

$0 < a < 1 \Rightarrow a^{+\infty} = 0$

$a^{-\infty} = +\infty$

11. Belirsizlik durumları

$\frac{\mp\infty}{\mp\infty}$, $\infty - \infty$, $\frac{0}{0}$, $0 \cdot \mp\infty$, $1^{\mp\infty}$, $0^{\mp\infty}$, $(\mp\infty)^0$

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ENLARGED REAL NUMBERS

Definition: For every $x \in R$, the set of real numbers can be represented $-\infty < x < +\infty$ or $R = (-\infty, +\infty)$. Here, $-\infty$ and $+\infty$ are not included by the set of real numbers. The set $R \cup \{-\infty, +\infty\}$ is called enlarged real number set.

Conclusions

1. $a \in R$

$a \mp \infty = \mp \infty$, $a \mp (-\infty) = \pm \infty$

2. $a \in R^+ \Rightarrow a \cdot (+\infty) = \infty$, $a \cdot (-\infty) = -\infty$

$a \in R^- \Rightarrow a \cdot (+\infty) = -\infty$, $a \cdot (-\infty) = +\infty$

3. $a \in R^+ \Rightarrow \frac{+\infty}{a} = +\infty$, $\frac{-\infty}{a} = -\infty$

$a \in R^- \Rightarrow \frac{+\infty}{a} = -\infty$, $\frac{-\infty}{a} = +\infty$

4. $a \in R \Rightarrow \frac{a}{\pm\infty} = 0$, $a \neq 0$

5. $a \in R^+ \Rightarrow \frac{a}{0^+} = +\infty$, $\frac{a}{0^-} = -\infty$

$a \in R^- \Rightarrow \frac{a}{0^+} = -\infty$, $\frac{a}{0^-} = +\infty$

6. $\infty + \infty = \infty$, $(-\infty) + (-\infty) = -\infty$

7. $(\mp\infty) \cdot (\mp\infty) = \infty$, $(\pm\infty) \cdot (\mp\infty) = -\infty$

8. $n \in Z$

$\sqrt[2n-1]{\pm\infty} = \pm\infty$

$\sqrt[2n]{+\infty} = \infty$, $\sqrt[2n]{-\infty}$ =indefinite

9. $n \in Z$

$(+\infty)^n = +\infty$

$(-\infty)^{2n} = +\infty$

$(-\infty)^{2n-1} = -\infty$

10. $a > 1 \Rightarrow a^{+\infty} = \infty$

$a^{-\infty} = 0$

$0 < a < 1 \Rightarrow a^{+\infty} = 0$

$a^{-\infty} = +\infty$

11. Indeterminate forms

$\frac{\mp\infty}{\mp\infty}$, $\infty - \infty$, $\frac{0}{0}$, $0 \cdot \mp\infty$, $1^{\mp\infty}$, $0^{\mp\infty}$, $(\mp\infty)^0$

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ÖRNEK 6

$\lim_{x\to\infty} 5^{\frac{2}{x}} = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM

Yanıt D

$\lim_{x\to\infty} 5^{\frac{2}{x}} = 5^{\frac{2}{\infty}}$

$= 5^0 = 1$

PEKİŞTİRME

$\lim_{x\to-\infty} 35^{\frac{1}{x}} = ?$

YANIT : 1

ÖRNEK 7

$\lim_{x\to-\infty} \left(11^{\frac{1}{x}} + 5^x + 5\right) = ?$

A) 1 B) 3 C) 5 D) 6 E) 8

ÇÖZÜM

Yanıt D

$11^{\frac{1}{-\infty}} = 11^0 = 1$

$5^{-\infty} = \frac{1}{5^{\infty}} = \frac{1}{\infty} = 0$

$\Rightarrow \lim_{x\to-\infty} \left(11^{\frac{1}{x}} + 5^x + 5\right) = 1 + 0 + 5 = 6$

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REINFORCE

$\lim_{x\to-\infty} 35^{\frac{1}{x}} = ?$

ANSWER : 1

EXAMPLE 6

$\lim_{x\to\infty} 5^{\frac{2}{x}} = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

SOLUTION

The answer is **D**

$\lim_{x\to\infty} 5^{\frac{2}{x}} = 5^{\frac{2}{\infty}}$

$= 5^0 = 1$

EXAMPLE 7

$\lim_{x\to-\infty} \left(11^{\frac{1}{x}} + 5^x + 5\right) = ?$

A) 1 B) 3 C) 5 D) 6 E) 8

SOLUTION

The answer is **D**

$11^{\frac{1}{-\infty}} = 11^0 = 1$

$5^{-\infty} = \frac{1}{5^{\infty}} = \frac{1}{\infty} = 0$

$\Rightarrow \lim_{x\to-\infty} \left(11^{\frac{1}{x}} + 5^x + 5\right) = 1 + 0 + 5 = 6$

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ÖRNEK 8

$$\lim_{x\to 0^-}\left(e^{\frac{1}{x}}+\pi^x+2\right)=?$$

A) 1 B) 2 C) 3 D) e + 1 E) π + 1

ÇÖZÜM

Yanıt C

$$e^{\frac{1}{0^-}}=e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0$$

$$\pi^{0^-}=1$$

$$\Rightarrow \lim_{x\to 0^-}\left(e^{\frac{1}{x}}+\pi^x+2\right)=0+1+2=3$$

▼ $x \to +\infty, -\infty$ için polinom fonksiyonlarında limit bulunurken derecesi en büyük olan terimin limitine bakılması yeterlidir.

$$\lim_{x\to\infty} f(x)=4x^3+5x^2+7x+1=\lim_{x\to\infty} 4x^3=4\cdot(\infty)=\infty$$

Mutlak Değer Fonksiyonunun Limiti

$$\lim_{x\to a}|f(x)|=|f(a)|$$

ÖRNEK 9

$$\lim_{x\to 4^-}\frac{|2x-8|}{x-4}=?$$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM

Yanıt A

$$\lim_{x\to 4^-}\frac{|2x-8|}{x-4}=\lim_{x\to 4^-}\frac{2\cdot|x-4|}{x-4}$$

$$=\frac{2(4-x)}{x-4}=-2$$

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EXAMPLE 8

$$\lim_{x\to 0^-}\left(e^{\frac{1}{x}}+\pi^x+2\right)=?$$

A) 1 B) 2 C) 3 D) e + 1 E) π + 1

SOLUTION

The answer is C

$$e^{\frac{1}{0^-}}=e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0$$

$$\pi^{0^-}=1$$

$$\Rightarrow \lim_{x\to 0^-}\left(e^{\frac{1}{x}}+\pi^x+2\right)=0+1+2=3$$

▼ To find the limit of the polynomial function for $x \to +\infty, -\infty$, only we can see term with maximum degree.

$$\lim_{x\to\infty} f(x)=4x^3+5x^2+7x+1=\lim_{x\to\infty}4x^3=4\cdot(\infty)=\infty$$

Limit of the Absolute Value Function

$$\lim_{x\to a}|f(x)|=|f(a)|$$

EXAMPLE 9

$$\lim_{x\to 4^-}\frac{|2x-8|}{x-4}=?$$

A) –2 B) –1 C) 0 D) 1 E) 2

SOLUTION

The answer is A

$$\lim_{x\to 4^-}\frac{|2x-8|}{x-4}=\lim_{x\to 4^-}\frac{2\cdot|x-4|}{x-4}$$

$$=\frac{2(4-x)}{x-4}=-2$$

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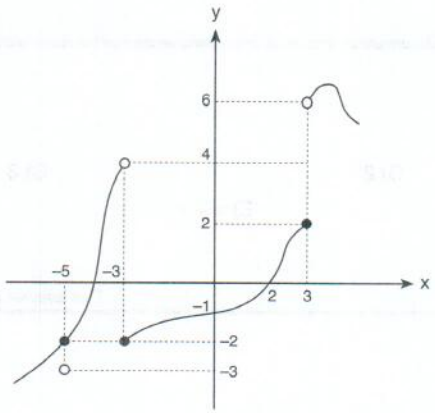
1.



$\lim_{x\to -3^-} f(x) + \lim_{x\to 0^+} f(x) + \lim_{x\to -3^+} f(x) = ?$

A) –2 B) –1 C) 0

D) 1 E) 2

2. $\lim_{x\to 1} (4x - \ln x) = ?$

A) 1

B) 2

C) 3

D) 4

E) 5

3. $\lim_{x\to \pi} (\cos(\sin x)) = ?$

A) 1

B) 0

C) –1

D) $-\frac{1}{2}$

E) $-\frac{1}{3}$

4. $f(x) = \begin{cases} 2x + a & x > 3 \\ x^2 - bx + 2 & x \le 3 \end{cases}$

$\lim_{x\to 3} f(x) = 2 \Rightarrow b - a = ?$

A) 7

B) 6

C) 5

D) 4

E) 3

5. $f(x) = \begin{cases} 5x - 4 & x < 0 \\ x^2 + 3 & 0 \le x < 2 \\ x^2 - 3 & x \ge 2 \end{cases}$

$\Rightarrow \lim_{x\to 0^-} f(x) + \lim_{x\to 1^+} f(x) + \lim_{x\to 2^+} f(x) = ?$

A) 1

B) 2

C) 3

D) 4

E) 5

6. $f(x) = \begin{cases} x + 3 & x > 1 \\ x^2 + 2 & x < 1 \end{cases}$

$\Rightarrow \lim_{x\to 1^+} f(x) + \lim_{x\to -1} f(x) + \lim_{x\to 1^-} f(x) = ?$

A) 6

B) 8

C) 10

D) 12

E) 15

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7. $f(x)=\sqrt{x+11}$, $g(x)=x^2-8 \Rightarrow \lim\limits_{x\to 3}(gof)(x)=?$

A) 3
B) 4
C) 5
D) 6
E) 7

8. $\lim\limits_{a\to x}\dfrac{a^2+2x^2}{a+2x}=?$

A) 3a
B) 3x
C) a
D) x
E) 0

9. $\lim\limits_{x\to 2}(f(x)+x-5)=4 \Rightarrow \lim\limits_{x\to 2} f(x)=?$

A) 3
B) 4
C) 5
D) 6
E) 7

10. $f=\begin{pmatrix} a & b & c & d & e \\ b & c & d & a & e \end{pmatrix}$, $g=\begin{pmatrix} a & b & c & d & e \\ a & d & b & c & e \end{pmatrix}$

$\Rightarrow \lim\limits_{x\to a}(fog)(x)=?$

A) a
B) b
C) c
D) d
E) e

11. $\lim\limits_{x\to 3}\left(\log_5\sqrt{x+2}\right)=?$

A) $-\dfrac{5}{2}$
B) $-\dfrac{1}{2}$
C) $\dfrac{5}{2}$
D) $\dfrac{1}{2}$
E) 2

12. $\lim\limits_{x\to 2^+}\dfrac{|x-2|}{x-2}+\lim\limits_{x\to 4^-}\dfrac{|x-4|}{x-4}=?$

A) –2
B) –1
C) 0
D) 1
E) 2

YANITLAR / ANSWERS							
1. D	2. D	3. A	4. A	5. A	6. C	7. D	8. D
9. E	10. B	11. D	12. C				

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1. $\lim\limits_{x\to -3^-} f(x) = 4$

$\lim\limits_{x\to 0^+} f(x) = -1$

$\lim\limits_{x\to -3^+} f(x) = -2$

$\lim\limits_{x\to -3^-} f(x) + \lim\limits_{x\to 0^+} f(x) + \lim\limits_{x\to -3^+} f(x) = 4 - 1 - 2 = 1$

Yanıt / The Answer is D

2. $\lim\limits_{x\to 1} (4x - \ln x) = 4\cdot 1 - \ln 1$

$= 4 - 0$

$= 4$

Yanıt / The Answer is D

3. $\lim\limits_{x\to \pi} (\cos(\sin x)) = \cos(\sin \pi)$

$= \cos 0$

$= 1$

Yanıt / The Answer is A

4. $\lim\limits_{x\to 3^+} f(x) = 6 + a$

$\lim\limits_{x\to 3^-} f(x) = 9 - 3b + 2$

$\lim\limits_{x\to 3} f(x) = 6 + a = 9 - 3b + 2 = 2$

$a = -4 \ , \ 11 - 3b = 2$

$b = 3$

$\Rightarrow b - a = 4 + 3 = 7$

Yanıt / The Answer is A

5. $\lim\limits_{x\to 0^-} f(x) = 5\cdot 0 - 4 = -4$

$\lim\limits_{x\to 1^+} f(x) = 1^2 + 3 = 4$

$\lim\limits_{x\to 2^+} f(x) = 2^2 - 3 = 1$

$\Rightarrow \lim\limits_{x\to 0^-} f(x) + \lim\limits_{x\to 1^+} f(x) + \lim\limits_{x\to 2^+} f(x) = -4 + 4 + 1 = 1$

Yanıt / The Answer is A

6. $\lim\limits_{x\to 1^+} f(x) = 1 + 3 = 4$

$\lim\limits_{x\to -1} f(x) = (-1)^2 + 2 = 3$

$\lim\limits_{x\to 1^-} f(x) = 1^2 + 2 = 3$

$\Rightarrow \lim\limits_{x\to 1^+} f(x) + \lim\limits_{x\to -1} f(x) + \lim\limits_{x\to 1^-} f(x) = 4 + 3 + 3 = 10$

Yanıt / The Answer is C

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7. $\lim_{x\to 3}(gof)(x) = g(f(3))$

$= g\left(\sqrt{3+11}\right)$

$= g(\sqrt{14})$

$= (\sqrt{14})^2 - 8$

$= 14 - 8 = 6$

Yanıt / The Answer is D

8. $\lim_{a\to x}\frac{a^2+2x^2}{a+2x} = \lim_{a\to x}\frac{x^2+2x^2}{x+2x}$

$= \frac{3x^2}{3x}$

$= x$

Yanıt / The Answer is D

9. $\lim_{x\to 2}(f(x)+x-5) = 4$

$\lim_{x\to 2} f(x) + \lim_{x\to 2}(x-5) = 4$

$\lim_{x\to 2} f(x) - 3 = 4$

$\lim_{x\to 2} f(x) = 7$

Yanıt / The Answer is E

10. $\lim_{x\to a}(fog)(x) = fog(a) = f(a) = b$

Yanıt / The Answer is B

11. $\lim_{x\to 3}\left(\log_5\sqrt{x+2}\right) = \log_5 \lim_{x\to 3}\sqrt{x+2}$

$= \log_5\sqrt{5}$

$= \log_5 5^{\frac{1}{2}}$

$= \frac{1}{2}$

Yanıt / The Answer is D

12. $\lim_{x\to 2^+}\frac{|x-2|}{x-2} = \frac{x-2}{x-2} = 1$

$\lim_{x\to 4^-}\frac{|x-4|}{x-4} = \frac{-(x-4)}{x-4} = -1$

$\Rightarrow \lim_{x\to 2^+}\frac{|x-2|}{x-2} + \lim_{x\to 4^-}\frac{|x-4|}{x-4} = 1-1 = 0$

Yanıt / The Answer is C

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YÖS Soruları / YÖS Questions

1. $\lim\limits_{h\to 0} \dfrac{(h+1)(2h+1)}{\left(\dfrac{h}{h+3}\right)} = ?$

A) 0 B) −1 C) $\frac{1}{5}$ D) 2 E) ∞

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

2. $x_n = \sqrt[n]{5}$, $y_n = 2+\frac{1}{n}$

$\Rightarrow \lim\limits_{n\to\infty} (x_n y_n) = ?$

A) 2 B) −1 C) 0 D) ∞ E) n

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

3. $\lim\limits_{x\to \frac{\pi}{3}} \dfrac{2\sin x - 1}{\sqrt{3} - 2\cos x} = ?$

A) −1 B) 0 C) $\frac{1}{4}$ D) $\frac{\sqrt{3}}{4}$ E) 1

[MARMARA ÜNİV. YÖS 2013]

4. $\lim\limits_{x\to 1} \dfrac{x\cdot\sqrt{2x-1}}{x-1} = ?$

A) $-\infty$ B) −1 C) 0 D) 1 E) ∞

[ANKARA ÜNİV. YÖS 2013]

5. $\lim\limits_{x\to 3} f(x) = 4$

$\Rightarrow \lim\limits_{x\to 3} \dfrac{(f(x)-3)(f^2(x)-4f(x)+1)}{x+3} = ?$

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 5

[İSTANBUL ÜNİV. YÖS 2013]

6. $\lim\limits_{x\to\infty} (e^{-x} - \pi^{\frac{1}{x}}) = ?$

A) −1 B) 0 C) 1 D) e E) π

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

YANITLAR / ANSWERS

1. E	2. A	3. E	4. E	5. B	6. A

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1. $\lim_{h\to 0} \frac{(h+1)(2h+1)}{\left(\frac{h}{h+3}\right)} = \lim_{h\to 0} \frac{(h+1)(2h+1)\cdot(h+3)}{h}$

$= \frac{(0+1)(0+1)(0+3)}{0}$

$= \frac{3}{0} = \infty$

Yanıt / The Answer is E

2. $x_n = \sqrt[n]{5}$, $y_n = 2+\frac{1}{n} \Rightarrow$

$\lim_{n\to\infty} (x_n y_n) = 5^{\frac{1}{\infty}} \cdot \left(2+\frac{1}{\infty}\right)$

$= 5^0 (2+0)$

$= 2$

Yanıt / The Answer is A

3. $\lim_{x\to\frac{\pi}{3}} \frac{2\sin x - 1}{\sqrt{3} - 2\cos x} = \frac{2\cdot\sin\frac{\pi}{3} - 1}{\sqrt{3} - 2\cos\frac{\pi}{3}}$

$= \frac{2\cdot\frac{\sqrt{3}}{2} - 1}{\sqrt{3} - 2\cdot\frac{1}{2}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}-1} = 1$

Yanıt / The Answer is E

4. $\lim_{x\to 1} \frac{x\cdot\sqrt{2x-1}}{x-1} = \frac{1\cdot\sqrt{1}}{0} = \infty$

Yanıt / The Answer is E

5. $\lim_{x\to 3} \frac{(4-3)\cdot(4^2-4\cdot 4+1)}{3+3} = \frac{1\cdot 1}{6}$

$= \frac{1}{6}$

Yanıt / The Answer is B

6. $\lim_{x\to\infty} (e^{-x} - \pi^{\frac{1}{x}}) = \frac{1}{e^{\infty}} - \pi^{\frac{1}{\infty}}$

$= \frac{1}{\infty} - \pi^0$

$= 0 - 1$

$= -1$

Yanıt / The Answer is A

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1. $\lim_{x \to 2^+} (x^2 - 4x + 1) = ?$

Yanıt / The Answer is **–3**

2. $\lim_{x \to e^2} \left(\ln x^2\right) = ?$

Yanıt / The Answer is **4**

3. $\lim_{x \to 3} \left[4 \cdot (3x - 2) \cdot (x + 2)\right] = ?$

Yanıt / The Answer is **140**

4. $\lim_{x \to 2^+} (9x^2 - 3x + 2) = ?$

Yanıt / The Answer is **32**

5. $\lim_{x \to 5} \frac{x^2 - x + 1}{x - 4} = ?$

Yanıt / The Answer is **21**

6. $\lim_{x \to 2^+} \frac{|x - 2|}{x - 2} = ?$

Yanıt / The Answer is **1**

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1. **Aşağıdakilerden hangisi yanlıştır?**
Which of the following is not true?

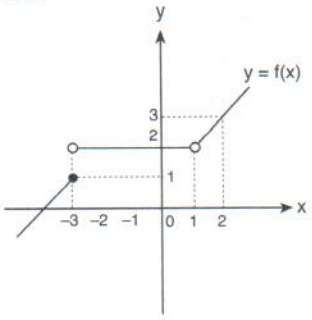
A) $\lim_{x \to -3^-} f(x) = 1$

B) $\lim_{x \to -3^+} f(x) = 2$

C) $\lim_{x \to 2} f(x) = 2$

D) $\lim_{x \to 0^+} f(x) = 2$

E) $\lim_{x \to 1} f(x) = 2$



2. $\lim_{x-y \to 0} \frac{x^2+y^2}{x+y} = ?$

A) $-2y$

B) $-y$

C) 0

D) y

E) 2y

3. $\lim_{x \to \frac{\pi}{6}} \left(\sin x + \cos^2 x\right) = ?$

A) $\frac{5}{4}$

B) $\frac{3}{2}$

C) $\frac{7}{4}$

D) 2

E) $\frac{9}{4}$

4. $\lim_{x \to 6} \left(\log_2 \sqrt{x^2-4}\right) = ?$

A) -2

B) $\frac{9}{2}$

C) 2

D) 4

E) $\frac{5}{2}$

5. $\lim_{x \to -5} \left[(x+4)^{100} \cdot (x+2)\right] = ?$

A) -3

B) -2

C) -1

D) 2

E) 3

6. $f(x) = \begin{cases} 2x^2 - x & x \le 2 \\ 3x+5 & x > 2 \end{cases}$

$\Rightarrow \lim_{x \to 2^+} f(x) = ?$

A) 5

B) 7

C) 9

D) 10

E) 11

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7. $\lim_{x \to 1} \frac{x^2+3x-4}{x^2-5x+6} = ?$

A) $-\frac{1}{3}$

B) $-\frac{2}{3}$

C) -1

D) $-\frac{4}{3}$

E) 0

8. $\lim_{x \to 4} \frac{\sqrt{x}-2}{x^2-x+12} = ?$

A) $\frac{1}{22}$

B) 0

C) $\frac{1}{26}$

D) $\frac{1}{28}$

E) $\frac{1}{30}$

9. $f(x) = \begin{cases} x^2-a & x>2 \\ 3x^2+ax+b & x \le 2 \end{cases}$

$\lim_{x \to 2} f(x) = 6 \Rightarrow b = ?$

A) –2

B) –1

C) 0

D) 1

E) 2

10. $f(x) = x^2 - 2x + \cos(x-5) \Rightarrow \lim_{x \to 5^+} f(x) = ?$

A) 15

B) 16

C) 17

D) 18

E) 19

11. $f(x) = x^2 - 3x + 5 \Rightarrow \lim_{x \to 2} f(x) = ?$

A) 3

B) 4

C) 5

D) 6

E) 7

12. $\lim_{\frac{a}{b} \to 2} \frac{b+3a}{b+2a} = ?$

A) $\frac{1}{2}$ B) $\frac{5}{2}$ C) $\frac{7}{5}$

D) 2 E) 3

YANITLAR / ANSWERS							
1. C	2. D	3. A	4. E	5. A	6. E	7. E	8. B
9. A	10. B	11. A	12. C				

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ZTC زنگنه مجموعه موسسات

BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Kolmogorov, Andrei Nikolaevich [1903 – 1987]

Rus matematikçi. Matematikte olasılığın kullanılmasına olan katkıları ile bilinir. Küme teorisinin gösterilmesi, dil ile ifade edilmesine önemli katkıları olmuştur.

Russian mathematician who worked in a number of areas of mathematics, including probability. In this, his important contribution was to give the subject a rigorous foundation by using the language and notation of set theory.

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LİMİTTE BELİRSİZLİKLER
INDETERMINATE FORMS

BÖLÜM 2
CHAPTER

Tanım:

$\lim_{x\to a} f(x)$ limitinin sonucu $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty-\infty$, $0\cdot\infty$, 1^{∞}, 0^{0}, ∞^{0} olabilir.

Bu sonuçların herbirine *belirsizlik* denir.

1. $\frac{0}{0}$ Belirsizliği

$\lim_{x\to a} \frac{f(x)}{g(x)}$ limitinin sonucu $\frac{0}{0}$ belirsizliği oluyorsa, f(x) ve g(x) fonksiyonlarında (x – a) çarpanı vardır.

$\frac{0}{0}$ belirsizlikleri çözülürken,

$$f(x) = (x-a)\cdot f_1(x) \quad , \quad g(x) = (x-a)\cdot g_1(x)$$

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{(x-a)\cdot f_1(x)}{(x-a)\cdot g_1(x)} = \lim_{x\to a} \frac{f_1(x)}{g_1(x)} = \lim_{x\to a} \frac{f_1(a)}{g_1(a)}$ yolu izlenir.

Eğer belirsizlik kalkmıyorsa bu işleme devam edilir.

ÖRNEK 1

$\lim_{x\to -2} \frac{x^3+2x^2+x+2}{x^2-4} = ?$

A) $-\frac{1}{4}$ B) $-\frac{3}{4}$ C) $-\frac{5}{4}$ D) $-\frac{7}{4}$ E) $-\frac{9}{4}$

ÇÖZÜM

Yanıt C

$\lim_{x\to -2} \frac{x^3+2x^2+x+2}{x^2-4} = \frac{-8+8-2+2}{4-4} = \frac{0}{0}$

$\lim_{x\to -2} \frac{x^3+2x^2+x+2}{x^2-4} = \lim_{x\to -2} \frac{x^2(x+2)+(x+2)}{(x-2)(x+2)}$

$= \lim_{x\to -2} \frac{(x+2)\cdot(x^2+1)}{(x-2)(x+2)}$

$= \lim_{x\to -2} \frac{x^2+1}{x-2}$

$= \frac{(-2)^2+1}{-2-2} = \frac{5}{-4} = \frac{-5}{4}$

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Definition:

Result of $\lim_{x\to a} f(x)$ can be $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0\cdot\infty$, 1^{∞}, 0^0, ∞^0. Each of the results is called *indeterminate.*

1. Indeterminate Form $\frac{0}{0}$

To find $\lim_{x\to a} \frac{f(x)}{g(x)}$, when you apply direct substitution and obtain $\frac{0}{0}$ you can conclude that $(x - a)$ must be a common factor to both $f(x)$ and $g(x)$.

While we solve indeterminate form $\frac{0}{0}$,

$$f(x) = (x - a)\cdot f_1(x) \quad , \quad g(x) = (x - a)\cdot g_1(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{(x-a)\cdot f_1(x)}{(x-a)\cdot g_1(x)} = \lim_{x\to a}\frac{f_1(x)}{g_1(x)} = \lim_{x\to a}\frac{f_1(a)}{g_1(a)}$$ is written.

If indeterminate continue, apply the similar process one more time.

EXAMPLE 1

$$\lim_{x\to -2}\frac{x^3+2x^2+x+2}{x^2-4} = ?$$

A) $-\frac{1}{4}$ B) $-\frac{3}{4}$ C) $-\frac{5}{4}$ D) $-\frac{7}{4}$ E) $-\frac{9}{4}$

SOLUTION

The answer is **C**

$$\lim_{x\to -2}\frac{x^3+2x^2+x+2}{x^2-4} = \frac{-8+8-2+2}{4-4} = \frac{0}{0}$$

$$\lim_{x\to -2}\frac{x^3+2x^2+x+2}{x^2-4} = \lim_{x\to -2}\frac{x^2(x+2)+(x+2)}{(x-2)(x+2)}$$

$$= \lim_{x\to -2}\frac{(x+2)\cdot(x^2+1)}{(x-2)(x+2)}$$

$$= \lim_{x\to -2}\frac{x^2+1}{x-2}$$

$$= \frac{(-2)^2+1}{-2-2} = \frac{5}{-4} = \frac{-5}{4}$$

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ÖRNEK 2

$$\lim_{x \to -1} \frac{x^4 - x^3 + 3x^2 - 2x - 7}{x+1} = ?$$

A) –12 B) –13 C) –14 D) –15 E) –16

ÇÖZÜM

Yanıt D

$$\lim_{x \to -1} \frac{x^4 - x^3 + 3x^2 - 2x - 7}{x+1} = \frac{1+1+3+2-7}{-1+1} = \frac{0}{0}$$

$$\begin{array}{r|l} x^4 - x^3 + 3x^2 - 2x - 7 & x+1 \\ \hline -x^4 \mp x^3 & x^3 - 2x^2 + 5x - 7 \\ \hline -2x^3 + 3x^2 - 2x - 7 & \\ \mp 2x^3 \mp 2x^2 & \\ \hline 5x^2 - 2x - 7 & \\ -5x^2 \mp 5x & \\ \hline -7x - 7 & \\ -7x - 7 & \\ \hline 0 & \end{array}$$

$$\lim_{x \to -1} \frac{x^4 - x^3 + 3x^2 - 2x - 7}{x+1} = \lim_{x \to -1} (x^3 - 2x^2 + 5x - 7) = (-1)^3 - 2(-1)^2 + 5(-1) - 7$$

$$= -1 - 2 - 5 - 7 = -15$$

Trigonometrik Fonksiyonların Limitleri

1. $\lim_{x \to a} \sin x = \sin a$
2. $\lim_{x \to a} \cos x = \cos a$
3. $\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b}$
4. $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$
5. $\lim_{x \to 0} \frac{\tan ax}{bx} = \frac{a}{b}$
6. $\lim_{x \to 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$
7. $\lim_{x \to 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$
8. $\lim_{x \to 0} \frac{\tan ax}{\sin bx} = \frac{a}{b}$

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EXAMPLE 2

$$\lim_{x\to -1}\frac{x^4-x^3+3x^2-2x-7}{x+1}=?$$

A) −12 B) −13 C) −14 D) −15 E) −16

SOLUTION

The answer is D

$$\lim_{x\to -1}\frac{x^4-x^3+3x^2-2x-7}{x+1}=\frac{1+1+3+2-7}{-1+1}=\frac{0}{0}$$

$$\begin{array}{r|l} x^4-x^3+3x^2-2x-7 & x+1 \\ \hline -x^4\mp x^3 & x^3-2x^2+5x-7 \\ \hline -2x^3+3x^2-2x-7 & \\ \mp 2x^3\mp 2x^2 & \\ \hline 5x^2-2x-7 & \\ -5x^2\mp 5x & \\ \hline -7x-7 & \\ -7x-7 & \\ \hline 0 & \end{array}$$

$$\lim_{x\to -1}\frac{x^4-x^3+3x^2-2x-7}{x+1}=\lim_{x\to -1}(x^3-2x^2+5x-7)=(-1)^3-2(-1)^2+5(-1)-7$$

$$=-1-2-5-7=-15$$

Limits of Trigonometric Functions

1. $\lim_{x\to a}\sin x=\sin a$
2. $\lim_{x\to a}\cos x=\cos a$
3. $\lim_{x\to 0}\frac{\sin ax}{bx}=\frac{a}{b}$
4. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}=\frac{a}{b}$
5. $\lim_{x\to 0}\frac{\tan ax}{bx}=\frac{a}{b}$
6. $\lim_{x\to 0}\frac{\tan ax}{\tan bx}=\frac{a}{b}$
7. $\lim_{x\to 0}\frac{\sin ax}{\tan bx}=\frac{a}{b}$
8. $\lim_{x\to 0}\frac{\tan ax}{\sin bx}=\frac{a}{b}$

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ÖRNEK 3

$$\lim_{x\to 0}\frac{\sin 4x}{x} = ?$$

A) 1 B) 2 C) 4 D) $\frac{1}{4}$ E) $\frac{1}{8}$

ÇÖZÜM

Yanıt C

$$\lim_{x\to 0}\frac{\sin 4x}{x} = \frac{4}{1} = 4$$

ÖRNEK 4

$$\lim_{x\to 2}\frac{\sin(x-2)}{x^2-4} = ?$$

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

ÇÖZÜM

Yanıt B

$$\lim_{x\to 2}\frac{\sin(x-2)}{x^2-4} = \lim_{x\to 2}\frac{\sin(x-2)}{x-2}\cdot\lim_{x\to 2}\frac{1}{x+2}$$

$$= 1\cdot\frac{1}{4} = \frac{1}{4}$$

PEKİŞTİRME

$$\lim_{x\to 5}\frac{\sin(5x-25)}{125x-625} = ?$$

YANIT : 1/25

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EXAMPLE 3

$\lim_{x\to 0} \frac{\sin 4x}{x} = ?$

A) 1 B) 2 C) 4 D) $\frac{1}{4}$ E) $\frac{1}{8}$

SOLUTION

The answer is C

$\lim_{x\to 0} \frac{\sin 4x}{x} = \frac{4}{1} = 4$

REINFORCE

$\lim_{x\to 5} \frac{\sin(5x-25)}{125x-625} = ?$

ANSWER : 1/25

EXAMPLE 4

$\lim_{x\to 2} \frac{\sin(x-2)}{x^2-4} = ?$

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

SOLUTION

The answer is B

$\lim_{x\to 2} \frac{\sin(x-2)}{x^2-4} = \lim_{x\to 2} \frac{\sin(x-2)}{x-2} \cdot \lim_{x\to 2} \frac{1}{x+2}$

$= 1 \cdot \frac{1}{4} = \frac{1}{4}$

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2. $\frac{\infty}{\infty}$ Belirsizliği

$\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to\infty} f(x)}{\lim\limits_{x\to\infty} g(x)} = \pm\frac{\infty}{\infty}$ olsun.

Bu durumda, f(x) ve g(x) fonksiyonlarındaki en yüksek dereceli terimden parantez alınır ve sonra sadeleştirilir.

m, n ∈ N için,

$$\lim_{x\to\pm\infty} \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_0} = \begin{cases} 0 & , \quad n < m \\ \frac{a_n}{b_m} & , \quad n = m \\ \pm\infty & , \quad n > m \end{cases}$$

Etkinlikler

✓ $\lim\limits_{x\to\infty} \frac{x^2+x-1}{x^3-x+1} = 0 \qquad (2<3)$

✓ $\lim\limits_{x\to\infty} \frac{x^3+x-1}{2x^3+x-1} = \frac{1}{2} \qquad (3=3)$

✓ $\lim\limits_{x\to\infty} \frac{x^4+x^2-1}{x^3+3x} = +\infty \qquad (4>3)$

✓ $\lim\limits_{x\to-\infty} \frac{x^2-x+1}{x+1} = -\infty \qquad (2>1)$

ÖRNEK 5

$$\lim_{x\to\infty} \frac{\sqrt{4x^2-x+5}+\sqrt{x^2-3x}}{2x} = ?$$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

ÇÖZÜM

Yanıt D

$$\lim_{x\to\infty} \frac{\sqrt{4x^2-x+5}+\sqrt{x^2-3x}}{2x} = \frac{\infty+\infty}{\infty} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\sqrt{4x^2-x+5}+\sqrt{x^2-3x}}{2x} = \lim_{x\to\infty} \frac{\sqrt{4x^2\left(1-\frac{1}{4x}+\frac{5}{4x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x}\right)}}{2x}$$

$$= \lim_{x\to\infty} \frac{|2x|\sqrt{(1-0+0)}+|x|\sqrt{1-0}}{2x}$$

$$= \lim_{x\to\infty} \frac{2x+x}{2x} = \frac{3}{2}$$

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2. Indeterminate Form $\frac{\infty}{\infty}$

Let $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\frac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)}=\pm\frac{\infty}{\infty}$

In this case, factor out the terms with highest degree in f(x) and g(x) and then simplify them.

For $m,n\in N$,

$$\lim_{x\to\pm\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_mx^m+b_{m-1}x^{m-1}+\ldots+b_0}=\begin{cases}0, & n<m\\ \frac{a_n}{b_m}, & n=m\\ \pm\infty, & n>m\end{cases}$$

Activities

✓ $\lim_{x\to\infty}\frac{x^2+x-1}{x^3-x+1}=0 \quad (2<3)$

✓ $\lim_{x\to\infty}\frac{x^3+x-1}{2x^3+x-1}=\frac{1}{2} \quad (3=3)$

✓ $\lim_{x\to\infty}\frac{x^4+x^2-1}{x^3+3x}=+\infty \quad (4>3)$

✓ $\lim_{x\to-\infty}\frac{x^2-x+1}{x+1}=-\infty \quad (2>1)$

EXAMPLE 5

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-x+5}+\sqrt{x^2-3x}}{2x}=?$$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

SOLUTION

The answer is **D**

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-x+5}+\sqrt{x^2-3x}}{2x}=\frac{\infty+\infty}{\infty}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-x+5}+\sqrt{x^2-3x}}{2x}=\lim_{x\to\infty}\frac{\sqrt{4x^2\left(1-\frac{1}{4x}+\frac{5}{4x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x}\right)}}{2x}$$

$$=\lim_{x\to\infty}\frac{|2x|\sqrt{(1-0+0)}+|x|\sqrt{1-0}}{2x}$$

$$=\lim_{x\to\infty}\frac{2x+x}{2x}=\frac{3}{2}$$

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ÖRNEK 6

$$\lim_{x\to\infty}\frac{(a-3)x^3-4x^2+x-5}{(b-2)x^2+3}=4 \Rightarrow a+b=?$$

A) 2 B) 3 C) 4 D) 5 E) 6

ÇÖZÜM

Yanıt C

$$\lim_{x\to\infty}\frac{(a-3)x^3-4x^2+x-5}{(b-2)x^2+3}=4$$

Sonuç reel sayı olduğundan pay ve paydanın dereceleri aynı olmalıdır.

$\Rightarrow a-3=0$, $\frac{-4}{b-2}=4$

$a=3$ $b-2=-1$

$b=1$

$a+b=3+1=4$

ÖRNEK 7

$$\lim_{x\to\infty}\left(\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}\right)=?$$

A) $-\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{6}{5}$ E) $\frac{1}{2}$

ÇÖZÜM

Yanıt D

$$\lim_{x\to\infty}\left(\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}\right)=\frac{4^\infty+6^\infty}{5\cdot6^\infty+2^\infty}=\frac{\infty+\infty}{\infty+\infty}=\frac{\infty}{\infty}$$

Çözüm 1:

$$\lim_{x\to\infty}\left(\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}\right)=\lim_{x\to\infty}\frac{4^x\cdot4^2+6^x\cdot6}{5\cdot6^x+2^x}=\lim_{x\to\infty}\frac{6^x\left(\frac{16\cdot4^x}{6^x}+6\right)}{6^x\left(5+\frac{2^x}{6^x}\right)}$$

$$=\lim_{x\to\infty}\frac{16\left(\frac{4}{6}\right)^x+6}{5+\left(\frac{2}{6}\right)^x}=\frac{0+6}{5}=\frac{6}{5}$$

Çözüm 2:

$$\lim_{x\to\infty}\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}=\lim_{x\to\infty}\frac{6^{x+1}}{5\cdot6^x}=\lim_{x\to\infty}\frac{6^x\cdot6}{5\cdot6^x}=\frac{6}{5}$$

PEKİŞTİRME

$$\lim_{x\to\infty}\frac{3^{x-1}+4^x}{2^{x+1}}=?$$

YANIT : ∞

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EXAMPLE 6

$$\lim_{x\to\infty}\frac{(a-3)x^3-4x^2+x-5}{(b-2)x^2+3}=4 \Rightarrow a+b=?$$

A) 2 B) 3 C) 4 D) 5 E) 6

SOLUTION

The answer is C

$$\lim_{x\to\infty}\frac{(a-3)x^3-4x^2+x-5}{(b-2)x^2+3}=4$$

Since the result is a real number the degrees of numerator and denominator must be same.

$\Rightarrow a-3=0$, $\frac{-4}{b-2}=4$

$a=3$ $b-2=-1$

$b=1$

$a+b=3+1=4$

REINFORCE

$$\lim_{x\to-\infty}\frac{3^{x-1}+4^x}{2^{x+1}}=?$$

ANSWER : ∞

EXAMPLE 7

$$\lim_{x\to\infty}\left(\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}\right)=?$$

A) $-\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{6}{5}$ E) $\frac{1}{2}$

SOLUTION

The answer is D

$$\lim_{x\to\infty}\left(\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}\right)=\frac{4^\infty+6^\infty}{5\cdot6^\infty+2^\infty}=\frac{\infty+\infty}{\infty+\infty}=\frac{\infty}{\infty}$$

Solution 1:

$$\lim_{x\to\infty}\left(\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}\right)=\lim_{x\to\infty}\frac{4^x\cdot4^2+6^x\cdot6}{5\cdot6^x+2^x}=\lim_{x\to\infty}\frac{6^x\left(\frac{16\cdot4^x}{6^x}+6\right)}{6^x\left(5+\frac{2^x}{6^x}\right)}$$

$$=\lim_{x\to\infty}\frac{16\left(\frac{4}{6}\right)^x+6}{5+\left(\frac{2}{6}\right)^x}=\frac{0+6}{5}=\frac{6}{5}$$

Solution 2:

$$\lim_{x\to\infty}\frac{4^{x+2}+6^{x+1}}{5\cdot6^x+2^x}=\lim_{x\to\infty}\frac{6^{x+1}}{5\cdot6^x}=\lim_{x\to\infty}\frac{6^x\cdot6}{5\cdot6^x}=\frac{6}{5}$$

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3. $\infty - \infty$ Belirsizliği

$\lim\limits_{x\to a}(f(x)-g(x))$ limitinin sonucu $\infty-\infty$ belirsizliği oluyorsa, $\frac{0}{0}$ ya da $\frac{\infty}{\infty}$ belirsizliklerine dönüştürülerek çözülür.

NOT

$a>0$

$$\lim_{x\to\pm\infty}\sqrt{ax^2+bx+c}=\sqrt{a}\lim_{x\to\pm\infty}\left|x+\frac{b}{2a}\right|$$

KANIT

$$\lim_{x\to\infty}\sqrt{a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)}$$

$$=\lim_{x\to\infty}\sqrt{a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}\right]}$$

$$=\lim_{x\to\infty}\sqrt{\left(x+\frac{b}{2a}\right)^2\cdot\left[a+\frac{4ac-b^2}{4a}\right]:\left(x+\frac{b}{2a}\right)^2}$$

$$=\lim_{x\to\infty}\left|x+\frac{b}{2a}\right|\sqrt{\frac{a+\frac{4ac-b^2}{4a^2}}{\left(x+\frac{b}{2a}\right)^2}}$$

$$=\lim_{x\to\infty}\left|x+\frac{b}{2a}\right|\sqrt{a}$$

$$=\lim_{x\to\infty}\sqrt{a}\left|x+\frac{b}{2a}\right|$$

ÖRNEK 8

$\lim\limits_{x\to2}\left(\frac{1}{x-2}-\frac{1}{x^2-3x+2}\right)=?$

A) -2 B) -1 C) 0 D) 1 E) 2

ÇÖZÜM

Yanıt **D**

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{1}{x^2-3x+2}\right)=\lim_{x\to2}\left(\frac{1}{\underset{(x-1)}{x-2}}-\frac{1}{(x-2)(x-1)}\right)$$

$$=\lim_{x\to2}\frac{x-1-1}{(x-2)(x-1)}=\lim_{x\to2}\frac{1}{x-1}=\frac{1}{1}=1$$

ÖRNEK 9

$f(x)=2\sqrt{x^2+1}-\sqrt{4x^2+2x+3}\Rightarrow\lim\limits_{x\to\infty}f(x)=?$

A) $-\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{6}{5}$ E) $\frac{1}{2}$

ÇÖZÜM

Yanıt **A**

Çözüm 1:

$$\lim_{x\to\infty}\frac{(2\sqrt{x^2+1}-\sqrt{4x^2+2x+3})(2\sqrt{x^2+1}+\sqrt{4x^2+2x+3})}{2\sqrt{x^2+1}+\sqrt{4x^2+2x+3}}$$

$$=\lim_{x\to\infty}\frac{4x^2+4-4x^2-2x-3}{x\left(2\sqrt{1+\frac{1}{x^2}}+\sqrt{4+\frac{2}{x}+\frac{3}{x^2}}\right)}=\lim_{x\to\infty}\frac{x(-2+\frac{1}{x})}{x(2+2)}=-\frac{1}{2}$$

Çözüm 2:

$$\lim_{x\to\infty}2\sqrt{x^2+1}-\sqrt{4x^2+2x+3}=\lim_{x\to\infty}2\sqrt{1}\left|x+\frac{0}{2}\right|-\sqrt{4}\left|x+\frac{2}{4\cdot2}\right|$$

$$=\lim_{x\to\infty}2x-2x-\frac{1}{2}=-\frac{1}{2}$$

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NOTE

$a > 0$

$$\lim_{x\to\pm\infty}\sqrt{ax^2+bx+c}=\sqrt{a}\lim_{x\to\pm\infty}\left|x+\frac{b}{2a}\right|$$

PROOF

$$\lim_{x\to\infty}\sqrt{a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)}$$

$$=\lim_{x\to\infty}\sqrt{a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}\right]}$$

$$=\lim_{x\to\infty}\sqrt{\left(x+\frac{b}{2a}\right)^2\cdot\left[a+\frac{4ac-b^2}{4a}\right]:\left(x+\frac{b}{2a}\right)^2}$$

$$=\lim_{x\to\infty}\left|x+\frac{b}{2a}\right|\sqrt{\frac{a+\frac{4ac-b^2}{4a^2}}{\left(x+\frac{b}{2a}\right)^2}}$$

$$=\lim_{x\to\infty}\left|x+\frac{b}{2a}\right|\sqrt{a}$$

$$=\lim_{x\to-\infty}\sqrt{a}\left|x+\frac{b}{2a}\right|$$

3. Indeterminate Form $\infty-\infty$

An expression such as $\infty-\infty$ is called an indeterminate form becuse we cannot determine the limit. When we encounter this form, we must rewrite it in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

EXAMPLE 8

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{1}{x^2-3x+2}\right)=?$$

A) -2 B) -1 C) 0 D) 1 E) 2

SOLUTION

The answer is **D**

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{1}{x^2-3x+2}\right)=\lim_{x\to2}\left(\underset{(x-1)}{\frac{1}{x-2}}-\frac{1}{(x-2)(x-1)}\right)$$

$$=\lim_{x\to2}\frac{x-1-1}{(x-2)(x-1)}=\lim_{x\to2}\frac{1}{x-1}=\frac{1}{1}=1$$

EXAMPLE 9

$$f(x)=2\sqrt{x^2+1}-\sqrt{4x^2+2x+3}\Rightarrow\lim_{x\to\infty}f(x)=?$$

A) $-\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{6}{5}$ E) $\frac{1}{2}$

SOLUTION

The answer is **A**

Solution 1:

$$\lim_{x\to\infty}\frac{(2\sqrt{x^2+1}-\sqrt{4x^2+2x+3})(2\sqrt{x^2+1}+\sqrt{4x^2+2x+3})}{2\sqrt{x^2+1}+\sqrt{4x^2+2x+3}}$$

$$=\lim_{x\to\infty}\frac{4x^2+4-4x^2-2x-3}{x\left(2\sqrt{1+\frac{1}{x^2}}+\sqrt{4+\frac{2}{x}+\frac{3}{x^2}}\right)}=\lim_{x\to\infty}\frac{x(-2+\frac{1}{x})}{x(2+2)}=-\frac{1}{2}$$

Solution 2:

$$\lim_{x\to\infty}2\sqrt{x^2+1}-\sqrt{4x^2+2x+3}=\lim_{x\to\infty}2\sqrt{1}\left|x+\frac{0}{2}\right|-\sqrt{4}\left|x+\frac{2}{4\cdot2}\right|$$

$$=\lim_{x\to\infty}2x-2x-\frac{1}{2}=-\frac{1}{2}$$

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4. $0 \cdot \infty$ Belirsizliği

$\lim\limits_{x \to a} f(x) \cdot g(x) = 0 \cdot \infty$ belirsizliği, $\frac{0}{0}$ veya $\frac{\infty}{\infty}$ belirsizliklerine dönüştürülerek hesaplanır.

1. $\lim\limits_{x \to a} f(x) \cdot g(x) = \lim\limits_{x \to a} \dfrac{f(x)}{\frac{1}{g(x)}} \quad \left(\frac{0}{0}\right)$

2. $\lim\limits_{x \to a} f(x) \cdot g(x) = \lim\limits_{x \to a} \dfrac{g(x)}{\frac{1}{f(x)}} \quad \left(\frac{\infty}{\infty}\right)$

ÖRNEK 10

$\lim\limits_{x \to 0} \sin 5x \cdot \cot 3x = ?$

A) $\frac{4}{3}$ B) $\frac{3}{2}$ C) $\frac{5}{3}$ D) $\frac{7}{3}$ E) $\frac{5}{2}$

ÇÖZÜM

Yanıt C

$\lim\limits_{x \to 0} \sin 5x \cdot \cot 3x = 0 \cdot \infty$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\sin 5x}{\frac{1}{\cot 3x}} = \lim\limits_{x \to 0} \dfrac{\sin 5x}{\tan 3x} = \dfrac{5}{3}$

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4. Indeterminate Form $0 \cdot \infty$

The indeterminate $\lim_{x \to a} f(x) \cdot g(x) = 0 \cdot \infty$ is written in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and then calculated.

1. $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} \frac{f(x)}{\frac{1}{g(x)}} \left(\frac{0}{0}\right)$

2. $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} \frac{g(x)}{\frac{1}{f(x)}} \left(\frac{\infty}{\infty}\right)$

EXAMPLE 10

$\lim_{x \to 0} \sin 5x \cdot \cot 3x = ?$

A) $\frac{4}{3}$ B) $\frac{3}{2}$ C) $\frac{5}{3}$ D) $\frac{7}{3}$ E) $\frac{5}{2}$

SOLUTION

The answer is C

$\lim_{x \to 0} \sin 5x \cdot \cot 3x = 0 \cdot \infty$

$\Rightarrow \lim_{x \to 0} \frac{\sin 5x}{\frac{1}{\cot 3x}} = \lim_{x \to 0} \frac{\sin 5x}{\tan 3x} = \frac{5}{3}$

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5. 1^{∞} Belirsizliği

$$\left.\begin{array}{l}\lim\limits_{x\to\infty} f(x) = 0\\ \lim\limits_{x\to\infty} g(x) = +\infty\\ \lim\limits_{x\to\infty} (f(x)\cdot g(x)) = k\end{array}\right\} \Rightarrow \lim_{x\to\infty} [1+f(x)]^{g(x)} = e^{k}$$

ÖRNEK 11

$$\lim_{t\to\infty}\left(\frac{t^3-2}{t^3+1}\right)^{t^3+3} = ?$$

A) 1 B) 2 C) e D) e^{-3} E) ∞

ÇÖZÜM

Yanıt D

$$\lim_{t\to\infty}\left(\frac{t^3-2}{t^3+1}\right)^{t^3+3} = 1^{\infty}$$

$$\begin{array}{r|l} t^3-2 & t^3+1 \\ -t^3-1 & 1 \\ \hline -3 & \end{array} \Rightarrow \frac{t^3-2}{t^3+1} = 1+\frac{-3}{t^3+1}$$

$$\left.\begin{array}{l}\lim\limits_{t\to\infty} \dfrac{-3}{t^3+1} = \dfrac{-3}{\infty} = 0\\ \lim\limits_{t\to\infty} t^3+3 = \infty\end{array}\right\} \Rightarrow \lim_{t\to\infty}\left(1+\frac{-3}{t^3+1}\right)^{t^3+3} = e^{\lim\limits_{t\to\infty}\frac{-3(t^3+3)}{t^3+1}} = e^{-3}$$

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5. Indeterminate Form 1^{∞}

$$\left.\begin{array}{l}\lim\limits_{x\to\infty} f(x)=0\\ \lim\limits_{x\to\infty} g(x)=+\infty\\ \lim\limits_{x\to\infty}\left(f(x)\cdot g(x)\right)=k\end{array}\right\} \Rightarrow \lim_{x\to\infty}\left[1+f(x)\right]^{g(x)}=e^{k}$$

EXAMPLE 11

$$\lim_{t\to\infty}\left(\frac{t^3-2}{t^3+1}\right)^{t^3+3}=?$$

A) 1 B) 2 C) e D) e^{-3} E) ∞

SOLUTION

The answer is D

$$\lim_{t\to\infty}\left(\frac{t^3-2}{t^3+1}\right)^{t^3+3}=1^{\infty}$$

$$\begin{array}{r|l} t^3-2 & t^3+1 \\ \hline -(t^3-1) & 1 \\ \hline -3 & \end{array} \quad \Rightarrow \quad \frac{t^3-2}{t^3+1}=1+\frac{-3}{t^3+1}$$

$$\left.\begin{array}{l}\lim\limits_{t\to\infty}\dfrac{-3}{t^3+1}=\dfrac{-3}{\infty}=0\\ \lim\limits_{t\to\infty} t^3+3=\infty\end{array}\right\} \Rightarrow \lim_{t\to\infty}\left(1+\frac{-3}{t^3+1}\right)^{t^3+3}=e^{\lim\limits_{t\to\infty}\frac{-3(t^3+3)}{t^3+1}}=e^{-3}$$

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6. 0^0 ve ∞^0 Belirsizlikleri

$y = (f(x))^{g(x)}$ olmak üzere,

$\lim_{x\to a} (f(x))^{g(x)}$ limiti hesaplandığında 0^0 veya ∞^0 belirsizliklerinin oluşması durumunda her iki tarafında logaritması alınarak limit hesaplanır.

$\lim_{x\to a} \ln y = \lim_{x\to a} g(x) \cdot \ln f(x) = 0\cdot\infty$ belirsizliğine dönüştürülerek hesaplanır.

ÖRNEK 12

$\lim_{x\to \frac{\pi}{2}} \cos x^{\cos x} = ?$

A) 0 B) 1 C) $\frac{3}{2}$ D) 2 E) ∞

ÇÖZÜM

Yanıt **B**

$y = (\cos x)^{\cos x}$

$\ln y = \cos x \cdot \ln \cos x$

$\lim_{x\to \frac{\pi}{2}} \ln y = \lim_{x\to \frac{\pi}{2}} \cos x \cdot \ln \cos x \quad (0\cdot\infty)$

$\left.\begin{array}{l} \cos x = t \\ x \to \frac{\pi}{2} \end{array}\right\} \Rightarrow t \to 0$

$\lim_{t\to 0} t \cdot \ln t \quad (0\cdot\infty)$

$\lim_{t\to 0} \frac{\ln t}{\frac{1}{t}} \quad (\frac{\infty}{\infty})$

$\lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t\to 0} -t = 0$

$\lim_{x\to \frac{\pi}{2}} \ln y = 0 \Rightarrow \lim_{x\to \frac{\pi}{2}} y = \lim_{x\to \frac{\pi}{2}} (\cos x)^{\cos x} = 1$

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6. Indeterminate forms 0^0 and ∞^0

$y = (f(x))^{g(x)}$,

If we calculate $\lim_{x\to a} (f(x))^{g(x)}$ and we obtain 0° or ∞^0, we can take logarithm both sides.

$\lim_{x\to a} \ln y = \lim_{x\to a} g(x) \cdot \ln f(x) = 0\cdot\infty$ is obtained and evaluate.

EXAMPLE 12

$\lim_{x\to\frac{\pi}{2}} \cos x^{\cos x} = ?$

A) 0 B) 1 C) $\frac{3}{2}$ D) 2 E) ∞

SOLUTION

The answer is **B**

$y = (\cos x)^{\cos x}$

$\ln y = \cos x \cdot \ln\cos x$

$\lim_{x\to\frac{\pi}{2}} \ln y = \lim_{x\to\frac{\pi}{2}} \cos x \cdot \ln\cos x \quad (0\cdot\infty)$

$\left.\begin{array}{l}\cos x = t\\ x\to\frac{\pi}{2}\end{array}\right\} \Rightarrow t\to 0$

$\lim_{t\to 0} t\cdot\ln t \quad (0\cdot\infty)$

$\lim_{t\to 0} \frac{\ln t}{\frac{1}{t}} \quad (\frac{\infty}{\infty})$

$\lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t\to 0} -t = 0$

$\lim_{x\to\frac{\pi}{2}} \ln y = 0 \Rightarrow \lim_{x\to\frac{\pi}{2}} y = \lim_{x\to\frac{\pi}{2}} (\cos x)^{\cos x} = 1$

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1. $f(x)=\dfrac{x^2-1}{x^3-1} \Rightarrow \lim\limits_{x\to 1} f(x)=?$

A) $\dfrac{1}{3}$

B) $\dfrac{2}{3}$

C) 1

D) $\dfrac{5}{3}$

E) $\dfrac{7}{3}$

2. $f(x)=\dfrac{\sqrt{x}-2}{x^3-64} \Rightarrow \lim\limits_{x\to 4} f(x)=?$

A) $\dfrac{1}{192}$

B) $\dfrac{1}{190}$

C) $\dfrac{1}{188}$

D) $\dfrac{1}{186}$

E) $\dfrac{1}{176}$

3. $\lim\limits_{x\to 3} \dfrac{\sin(x-3)}{x^3-27}=?$

A) $\dfrac{1}{30}$

B) $\dfrac{1}{27}$

C) $\dfrac{1}{24}$

D) $\dfrac{1}{18}$

E) $\dfrac{1}{15}$

4. $\lim\limits_{x\to 3} \dfrac{x^2-9}{x^3-27}=?$

A) $\dfrac{1}{3}$

B) $\dfrac{2}{9}$

C) $\dfrac{4}{9}$

D) $\dfrac{5}{9}$

E) $\dfrac{7}{9}$

5. $\lim\limits_{x\to \infty} \left[\ln\sqrt{e^3x^2+2}-\ln\sqrt{e^4x^2}\right]=?$

A) -1

B) $-\dfrac{1}{2}$

C) -2

D) $\dfrac{1}{2}$

E) 1

6. $\lim\limits_{x\to \infty} \dfrac{(2x+4)^5(x+2)^3}{8x^8-2}=?$

A) 2

B) 4

C) 6

D) 8

E) 12

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7. $\lim_{x \to y} \frac{\sin x - \sin y}{\sin(x-y)} = ?$

A) cosx

B) sinx

C) siny

D) cosy

E) 0

8. $\lim_{a \to 0} \frac{\cos 5a - \cos 3a}{\sin 5a + \sin 3a} = ?$

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

9. $\lim_{x \to 0} \frac{2\sin^2 x}{3 - 3\cos x} = ?$

A) -2

B) $-\frac{1}{2}$

C) 0

D) $\frac{4}{3}$

E) 2

10. $\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos^3 x} = ?$

A) $\frac{1}{2}$

B) $\frac{2}{3}$

C) 1

D) $\frac{3}{2}$

E) $\frac{5}{2}$

11. $\lim_{x \to 0} (\tan 3x \cdot \operatorname{cosec} 9x) = ?$

A) 0

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) 3

E) 9

12. $f(x) = \left(1 + \frac{4x^2}{3x^3 + 1}\right)^x \Rightarrow \lim_{x \to \infty} f(x) = ?$

A) e

B) 1

C) $\sqrt{e}$

D) $e^{\frac{4}{3}}$

E) $\sqrt[3]{e}$

YANITLAR / ANSWERS							
1. B	2. A	3. B	4. B	5. B	6. B	7. D	8. A
9. D	10. B	11. C	12. D				

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1. $\lim_{x\to1} f(x) = \lim_{x\to1} \frac{x^2-1}{x^3-1} = \frac{\lim_{x\to1}(x^2-1)}{\lim_{x\to1}(x^3-1)} = \frac{1-1}{1-1} = \frac{0}{0}$

$$\lim_{x\to1} f(x) = \lim_{x\to1} \frac{x^2-1}{x^3-1} = \lim_{x\to1} \frac{(x-1)(x+1)}{(x-1)(x^2+x+1)}$$

$$= \lim_{x\to1} \frac{x+1}{x^2+x+1} = \frac{1+1}{1+1+1} = \frac{2}{3}$$

Yanıt / The Answer is B

2. $\lim_{x\to4} f(x) = \lim_{x\to4} \frac{\sqrt{x}-2}{x^3-64} = \frac{\sqrt{4}-2}{4^3-64} = \frac{2-2}{64-64} = \frac{0}{0}$

$$\lim_{x\to4} f(x) = \lim_{x\to4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(x^2+4x+16)(\sqrt{x}+2)}$$

$$= \lim_{x\to4} \frac{x-4}{(x-4)(x^2+4x+16)(\sqrt{x}+2)}$$

$$= \lim_{x\to4} \frac{1}{(x^2+4x+16)(\sqrt{x}+2)}$$

$$= \frac{1}{(16+16+16)\cdot(2+2)} = \frac{1}{48\cdot4} = \frac{1}{192}$$

Yanıt / The Answer is A

3. $\lim_{x\to3} \frac{\sin(x-3)}{x^3-27} = \frac{\sin(3-3)}{27-27} = \frac{0}{0}$

$$\Rightarrow \lim_{x\to3} \frac{\sin(x-3)}{(x-3)(x^2+3x+9)} = \frac{1}{3^2+3\cdot3+9}$$

$$= \frac{1}{27}$$

Yanıt / The Answer is B

4. $\lim_{x\to3} \frac{x^2-9}{x^3-27} = \frac{9-9}{27-27} = \frac{0}{0}$

$$\lim_{x\to3} \frac{x^2-9}{x^3-27} = \lim_{x\to3} \frac{(x-3)(x+3)}{(x-3)(x^2+3x+9)}$$

$$= \frac{3+3}{3^2+3\cdot3+9}$$

$$= \frac{6}{27} = \frac{2}{9}$$

Yanıt / The Answer is B

5. $\lim_{x\to\infty} \left[\ln\sqrt{e^3x^2+2} - \ln\sqrt{e^4x^2}\right]$

$$= \lim_{x\to\infty} \ln\frac{\sqrt{e^3x^2+2}}{\sqrt{e^4x^2}}$$

$$= \frac{1}{2}\ln\lim_{x\to\infty}\left(\frac{e^3\cdot x^2+2}{e^4x^2}\right)$$

$$= \frac{1}{2}\ln\lim_{x\to\infty}\frac{x^2(e^3+\frac{2}{x^2})}{e^4x^2}$$

$$= \frac{1}{2}\ln\frac{e^3}{e^4} = -\frac{1}{2}$$

Yanıt / The Answer is B

6. $\lim_{x\to\infty} \frac{(2x+4)^5(x+2)^3}{8x^8-2} = \lim_{x\to\infty} \frac{(2x)^5\cdot x^3}{8x^8}$

$$= \lim_{x\to\infty} \frac{32x^5\cdot x^3}{8x^8}$$

$$= \frac{32}{8} = 4$$

Yanıt / The Answer is B

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7. $\lim_{x\to y}\frac{\sin x-\sin y}{\sin(x-y)}=\frac{0}{0}$

$$\Rightarrow \lim_{x\to y}\frac{\sin x-\sin y}{\sin(x-y)}=\lim_{x\to y}\frac{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{\sin(x-y)}$$

$$=2\lim_{x\to y}\frac{\sin\frac{x-y}{2}}{x-y}\cdot\lim_{x\to y}\cos\frac{x+y}{2}$$

$$=2\cdot\frac{1}{2}\cdot\cos y=\cos y$$

Yanıt / The Answer is D

8. $\lim_{a\to 0}\frac{\cos 5a-\cos 3a}{\sin 5a+\sin 3a}=\frac{1-1}{0-0}=\frac{0}{0}$

$$\Rightarrow \lim_{a\to 0}\frac{\cos 5a-\cos 3a}{\sin 5a+\sin 3a}=\lim_{a\to 0}\frac{-2\sin 4a\cdot\sin a}{2\sin 4a\cdot\sin a}$$

$$=\lim_{a\to 0}-\frac{\sin a}{\cos a}=-1$$

Yanıt / The Answer is A

9. $\lim_{x\to 0}\frac{2\sin^2 x}{3-3\cos x}=\frac{2\cdot\sin^2 0}{3-3\cdot\cos 0}=\frac{0}{0}$

$$\Rightarrow \lim_{x\to 0}\frac{2(1-\cos^2 x)}{3(1-\cos x)}=\lim_{x\to 0}\frac{2(1-\cos x)(1+\cos x)}{3(1-\cos x)}$$

$$=\lim_{x\to 0}\frac{2(1+\cos x)}{3}$$

$$=\frac{2(1+\cos 0)}{3}$$

$$=\frac{4}{3}$$

Yanıt / The Answer is D

10. $\lim_{x\to 0}\frac{\sin^2 x}{1-\cos^3 x}=\frac{0}{0}$

$$\Rightarrow \lim_{x\to 0}\frac{1-\cos^2 x}{1-\cos^3 x}=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x+\cos^2 x)}$$

$$=\lim_{x\to 0}\frac{1+\cos x}{1+\cos x+\cos^2 x}$$

$$=\frac{1+1}{1+1+1}=\frac{2}{3}$$

Yanıt / The Answer is B

11. $\lim_{x\to 0}(\tan 3x\cdot\operatorname{cosec} 9x)=0\cdot\infty$

$$\Rightarrow \lim_{x\to 0}(\tan 3x\cdot\operatorname{cosec} 9x)=\lim_{x\to 0}\frac{\tan 3x}{\sin 9x}$$

$$=\frac{3}{9}=\frac{1}{3}$$

Yanıt / The Answer is C

12. $\lim_{x\to\infty}\left(1+\frac{4x^2}{3x^3+1}\right)^x=(1+0)^{\infty}=1^{\infty}$

$$\Rightarrow \lim_{x\to\infty}\left(1+\frac{4x^2}{3x^3+1}\right)^x=e^{\lim_{x\to\infty}\left(\frac{4x^3}{3x^3+1}\right)}=e^{\frac{4}{3}}$$

Yanıt / The Answer is D

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YÖS Soruları / YÖS Questions

1. $\lim\limits_{x\to 9} \dfrac{9-x}{3-\sqrt{x}} = ?$

A) 0 B) 3 C) 6 D) 9 E) ∞

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

2. $\lim\limits_{x\to -\infty} \dfrac{2x^2+7x+5}{4x-x^2} = ?$

A) –2 B) –1 C) 0 D) 2 E) ∞

[MARMARA ÜNİV. YÖS 2013]

3. $\lim\limits_{x\to -1} \dfrac{(x^2+x)^3\cdot(x^3+x-1)}{(1-x^2)^3\cdot(3x+2)} = ?$

A) $-\dfrac{1}{8}$ B) –1 C) 0 D) ∞ E) 1

[ANKARA ÜNİV. YÖS 2013]

4. $\lim\limits_{x\to \infty} \dfrac{5-x^2}{x^2-x-2} = ?$

A) $\dfrac{2}{3}$ B) –2 C) –1 D) 2 E) 5

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

5. $\lim\limits_{x\to \frac{\pi}{2}} (\pi - 2x)\tan x = ?$

A) 2 B) –1 C) 0 D) ∞ E) π

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

6. $\lim\limits_{x\to a} \dfrac{x^3-a^3}{\sqrt{x}-\sqrt{a}} = ?$

A) $\dfrac{a^3}{\sqrt{a}}$ B) $3a\sqrt{a}$ C) $\dfrac{a^3-a}{2\sqrt{a}}$ D) $6\sqrt{a^3}$ E) $6a^2\sqrt{a}$

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

YANITLAR / ANSWERS					
1. C	2. A	3. A	4. C	5. A	6. E

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1. $\lim_{x\to 9} \frac{9-x}{3-\sqrt{x}} = \lim_{x\to 9} \frac{(3-\sqrt{x})\cdot(3+\sqrt{x})}{3-\sqrt{x}}$

$= \lim_{x\to 9} 3+\sqrt{x}$

$= 3+\sqrt{9} = 6$

Yanıt / The Answer is C

2. $\lim_{x\to -\infty} \frac{2x^2+7x+5}{4x-x^2} \Rightarrow$

$\lim_{x\to -\infty} \frac{2x^2+7x+5}{4x-x^2} = \frac{2x^2}{-x^2} = -2$

Yanıt / The Answer is A

3. $\lim_{x\to -1} \frac{(x^2+x)^3\cdot(x^3+x-1)}{(1-x^2)^3\cdot(3x+2)}$

$= \lim_{x\to -1} \frac{x^3\cdot(x+1)^3\cdot(x^3+x-1)}{(1-x)^3\cdot(1+x)^3\cdot(3x+2)}$

$= \lim_{x\to -1} \frac{x^3\cdot(x^3+x-1)}{(1-x)^3\cdot(3x+2)} = \frac{(-1)\cdot(-1)}{8\cdot(-1)} = -\frac{1}{8}$

Yanıt / The Answer is A

4. $\lim_{x\to \infty} \frac{5-x^2}{x^2-x-2} = -1$

Yanıt / The Answer is C

5. $\lim_{x\to \frac{\pi}{2}} (\pi - 2x)\tan x \quad (0\cdot\infty)$

$\lim_{x\to \frac{\pi}{2}} (\pi - 2x)\tan x = \lim_{x\to \frac{\pi}{2}} \frac{\pi-2x}{\frac{1}{\tan x}}$

$= \lim_{x\to \frac{\pi}{2}} \frac{\pi-2x}{\cot x}$

$= \lim_{x\to \frac{\pi}{2}} \frac{-2}{-(1+\cot^2 x)} = 2$

Yanıt / The Answer is A

6. $\lim_{x\to a} \frac{x^3-a^3}{\sqrt{x}-\sqrt{a}} = \frac{a^3-a^3}{\sqrt{a}-\sqrt{a}} = \frac{0}{0}$

$\lim_{x\to a} \frac{x^3-a^3}{\sqrt{x}-\sqrt{a}}$

$= \lim_{x\to a} \frac{(x-a)(x^2+ax+a^2)}{\sqrt{x}-\sqrt{a}}$

$= \lim_{x\to a} \frac{(\sqrt{x}-\sqrt{a})\cdot(\sqrt{x}+\sqrt{a})\cdot(x^2+ax+a^2)}{\sqrt{x}-\sqrt{a}}$

$= (\sqrt{a}+\sqrt{a})\cdot(a^2+a^2+a^2)$

$= 2\sqrt{a}\cdot 3a^2 = 6a^2\sqrt{a}$

Yanıt / The Answer is E

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1. $\lim_{x \to \infty} \frac{3x^2 - x + 5}{x^2 + x + 6} = ?$

Yanıt / The Answer is 3

2. $\lim_{x \to \infty} \frac{4x^2 - 2x + 1}{x^2 + 3} = ?$

Yanıt / The Answer is 4

3. $\lim_{x \to -\infty} \frac{6x^2 + 4x + 3}{x^2 - x^4} = ?$

Yanıt / The Answer is 0

4. $\lim_{x \to \infty} \frac{2x^5 - 3x + 1}{x^2 + 2x} = ?$

Yanıt / The Answer is ∞

5. $\lim_{x \to -\infty} \frac{4x^5 + 4x + 1}{x^3 - 3x + 1} = ?$

Yanıt / The Answer is ∞

6. $\lim_{x \to -\infty} \sqrt{9x^2 - 4x + 5} = ?$

Yanıt / The Answer is ∞

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7. $\lim_{x \to 0} \left(\frac{\sin 7x}{2x} + \frac{\tan 5x}{3x} \right) = ?$

Yanıt / The Answer is $\frac{31}{6}$

8. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = ?$

Yanıt / The Answer is **12**

9. $\lim_{x \to 3} \sqrt{\frac{x^3 - 27}{x^2 - 9}} = ?$

Yanıt / The Answer is $\frac{3}{\sqrt{2}}$

10. $\lim_{x \to 3} \frac{e^{\ln x} - 3}{x^2 - 9} = ?$

Yanıt / The Answer is $\frac{1}{6}$

11. $\lim_{x \to \frac{\pi}{2}} \frac{\sin^2 x - 1}{\sin x - 1} = ?$

Yanıt / The Answer is **2**

12. $\lim_{x \to \infty} \left(1 + \frac{3x}{x + 5} \right)^{x^2 + 5} = ?$

Yanıt / The Answer is ∞

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13. $\lim_{x \to \infty} \left(1+\frac{2}{x^4+2x^3+1}\right)^{x-2} = ?$

Yanıt / The Answer is **1**

14. $f(x)=\left(1+\frac{3x}{x^2+2}\right)^{2x} \Rightarrow \lim_{x \to \infty} f(x) = ?$

Yanıt / The Answer is e^6

15. $f(x)=\frac{\sin 5x}{x} \Rightarrow \lim_{x \to 0} f(x) = ?$

Yanıt / The Answer is **5**

16. $\lim_{x \to 2} \frac{x^3-2x-4}{x^3-8} = ?$

Yanıt / The Answer is $\frac{5}{6}$

17. $\lim_{x \to 2} \frac{x^2+x-6}{x^2-4} = ?$

Yanıt / The Answer is $\frac{5}{4}$

18. $\lim_{x \to 2} \frac{\sqrt{x+2}-2}{x-2} = ?$

Yanıt / The Answer is $\frac{1}{4}$

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1. $\lim_{x \to a} \frac{x^3 - a^3}{x - a} = ?$

A) $3a$
B) a^2
C) $3a^2$
D) $5a^2$
E) $6a^2$

2. $\lim_{x \to \pi} \frac{\cos 2x - 1}{\sin^2 x} = ?$

A) -2
B) -1
C) 0
D) 1
E) 2

3. $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 + x - 12} = ?$

A) $\frac{1}{7}$
B) $\frac{2}{7}$
C) $\frac{3}{7}$
D) $\frac{4}{7}$
E) $\frac{5}{7}$

4. $\lim_{x \to 2} \frac{\sin(x^2 - 4)}{x^4 - 16} = ?$

A) $\frac{1}{8}$
B) $\frac{1}{10}$
C) $\frac{1}{12}$
D) $\frac{1}{14}$
E) $\frac{1}{16}$

5. $\lim_{x \to \infty} \left(\sqrt{4x^2 + 6x + 5} - 2x \right) = ?$

A) $\frac{3}{2}$
B) 1
C) $\frac{1}{2}$
D) $-\frac{1}{2}$
E) $-\frac{3}{2}$

6. $\lim_{x \to \infty} \frac{ax^3 - x^2 + 3}{(b-2)x^4 + x^3 + 3x^2} = 1 \Rightarrow a + b = ?$

A) 4
B) 3
C) 2
D) 1
E) 0

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7. $\lim_{x\to\infty} \frac{(a-2)x^3-3x^2+x-5}{(b-3)x^4+x^3-2}=3 \Rightarrow a+b=?$

A) 4

B) 5

C) 6

D) 7

E) 8

8. $\lim_{x\to\infty} \frac{(a-2)x^2-5x+1}{(b-3)x^3+2x^2-3}=4 \Rightarrow a+b=?$

A) 5

B) 8

C) 10

D) 12

E) 13

9. $\lim_{x\to\infty} \frac{4^x+3^x}{4^{x+2}+3^x}=?$

A) 1

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{9}$

E) $\frac{1}{16}$

10. $\lim_{x\to 0} \frac{\cos(a+x)-\cos a}{x}=?$

A) 0

B) 1

C) cosa

D) sina

E) –sina

11. $\lim_{x\to-\infty} \left(1+\frac{2}{x+3}\right)^{x+5}=?$

A) e

B) e^2

C) e^3

D) e^4

E) e^5

12. $\lim_{x\to\infty} (x-3)\cdot\ln\left(\frac{x+3}{x}\right)=?$

A) –3

B) 0

C) 3

D) 4

E) ∞

YANITLAR / ANSWERS

1. C	2. A	3. D	4. A	5. A	6. B	7. E	8. E
9. E	10. E	11. B	12. C				

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1. $\lim_{x \to \pi} \frac{\sin^3 x}{\cos^2 x - 1} = ?$

A) $-\frac{3}{2}$

B) $-\frac{1}{2}$

C) $-\frac{1}{4}$

D) 0

E) $\frac{1}{2}$

2. $\lim_{x \to 1} \frac{x^2 + 3x - 4}{x^2 - 5x + 6} = ?$

A) $-\frac{1}{3}$

B) $-\frac{2}{3}$

C) -1

D) $-\frac{4}{3}$

E) 0

3. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x^2 - x - 12} = ?$

A) $\frac{1}{22}$

B) $\frac{1}{24}$

C) $\frac{1}{26}$

D) $\frac{1}{28}$

E) $\frac{1}{30}$

4. $\lim_{x \to y^+} \frac{x\sqrt{y} - y\sqrt{x}}{\sqrt{x} - \sqrt{y}} = ?$

A) $-y$

B) y

C) $\sqrt{y}$

D) $y\sqrt{y}$

E) 1

5. $\lim_{a \to b} \frac{a^2 - (b+1)a + b}{a^3 - b^3} = ?$

A) b

B) $\frac{1}{b}$

C) $\frac{b-1}{3b^2}$

D) $\frac{b+1}{b^2}$

E) $\frac{b+1}{3b^2}$

6. $\lim_{x \to -\infty} \frac{\sqrt{9x^2 + 3x + 2} - 2x + 1}{5x + 3} = ?$

A) -1

B) 0

C) 1

D) 2

E) 3

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7. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{ax^2+4x+5}-3x+2}{2x+3} = -3 \Rightarrow a = ?$

A) 1
B) 2
C) 3
D) 4
E) 9

8. $\lim\limits_{x\to\infty} \left(\dfrac{3x^2+2x+9}{3x^2+2x+5}\right)^{3x^2+1} = ?$

A) e
B) e^2
C) e^3
D) e^4
E) e^5

9. $\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{4x^2} = ?$

A) 1
B) $\dfrac{1}{2}$
C) $\dfrac{1}{3}$
D) $\dfrac{1}{6}$
E) $\dfrac{1}{36}$

10. $f(x) = \dfrac{\sin 4x \cdot \tan 2x}{1-\cos 2x} \Rightarrow \lim\limits_{x\to 0} f(x) = ?$

A) 2
B) 4
C) 6
D) 8
E) 10

11. $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{3x-\pi}{\cos\frac{9x}{2}} = ?$

A) $\dfrac{1}{6}$
B) $\dfrac{1}{3}$
C) $\dfrac{2}{3}$
D) 1
E) $\dfrac{4}{3}$

12. $\lim\limits_{x\to 0} \cot x\,(\operatorname{cosec} x - \cot x) = ?$

A) 1
B) $\dfrac{1}{2}$
C) $\dfrac{1}{4}$
D) $\dfrac{1}{8}$
E) $\dfrac{3}{4}$

YANITLAR / ANSWERS

1. D	2. E	3. D	4. B	5. C	6. A	7. E	8. D
9. B	10. B	11. C	12. B				

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ÜNİTE 6

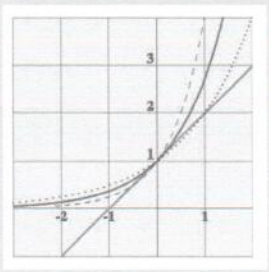
UNIT 6

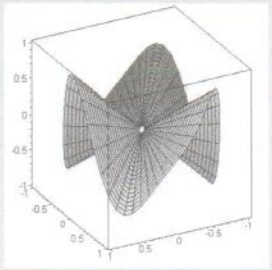
Bu ünitede, türev ayrıntılı olarak anlatılmıştır. Türev konusu karmaşık olduğundan iki bölüme ayrılmıştır. Birinci bölümde temel türev kuralları ve fonksiyonların türevi, ikinci bölümde türevin uygulamaları anlatılmıştır.

Bu ünite; 113 çözümlü örnek, 36 alıştırma ve 72 test sorusundan oluşmaktadır.

In this unit, derivative is explained in detailed. Derivative is consist of two chapters since this subject is complex. In the first chapter is consist of the basic definition of derivative and derivative of functions, in the second chapter is consist of applications of derivative.

This unit includes; 113 examples, 36 exercises, and 72 test questions.





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BÖLÜM 1
CHAPTER

TARİHSEL NOT / HISTORICAL NOTE

Fisher, Ronald Aylmer [1890 – 1962]

Ingiliz genetikçi ve istatistikçi. Deney tasarım yöntemlerinin kurulması ile uğraşmıştır. t testi ve uyumluluk tablolarını geliştirilmesi ile uğraşmış ve varyans analizi olarak bilinen metoda katkı sağlamıştır.

British geneticist and statistician who established methods of designing experiments. He developed the t-test and the use of contingency tables, and is responsible for the method known as analysis of variance.

TÜREVİN TANIMI ve FONKSİYONLARIN TÜREVİ
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TÜREVİN TANIMI ve FONKSİYONLARIN TÜREVİ
DEFINITION of DERIVATIVE and DERIVATIVE of FUNCTIONS

BÖLÜM 1
CHAPTER

Tanım: $f : A \to R$, $y = f(x)$ ve $x_0 \in A$ da sürekli olmak üzere,

$\lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$ limitine f(x) in x_0 noktasındaki türevi denir.

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

f(x) in x_0 noktasındaki türevi, $f'(x_0)$, $\dfrac{df(x_0)}{dx}$ simgelerinden biri ile gösterilir.

NOT

$x - x_0 = h \Rightarrow x \to x_0 \Leftrightarrow h \to 0$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

NOT

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

ÖRNEK 1

$f(x) = (x^2 - 3)^2 \Rightarrow \lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1} = ?$

A) –8 B) –4 C) 1 D) 3 E) 6

ÇÖZÜM

Yanıt A

$$\lim_{x \to 1} \frac{x^4 - 6x^2 + 9 - 4}{x - 1} = \lim_{x \to 1} \frac{x^4 - 6x^2 + 5}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x^2 - 5) \cdot (x^2 - 1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x^2 - 5)(x - 1)(x + 1)}{x - 1}$$

$$= \lim_{x \to 1} (x^2 - 5)(x + 1)$$

$$= (1 - 5) \cdot (1 + 1)$$

$$= -4 \cdot 2 = -8$$

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NOTE

$x - x_0 = h \Rightarrow x \to x_0 \Leftrightarrow h \to 0$

$f'(x_0) = \lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$

Definition: Let $f: A \to R$, $y = f(x)$. For any $x_0 \in A$

if $\lim_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$ exists, this limit is called the derivative of f at the point x_0.

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

The derivative of a function f at a point x_0 is denoted by $f'(x_0)$, $\dfrac{df(x_0)}{dx}$.

NOTE

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE 1

$f(x) = (x^2 - 3)^2 \Rightarrow \lim_{x \to 1} \dfrac{f(x) - f(1)}{x - 1} = ?$

A) –8 B) –4 C) 1 D) 3 E) 6

SOLUTION

The answer is A

$$\lim_{x \to 1} \frac{x^4 - 6x^2 + 9 - 4}{x - 1} = \lim_{x \to 1} \frac{x^4 - 6x^2 + 5}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x^2 - 5) \cdot (x^2 - 1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x^2 - 5)(x - 1)(x + 1)}{x - 1}$$

$$= \lim_{x \to 1} (x^2 - 5)(x + 1)$$

$$= (1 - 5) \cdot (1 + 1)$$

$$= -4 \cdot 2 = -8$$

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TÜREV ALMA KURALLARI

1. $c \in R$

$f(x) = c \Rightarrow f'(x) = 0$

Etkinlikler

a. $y = f(x)$, $f(x) = t \Rightarrow f'(x) = 0$

b. $y = f(x)$, $f(x) = 6 \Rightarrow f'(x) = 0$

c. $y = f(x)$, $f(x) = \frac{1}{3} \Rightarrow f'(x) = 0$

2. $n \in R$

$f(x) = x^n \Rightarrow f'(x) = n \cdot x^{n-1}$

Etkinlikler

a. $f(x) = x^2 \Rightarrow f'(x) = 2 \cdot x^{2-1} = 2x$

b. $f(x) = x^3 \Rightarrow f'(x) = 3 \cdot x^{3-1} = 3x^2$

3. $a, n \in R$

$f(x) = a \cdot x^n \Rightarrow f'(x) = a \cdot n \cdot x^{n-1}$

Etkinlikler

a. $f(x) = 2x^2 \Rightarrow f'(x) = 2 \cdot 2 \cdot x^{2-1} = 4x$

b. $f(x) = -3x^3 \Rightarrow f'(x) = -3 \cdot 3 \cdot x^{3-1} = -9x^2$

c. $f(x) = \frac{1}{4} x^{-4} \Rightarrow f'(x) = \frac{1}{4} \cdot (-4) \cdot x^{-4-1} = -x^{-5}$

4. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

ÖRNEK 2

$$f(x) = \begin{cases} x^3 - 2x & x \le 1 \\ 4x^2 + 5 & x > 1 \end{cases}$$

$\Rightarrow f'(-1) + f'(2) = ?$

A) 11 B) 15 C) 17 D) 19 E) 23

ÇÖZÜM

Yanıt C

$-1 \le 1 \Rightarrow f'(x) = 3x^2 - 2$

$f'(-1) = 3 \cdot (-1)^2 - 2 = 1$

$2 > 1 \Rightarrow f'(x) = 8x$

$f'(2) = 8 \cdot 2 = 16$

$\Rightarrow f'(-1) + f'(2) = 1 + 16 = 17$

PEKİŞTİRME

$f(x) = 3x^2 + 5x - 4 \Rightarrow f'(1) = ?$

YANIT : 11

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RULES for TAKING DERIVATIVE

1. $c \in R$

$f(x) = c \Rightarrow f'(x) = 0$

Activities

a. $y = f(x)$, $f(x) = t \Rightarrow f'(x) = 0$

b. $y = f(x)$, $f(x) = 6 \Rightarrow f'(x) = 0$

c. $y = f(x)$, $f(x) = \frac{1}{3} \Rightarrow f'(x) = 0$

2. $n \in R$

$f(x) = x^n \Rightarrow f'(x) = n \cdot x^{n-1}$

Activities

a. $f(x) = x^2 \Rightarrow f'(x) = 2 \cdot x^{2-1} = 2x$

b. $f(x) = x^3 \Rightarrow f'(x) = 3 \cdot x^{3-1} = 3x^2$

3. $a, n \in R$

$f(x) = a \cdot x^n \Rightarrow f'(x) = a \cdot n \cdot x^{n-1}$

Activities

a. $f(x) = 2x^2 \Rightarrow f'(x) = 2 \cdot 2 \cdot x^{2-1} = 4x$

b. $f(x) = -3x^3 \Rightarrow f'(x) = -3 \cdot 3 \cdot x^{3-1} = -9x^2$

c. $f(x) = \frac{1}{4} x^{-4} \Rightarrow f'(x) = \frac{1}{4} \cdot (-4) \cdot x^{-4-1} = -x^{-5}$

4. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

REINFORCE

$f(x) = 3x^2 + 5x - 4 \Rightarrow f'(1) = ?$

EXAMPLE 2

$$f(x) = \begin{cases} x^3 - 2x & x \le 1 \\ 4x^2 + 5 & x > 1 \end{cases}$$

$\Rightarrow$ **$f'(-1) + f'(2) = ?$**

A) 11 B) 15 C) 17 D) 19 E) 23

ANSWER : 11

SOLUTION

The answer is **C**

$-1 \le 1 \Rightarrow f'(x) = 3x^2 - 2$

$f'(-1) = 3 \cdot (-1)^2 - 2 = 1$

$2 > 1 \Rightarrow f'(x) = 8x$

$f'(2) = 8 \cdot 2 = 16$

$\Rightarrow f'(-1) + f'(2) = 1 + 16 = 17$

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5. $(f(x)\cdot g(x))' = f'(x)\cdot g(x) + g'(x)\cdot f(x)$

ÖRNEK 3

$f(x) = (x^3 - 2x + 3)\cdot(x^2 - 3x) \Rightarrow$ **f'(0) = ?**

A) –10 B) –9 C) –8 D) –7 E) –6

ÇÖZÜM

Yanıt **B**

$f(x) = (x^3 - 2x + 3)\cdot(x^2 - 3x)$

$\Rightarrow f'(x) = (3x^2 - 2)(x^2 - 3x) + (x^3 - 2x + 3)(2x - 3)$

$f'(0) = (3\cdot 0^2 - 2)(0^2 - 3\cdot 0) + (0^3 - 2\cdot 0 + 3)(2\cdot 0 - 3)$

$= (-2)\cdot 0 + 3\cdot(-3) = -9$

6. $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{(g(x))^2}$ $[\, g(x) \neq 0 \,]$

ÖRNEK 4

$f(x) = \frac{x^2 - 2x}{3x+1} \Rightarrow$ **f'(1) = ?**

A) $-\frac{3}{16}$ B) $-\frac{1}{16}$ C) $\frac{1}{16}$ D) $\frac{3}{16}$ E) $\frac{5}{16}$

ÇÖZÜM

Yanıt **D**

$f(x) = \frac{x^2 - 2x}{3x+1} \Rightarrow f'(x) = \frac{(x^2 - 2x)'\cdot(3x+1) - (x^2 - 2x)\cdot(3x+1)'}{(3x+1)^2}$

$= \frac{(2x-2)(3x+1) - (x^2 - 2x)\cdot 3}{(3x+1)^2}$

$\Rightarrow f'(1) = \frac{(2\cdot 1 - 2)\cdot(3\cdot 1 + 1) - (1^2 - 2\cdot 1)\cdot 3}{(3\cdot 1 + 1)^2}$

$= \frac{0+3}{16} = \frac{3}{16}$

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5. $(f(x)\cdot g(x))' = f'(x)\cdot g(x) + g'(x)\cdot f(x)$

EXAMPLE 3

$f(x) = (x^3 - 2x + 3)\cdot(x^2 - 3x) \Rightarrow$ **f'(0) = ?**

A) –10 B) –9 C) –8 D) –7 E) –6

SOLUTION

The answer is **B**

$f(x) = (x^3 - 2x + 3)\cdot(x^2 - 3x)$

$\Rightarrow f'(x) = (3x^2 - 2)(x^2 - 3x) + (x^3 - 2x + 3)(2x - 3)$

$f'(0) = (3\cdot 0^2 - 2)(0^2 - 3\cdot 0) + (0^3 - 2\cdot 0 + 3)(2\cdot 0 - 3)$

$= (-2)\cdot 0 + 3\cdot(-3) = -9$

6. $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{(g(x))^2}$ $[\, g(x) \neq 0 \,]$

EXAMPLE 4

$f(x) = \frac{x^2 - 2x}{3x+1} \Rightarrow$ **f'(1) = ?**

A) $-\frac{3}{16}$ B) $-\frac{1}{16}$ C) $\frac{1}{16}$ D) $\frac{3}{16}$ E) $\frac{5}{16}$

SOLUTION

The answer is **D**

$f(x) = \frac{x^2 - 2x}{3x+1} \Rightarrow f'(x) = \frac{(x^2 - 2x)'\cdot(3x+1) - (x^2 - 2x)\cdot(3x+1)'}{(3x+1)^2}$

$= \frac{(2x-2)(3x+1) - (x^2 - 2x)\cdot 3}{(3x+1)^2}$

$\Rightarrow f'(1) = \frac{(2\cdot 1 - 2)\cdot(3\cdot 1 + 1) - (1^2 - 2\cdot 1)\cdot 3}{(3\cdot 1 + 1)^2}$

$= \frac{0+3}{16} = \frac{3}{16}$

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7. $(f^m(x))' = m \cdot f^{m-1}(x) \cdot f'(x)$

ÖRNEK 5

$f(x) = \left(\frac{2x}{x+3}\right)^3 \Rightarrow f'(1) = ?$

A) $\frac{7}{32}$ B) $\frac{9}{32}$ C) $\frac{11}{32}$ D) $\frac{15}{32}$ E) $\frac{21}{32}$

ÇÖZÜM

Yanıt B

$f(x) = \left(\frac{2x}{x+3}\right)^3 \Rightarrow f'(x) = 3\cdot\left(\frac{2x}{x+3}\right)^2 \cdot \left(\frac{2x}{x+3}\right)'$

$= 3\cdot\left(\frac{2x}{x+3}\right)^2 \cdot \left(\frac{2(x+3)-2x\cdot 1}{(x+3)^2}\right)$

$f'(1) = 3\cdot\left(\frac{2\cdot 1}{1+3}\right)^2 \cdot \left(\frac{2\cdot 4-2\cdot 1}{(1+3)^2}\right)$

$= \frac{3}{4}\cdot\frac{6}{16} = \frac{9}{32}$

8. $f(x) = \sqrt{g(x)} \Rightarrow f'(x) = \frac{g'(x)}{2\cdot\sqrt{g(x)}}$

$f(x) = \sqrt[m]{g^n(x)} \Rightarrow f'(x) = \frac{n\cdot g'(x)}{m\cdot\sqrt[m]{g(x)^{m-n}}}$

NOT

$f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

ÖRNEK 6

$f(x) = \sqrt{x^2+8x} \Rightarrow f'(1) = ?$

A) $\frac{7}{3}$ B) $\frac{5}{3}$ C) 1 D) $\frac{1}{3}$ E) 0

ÇÖZÜM

Yanıt B

$f(x) = \sqrt{x^2+8x} \Rightarrow f'(x) = \frac{(x^2+8x)'}{2\sqrt{x^2+8x}}$

$= \frac{2x+8}{2\sqrt{x^2+8x}}$

$\Rightarrow f'(1) = \frac{2+8}{2\sqrt{1+8}} = \frac{10}{6} = \frac{5}{3}$

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7. $(f^m(x))' = m \cdot f^{m-1}(x) \cdot f'(x)$

EXAMPLE 5

$f(x) = \left(\frac{2x}{x+3}\right)^3 \Rightarrow f'(1) = ?$

A) $\frac{7}{32}$ B) $\frac{9}{32}$ C) $\frac{11}{32}$ D) $\frac{15}{32}$ E) $\frac{21}{32}$

SOLUTION

The answer is B

$f(x) = \left(\frac{2x}{x+3}\right)^3 \Rightarrow f'(x) = 3\left(\frac{2x}{x+3}\right)^2 \cdot \left(\frac{2x}{x+3}\right)'$

$= 3 \cdot \left(\frac{2x}{x+3}\right)^2 \cdot \left(\frac{2(x+3)-2x\cdot 1}{(x+3)^2}\right)$

$f'(1) = 3 \cdot \left(\frac{2\cdot 1}{1+3}\right)^2 \cdot \left(\frac{2\cdot 4 - 2\cdot 1}{(1+3)^2}\right)$

$= \frac{3}{4} \cdot \frac{6}{16} = \frac{9}{32}$

NOTE

$f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

8. $f(x) = \sqrt{g(x)} \Rightarrow f'(x) = \frac{g'(x)}{2\cdot\sqrt{g(x)}}$

$f(x) = \sqrt[m]{g^n(x)} \Rightarrow f'(x) = \frac{n\cdot g'(x)}{m\cdot\sqrt[m]{g(x)^{m-n}}}$

EXAMPLE 6

$f(x) = \sqrt{x^2+8x} \Rightarrow f'(1) = ?$

A) $\frac{7}{3}$ B) $\frac{5}{3}$ C) 1 D) $\frac{1}{3}$ E) 0

SOLUTION

The answer is B

$f(x) = \sqrt{x^2+8x} \Rightarrow f'(x) = \frac{(x^2+8x)'}{2\sqrt{x^2+8x}}$

$= \frac{2x+8}{2\sqrt{x^2+8x}}$

$\Rightarrow f'(1) = \frac{2+8}{2\sqrt{1+8}} = \frac{10}{6} = \frac{5}{3}$

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9. $m, n \in R$,

$f(x) = g(mx+n)$

$\Rightarrow f'(x) = (mx+n)' \cdot g'(mx+n) = mg'(mx+n)$

ÖRNEK 7

$f(x^3 - 2x) = 4x^5 - 8 \Rightarrow$ **f'(–1) = ?**

A) 12 B) 15 C) 18 D) 20 E) 25

ÇÖZÜM

Yanıt **D**

$f(x^3 - 2x) = 4x^5 - 8$

$f'(x^3 - 2x) \cdot (x^3 - 2x)' = 20x^4$

$x^3 - 2x = -1 \Rightarrow x = 1$

$f'(x^3 - 2x) = \dfrac{20x^4}{3x^2 - 2} \Rightarrow f'(-1) = \dfrac{20 \cdot 1^4}{3 \cdot 1^2 - 2} = \dfrac{20}{1} = 20$

10. $f(x) = |g(x)|$

$$f'(x) = \begin{cases} g'(x) & , \quad g(x) \geq 0 \\ -g'(x) & , \quad g(x) < 0 \end{cases}$$

ÖRNEK 8

$f(x) = |x^2 - 2x| + |3x - 4| + x^3 \Rightarrow$ **f'(–2) = ?**

A) –5 B) –1 C) 3 D) 5 E) 7

ÇÖZÜM

Yanıt **C**

$x = -2 \Rightarrow x^2 - 2x = 8 > 0$

$x = -2 \Rightarrow 3x - 4 = -10 < 0$

$f(x) = x^2 - 2x - 3x + 4 + x^3 \Rightarrow f'(x) = 2x - 5 + 3x^2$

$f'(-2) = -4 - 5 + 12 = 3$

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9. $m, n \in R$,

$f(x) = g(mx+n)$

$\Rightarrow f'(x) = (mx+n)' \cdot g'(mx+n) = mg'(mx+n)$

EXAMPLE 7

$f(x^3 - 2x) = 4x^5 - 8 \Rightarrow$ **$f'(-1) = ?$**

A) 12 B) 15 C) 18 D) 20 E) 25

SOLUTION

The answer is **D**

$f(x^3 - 2x) = 4x^5 - 8$

$f'(x^3 - 2x) \cdot (x^3 - 2x)' = 20x^4$

$x^3 - 2x = -1 \Rightarrow x = 1$

$f'(x^3 - 2x) = \dfrac{20x^4}{3x^2 - 2} \Rightarrow f'(-1) = \dfrac{20 \cdot 1^4}{3 \cdot 1^2 - 2} = \dfrac{20}{1} = 20$

10. $f(x) = |g(x)|$

$$f'(x) = \begin{cases} g'(x) & , \quad g(x) \geq 0 \\ -g'(x) & , \quad g(x) < 0 \end{cases}$$

EXAMPLE 8

$f(x) = |x^2 - 2x| + |3x - 4| + x^3 \Rightarrow$ **$f'(-2) = ?$**

A) –5 B) –1 C) 3 D) 5 E) 7

SOLUTION

The answer is **C**

$x = -2 \Rightarrow x^2 - 2x = 8 > 0$

$x = -2 \Rightarrow 3x - 4 = -10 < 0$

$f(x) = x^2 - 2x - 3x + 4 + x^3 \Rightarrow f'(x) = 2x - 5 + 3x^2$

$f'(-2) = -4 - 5 + 12 = 3$

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ÖRNEK 9

$f(x) = 5x^3 - 2x^2 + x + 4 \Rightarrow \lim\limits_{x \to -1} \dfrac{f(x) - f(-1)}{x+1} = ?$

A) 18 B) 20 C) 22 D) 24 E) 26

ÇÖZÜM

Yanıt **B**

$\lim\limits_{x \to -1} \dfrac{f(x) - f(-1)}{x+1} = f'(-1)$

$f'(x) = 15x^2 - 4x + 1 \Rightarrow f'(-1) = 15 + 4 + 1 = 20$

ÖRNEK 10

$f(x) = 8x^2 - 5x + 4 \Rightarrow \lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{3h} = ?$

A) 5 B) 6 C) 7 D) 9 E) 27

ÇÖZÜM

Yanıt **D**

$\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{3h} = \dfrac{f'(2)}{3}$

$f'(x) = 16x - 5 \Rightarrow f'(2) = 32 - 5 = 27$

$\Rightarrow \dfrac{f'(2)}{3} = \dfrac{27}{3} = 9$

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EXAMPLE 9

$f(x) = 5x^3 - 2x^2 + x + 4 \Rightarrow \lim_{x \to -1} \frac{f(x) - f(-1)}{x+1} = ?$

A) 18 B) 20 C) 22 D) 24 E) 26

SOLUTION

The answer is B

$\lim_{x \to -1} \frac{f(x) - f(-1)}{x+1} = f'(-1)$

$f'(x) = 15x^2 - 4x + 1 \Rightarrow f'(-1) = 15 + 4 + 1 = 20$

EXAMPLE 10

$f(x) = 8x^2 - 5x + 4 \Rightarrow \lim_{h \to 0} \frac{f(2+h) - f(2)}{3h} = ?$

A) 5 B) 6 C) 7 D) 9 E) 27

SOLUTION

The answer is D

$\lim_{h \to 0} \frac{f(2+h) - f(2)}{3h} = \frac{f'(2)}{3}$

$f'(x) = 16x - 5 \Rightarrow f'(2) = 32 - 5 = 27$

$\Rightarrow \frac{f'(2)}{3} = \frac{27}{3} = 9$

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BİLEŞKE FONKSİYONUN TÜREVİ (Zincir Kuralı)

$$\left.\begin{matrix} y = f(t) \\ t = g(x) \end{matrix}\right\} \Rightarrow y = f(g(x)) = (fog)(x)$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = f'(t) \cdot g'(x) \Rightarrow (fog)'(x) = f'(g(x)) \cdot g'(x)$$

ÖRNEK 11

$f(x) = 3x^2 + 2x$,

$g(x) = 3x - 4 \Rightarrow$ **(fog)'(–1) = ?**

A) –40 B) –60 C) –80 D) –100 E) –120

ÇÖZÜM

Yanıt **E**

$(fog)'(x) = f'(g(x)) \cdot g'(x)$

$f(x) = 3x^2 + 2x \Rightarrow f'(x) = 6x + 2$

$f'(g(x)) = 6(3x - 4) + 2 = 18x - 22$

$g(x) = 3x - 4 \Rightarrow g'(x) = 3$

$f'(g(-1)) \cdot g'(-1) = (18(-1) - 22) \cdot 3$

$= -40 \cdot 3 = -120$

ÖRNEK 12

$f(x) = 2x^2 - x$,

$g(x) = 1 - x^2 \Rightarrow$ **(gof)'(2) = ?**

A) –102 B) –84 C) –64 D) –48 E) –32

ÇÖZÜM

Yanıt **B**

$(gof)'(x) = g'(f(x)) \cdot f'(x)$

$= -2(2x^2 - x) \cdot (4x - 1)$

$\Rightarrow (gof)'(2) = -2(2 \cdot 2^2 - 2) \cdot (4 \cdot 2 - 1)$

$= -2(8 - 2) \cdot (8 - 1)$

$= -2(6) \cdot (7)$

$= -84$

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DERIVATIVE of a COMPOSITE FUNCTION (The Chain Rule)

$$\left.\begin{matrix} y = f(t) \\ t = g(x) \end{matrix}\right\} \Rightarrow y = f(g(x)) = (fog)(x)$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = f'(t) \cdot g'(x) \Rightarrow (fog)'(x) = f'(g(x)) \cdot g'(x)$$

EXAMPLE 11

$f(x) = 3x^2 + 2x$,

$g(x) = 3x - 4 \Rightarrow$ **(fog)'(–1) = ?**

A) –40 B) –60 C) –80 D) –100 E) –120

SOLUTION

The answer is **E**

$(fog)'(x) = f'(g(x)) \cdot g'(x)$

$f(x) = 3x^2 + 2x \Rightarrow f'(x) = 6x + 2$

$f'(g(x)) = 6(3x - 4) + 2 = 18x - 22$

$g(x) = 3x - 4 \Rightarrow g'(x) = 3$

$f'(g(-1)) \cdot g'(-1) = (18(-1) - 22) \cdot 3$

$= -40 \cdot 3 = -120$

EXAMPLE 12

$f(x) = 2x^2 - x$,

$g(x) = 1 - x^2 \Rightarrow$ **(gof)'(2) = ?**

A) –102 B) –84 C) –64 D) –48 E) –32

SOLUTION

The answer is **B**

$(gof)'(x) = g'(f(x)) \cdot f'(x)$

$= -2(2x^2 - x) \cdot (4x - 1)$

$\Rightarrow (gof)'(2) = -2(2 \cdot 2^2 - 2) \cdot (4 \cdot 2 - 1)$

$= -2(8 - 2) \cdot (8 - 1)$

$= -2(6) \cdot (7)$

$= -84$

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TRİGONOMETRİK FONKSİYONLARIN TÜREVİ

A. Sinüs Fonksiyonunun Türevi

1. $f(x) = \sin x \Rightarrow f'(x) = \cos x$
2. $f(x) = \sin(u(x)) \Rightarrow f'(x) = u'(x) \cdot \cos(u(x))$
3. $f(x) = \sin^n x \Rightarrow f'(x) = n \cdot \sin^{n-1} x \cdot \cos x$
4. $f(x) = \sin^n(u(x)) \Rightarrow f'(x) = n \cdot \sin^{n-1}(u(x)) \cdot \cos(u(x)) \cdot u'(x)$

ÖRNEK 13

$$\frac{1}{2 \cdot \cos 2x} \cdot \frac{d}{dx}(\sin^2 x) = ?$$

A) $\frac{\cot 2x}{\tan x}$ B) $\frac{\cot x}{2}$ C) $\frac{\cot 2x}{2}$ D) $\frac{\tan 2x}{2}$ E) $\frac{\tan x}{2}$

ÇÖZÜM

Yanıt **D**

$$\frac{d}{dx}(\sin^2 x) = 2\sin x \cdot (\sin x)' = 2 \sin x \cos x = \sin 2x$$

$$\frac{1}{2\cos 2x} \frac{d}{dx}(\sin^2 2x) = \frac{1}{2\cos 2x} \cdot \sin 2x = \frac{\tan 2x}{2}$$

ÖRNEK 14

$$f(x) = \frac{3 - \sin x^3}{1 + \sin x} \Rightarrow f'(0) = ?$$

A) –3 B) –2 C) –1 D) 0 E) 1

ÇÖZÜM

Yanıt **A**

$$f(x) = \frac{3 - \sin x^3}{1 + \sin x} \Rightarrow f'(x) = \frac{-\cos x^3 \cdot 3x^2 (1 + \sin x) - (3 - \sin x^3) \cdot \cos x}{(1 + \sin x)^2}$$

$$f'(0) = \frac{0 - 3 \cdot \cos 0^\circ}{1^2} = -3$$

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DERIVATIVE of TRIGONOMETRIC FUNCTIONS

A. The Derivative of The Sine Function

1. $f(x) = \sin x \Rightarrow f'(x) = \cos x$
2. $f(x) = \sin(u(x)) \Rightarrow f'(x) = u'(x) \cdot \cos(u(x))$
3. $f(x) = \sin^n x \Rightarrow f'(x) = n \cdot \sin^{n-1} x \cdot \cos x$
4. $f(x) = \sin^n(u(x)) \Rightarrow f'(x) = n \cdot \sin^{n-1}(u(x)) \cdot \cos(u(x)) \cdot u'(x)$

EXAMPLE 13

$$\frac{1}{2 \cdot \cos 2x} \cdot \frac{d}{dx}(\sin^2 x) = ?$$

A) $\frac{\cot 2x}{\tan x}$ B) $\frac{\cot x}{2}$ C) $\frac{\cot 2x}{2}$ D) $\frac{\tan 2x}{2}$ E) $\frac{\tan x}{2}$

SOLUTION

The answer is D

$$\frac{d}{dx}(\sin^2 x) = 2\sin x \cdot (\sin x)' = 2 \sin x \cos x = \sin 2x$$

$$\frac{1}{2\cos 2x} \frac{d}{dx}(\sin^2 2x) = \frac{1}{2\cos 2x} \cdot \sin 2x = \frac{\tan 2x}{2}$$

EXAMPLE 14

$$f(x) = \frac{3 - \sin x^3}{1 + \sin x} \Rightarrow f'(0) = ?$$

A) –3 B) –2 C) –1 D) 0 E) 1

SOLUTION

The answer is A

$$f(x) = \frac{3 - \sin x^3}{1 + \sin x} \Rightarrow f'(x) = \frac{-\cos x^3 \cdot 3x^2 (1 + \sin x) - (3 - \sin x^3) \cdot \cos x}{(1 + \sin x)^2}$$

$$f'(0) = \frac{0 - 3 \cdot \cos 0^\circ}{1^2} = -3$$

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B. Kosinüs Fonksiyonunun Türevi

1. $f(x) = \cos x \Rightarrow f'(x) = -\sin x$
2. $f(x) = \cos(u(x)) \Rightarrow f'(x) = -u'(x) \cdot \sin(u(x))$
3. $f(x) = \cos^n x \Rightarrow f'(x) = -n \cdot \cos^{n-1} x \cdot \sin x$
4. $f(x) = \cos^n(u(x)) \Rightarrow f'(x) = -n \cdot \cos^{n-1}(u(x)) \cdot \sin(u(x)) \cdot u'(x)$

ÖRNEK 15

$f(x) = \cos^4 3x \Rightarrow$ **f'(0) = ?**

A) 1 B) 0 C) –1 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

ÇÖZÜM

Yanıt **B**

$f'(x) = 4 \cdot \cos^3 3x \cdot (-\sin 3x) \cdot 3$

$f'(0) = 4 \cdot \cos^3 0 \cdot (-\sin 0) \cdot 3$

$= 0$

ÖRNEK 16

$f(x) = \cos^5(x^2 + 3) \Rightarrow$ **f'(x) = ?**

A) $-5\cos^4(x^2 + 3)$

B) $-5\cos^4 2x$

C) $-5\sin^4 4x$

D) $-5\cos^4(x^2 + 3) \cdot \sin(x^2 + 3)2x$

E) $-5\sin^4(x^2 + 3) \cdot 2x$

ÇÖZÜM

Yanıt **D**

$f(x) = \cos^5(x^2 + 3)$

$\Rightarrow f'(x) = 5\cos^4(x^2 + 3)\,(-\sin(x^2 + 3)) \cdot 2x$

$= -5\cos^4(x^2 + 3) \cdot \sin(x^2 + 3)2x$

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B. The Derivative of The Cosine Function

1. $f(x) = \cos x \Rightarrow f'(x) = -\sin x$
2. $f(x) = \cos(u(x)) \Rightarrow f'(x) = -u'(x) \cdot \sin(u(x))$
3. $f(x) = \cos^n x \Rightarrow f'(x) = -n \cdot \cos^{n-1} x \cdot \sin x$
4. $f(x) = \cos^n(u(x)) \Rightarrow f'(x) = -n \cdot \cos^{n-1}(u(x)) \cdot \sin(u(x)) \cdot u'(x)$

EXAMPLE 15

$f(x) = \cos^4 3x \Rightarrow$ **$f'(0) = ?$**

A) 1 B) 0 C) -1 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

SOLUTION

The answer is **B**

$f'(x) = 4 \cdot \cos^3 3x \cdot (-\sin 3x) \cdot 3$

$f'(0) = 4 \cdot \cos^3 0 \cdot (-\sin 0) \cdot 3$

$= 0$

EXAMPLE 16

$f(x) = \cos^5(x^2 + 3) \Rightarrow$ **$f'(x) = ?$**

A) $-5\cos^4(x^2 + 3)$

B) $-5\cos^4 2x$

C) $-5\sin^4 4x$

D) $-5\cos^4(x^2 + 3) \cdot \sin(x^2 + 3)2x$

E) $-5\sin^4(x^2 + 3) \cdot 2x$

SOLUTION

The answer is **D**

$f(x) = \cos^5(x^2 + 3)$

$\Rightarrow f'(x) = 5\cos^4(x^2 + 3)\,(-\sin(x^2 + 3)) \cdot 2x$

$= -5\cos^4(x^2 + 3) \cdot \sin(x^2 + 3)2x$

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C. Tanjant Fonksiyonunun Türevi

1. $f(x) = \tan x \Rightarrow f'(x) = 1 + \tan^2 x = \frac{1}{\cos^2 x} = \sec^2 x$
2. $f(x) = \tan(u(x)) \Rightarrow f'(x) = u'(x) \cdot (1 + \tan^2(u(x)))$
3. $f(x) = \tan^n x \Rightarrow f'(x) = n \cdot \tan^{n-1} x \cdot (1 + \tan^2 x)$
4. $f(x) = \tan^n(u(x)) \Rightarrow f'(x) = n \cdot \tan^{n-1}(u(x)) \cdot (1 + \tan^2(u(x))) \cdot u'(x)$

ÖRNEK 17

$f(x) = \sin(\tan x) \Rightarrow f'(\pi) = ?$

A) -1 B) 0 C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

ÇÖZÜM

Yanıt **D**

$f'(x) = \cos(\tan x) \cdot (1 + \tan^2 x)$

$\Rightarrow f'(\pi) = \cos(\tan\pi)(1 + \tan^2\pi)$

$= \cos 0° \ (1 + 0)$

$= 1$

ÖRNEK 18

$f(x) = \tan^3(x^2) \Rightarrow f'\left(\sqrt{\frac{\pi}{4}}\right) = ?$

A) $\sqrt{\pi}$ B) $2\sqrt{\pi}$ C) $3\sqrt{\pi}$ D) $4\sqrt{\pi}$ E) $6\sqrt{\pi}$

ÇÖZÜM

Yanıt **E**

$f(x) = \tan^3(x^2) \Rightarrow f'(x) = 3\tan^2(x^2) \cdot (1 + \tan^2(x^2)) \cdot 2x$

$\Rightarrow f'\left(\sqrt{\frac{\pi}{4}}\right) = 6 \cdot \frac{\sqrt{\pi}}{2} \cdot 1 \cdot (1 + 1) = 6\sqrt{\pi}$

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C. The Derivative of The Tangent Function

1. $f(x) = \tan x \Rightarrow f'(x) = 1 + \tan^2 x = \dfrac{1}{\cos^2 x} = \sec^2 x$
2. $f(x) = \tan(u(x)) \Rightarrow f'(x) = u'(x) \cdot (1 + \tan^2(u(x)))$
3. $f(x) = \tan^n x \Rightarrow f'(x) = n \cdot \tan^{n-1} x \cdot (1 + \tan^2 x)$
4. $f(x) = \tan^n(u(x)) \Rightarrow f'(x) = n \cdot \tan^{n-1}(u(x)) \cdot (1 + \tan^2(u(x))) \cdot u'(x)$

EXAMPLE 17

$f(x) = \sin(\tan x) \Rightarrow f'(\pi) = ?$

A) -1 B) 0 C) $\dfrac{1}{2}$ D) 1 E) $\dfrac{3}{2}$

SOLUTION

The answer is D

$f'(x) = \cos(\tan x) \cdot (1 + \tan^2 x)$

$\Rightarrow f'(\pi) = \cos(\tan\pi)(1 + \tan^2\pi)$

$= \cos 0° \ (1 + 0)$

$= 1$

EXAMPLE 18

$f(x) = \tan^3(x^2) \Rightarrow f'\left(\sqrt{\dfrac{\pi}{4}}\right) = ?$

A) $\sqrt{\pi}$ B) $2\sqrt{\pi}$ C) $3\sqrt{\pi}$ D) $4\sqrt{\pi}$ E) $6\sqrt{\pi}$

SOLUTION

The answer is E

$f(x) = \tan^3(x^2) \Rightarrow f'(x) = 3\tan^2(x^2) \cdot (1 + \tan^2(x^2)) \cdot 2x$

$\Rightarrow f'\left(\sqrt{\dfrac{\pi}{4}}\right) = 6 \cdot \dfrac{\sqrt{\pi}}{2} \cdot 1 \cdot (1 + 1) = 6\sqrt{\pi}$

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D. Kotanjant Fonksiyonunun Türevi

1. $f(x) = \cot x \Rightarrow f'(x) = -(1+\cot^2 x) = -\frac{1}{\sin^2 x} = -\text{cosec}^2 x$
2. $f(x) = \cot(u(x)) \Rightarrow f'(x) = -u'(x)\cdot(1+\cot^2(u(x)))$
3. $f(x) = \cot^n x \Rightarrow f'(x) = -n\cdot\cot^{n-1}x\cdot(1+\cot^2 x)$
4. $f(x) = \cot^n(u(x)) \Rightarrow f'(x) = -n\cdot\cot^{n-1}(u(x))\cdot(1+\cot^2(u(x)))\cdot u'(x)$

Düşünün

$3\cdot f(x)+f\left(\frac{\pi}{2}-x\right)=\cot x \Rightarrow f'\left(\frac{\pi}{4}\right)=?$

ÖRNEK 19

$f(x)=3\cot^2 x \Rightarrow f'\left(\frac{\pi}{4}\right)=?$

A) -12 B) -6 C) $-3\sqrt{2}$ D) $6\sqrt{2}$ E) 12

ÇÖZÜM

Yanıt **A**

$f(x) = 3\cot^2 x$

$f'(x) = -3\cdot 2\cot x\,(1+\cot^2 x)$

$f'\left(\frac{\pi}{4}\right) = -6\cdot 1\cdot(1+1) = -12$

ÖRNEK 20

$f(x)=\cot(3x-1) \Rightarrow f'\left(\frac{\pi+4}{12}\right)=?$

A) -2 B) -4 C) -6 D) -8 E) -9

ÇÖZÜM

Yanıt **C**

$f'(x) = -(1+\cot^2(3x-1))\cdot 3$

$f'\left(\frac{\pi+4}{12}\right) = -3\left(1+\cot^2\left(\frac{\pi+4}{4}-1\right)\right)$

$= -3(1+1)$

$= -6$

Yanıt : -1

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Think about it

$3\cdot f(x)+f\left(\frac{\pi}{2}-x\right)=\cot x \Rightarrow f'\left(\frac{\pi}{4}\right)=?$

D. The Derivative of The Cotangent Function

1. $f(x)=\cot x \Rightarrow f'(x)=-(1+\cot^2 x)=-\frac{1}{\sin^2 x}=-\text{cosec}^2 x$
2. $f(x)=\cot(u(x)) \Rightarrow f'(x)=-u'(x)\cdot(1+\cot^2(u(x)))$
3. $f(x)=\cot^n x \Rightarrow f'(x)=-n\cdot\cot^{n-1}x\cdot(1+\cot^2 x)$
4. $f(x)=\cot^n(u(x)) \Rightarrow f'(x)=-n\cdot\cot^{n-1}(u(x))\cdot(1+\cot^2(u(x)))\cdot u'(x)$

EXAMPLE 19

$f(x)=3\cot^2 x \Rightarrow f'\left(\frac{\pi}{4}\right)=?$

A) -12 B) -6 C) $-3\sqrt{2}$ D) $6\sqrt{2}$ E) 12

SOLUTION

The answer is **A**

$f(x)=3\cot^2 x$

$f'(x)=-3\cdot 2\cot x\,(1+\cot^2 x)$

$f'\left(\frac{\pi}{4}\right)=-6\cdot 1\cdot(1+1)=-12$

EXAMPLE 20

$f(x)=\cot(3x-1) \Rightarrow f'\left(\frac{\pi+4}{12}\right)=?$

A) -2 B) -4 C) -6 D) -8 E) -9

SOLUTION

The answer is **C**

$f'(x)=-(1+\cot^2(3x-1))\cdot 3$

$f'\left(\frac{\pi+4}{12}\right)=-3\left(1+\cot^2\left(\frac{\pi+4}{4}-1\right)\right)$

$=-3(1+1)$

$=-6$

Answer : –1

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TERS TRİGONOMETRİK FONKSİYONLARIN TÜREVİ

1. $f(x) = \arcsin x \Rightarrow f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

   $f(x) = \arcsin(u(x)) \Rightarrow f'(x) = \dfrac{u'(x)}{\sqrt{1-(u(x))^2}}$

2. $f(x) = \arccos x \Rightarrow f'(x) = \dfrac{-1}{\sqrt{1-x^2}}$

   $f(x) = \arccos(u(x)) \Rightarrow f'(x) = \dfrac{-u'(x)}{\sqrt{1-(u(x))^2}}$

3. $f(x) = \arctan x \Rightarrow f'(x) = \dfrac{1}{1+x^2}$

   $f(x) = \arctan(u(x)) \Rightarrow f'(x) = \dfrac{u'(x)}{1+(u(x))^2}$

4. $f(x) = \operatorname{arccot} x \Rightarrow f'(x) = \dfrac{-1}{1+x^2}$

   $f(x) = \operatorname{arccot}(u(x)) \Rightarrow f'(x) = \dfrac{-u'(x)}{1+(u(x))^2}$

ÖRNEK 21

$f(x) = \arctan(x^2 + 2x) - \arccos(2x) \Rightarrow$ **f'(0) = ?**

A) 2 B) 4 C) 6 D) 8 E) 10

ÇÖZÜM

Yanıt **A**

$$f'(x) = \frac{2x+2}{1+(x^2+2x)^2} - \frac{-2x}{\sqrt{1-(2x)^2}}$$

$$f'(x) = \frac{2(x+1)}{1+(x^2+2x)^2} + \frac{2x}{\sqrt{1-4x^2}}$$

$$f'(0) = \frac{2(0+1)}{1+0^2} + \frac{2\cdot 0}{\sqrt{1-0}} \Rightarrow f'(0) = 2$$

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DERIVATIVE of INVERSE TRIGONOMETRIC FUNCTIONS

1. $f(x) = \arcsin x \Rightarrow f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

   $f(x) = \arcsin(u(x)) \Rightarrow f'(x) = \dfrac{u'(x)}{\sqrt{1-(u(x))^2}}$

2. $f(x) = \arccos x \Rightarrow f'(x) = \dfrac{-1}{\sqrt{1-x^2}}$

   $f(x) = \arccos(u(x)) \Rightarrow f'(x) = \dfrac{-u'(x)}{\sqrt{1-(u(x))^2}}$

3. $f(x) = \arctan x \Rightarrow f'(x) = \dfrac{1}{1+x^2}$

   $f(x) = \arctan(u(x)) \Rightarrow f'(x) = \dfrac{u'(x)}{1+(u(x))^2}$

4. $f(x) = \operatorname{arccot} x \Rightarrow f'(x) = \dfrac{-1}{1+x^2}$

   $f(x) = \operatorname{arccot}(u(x)) \Rightarrow f'(x) = \dfrac{-u'(x)}{1+(u(x))^2}$

EXAMPLE 21

$f(x) = \arctan(x^2 + 2x) - \arccos(2x) \Rightarrow f'(0) = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

SOLUTION

The answer is A

$$f'(x) = \frac{2x+2}{1+(x^2+2x)^2} - \frac{-2x}{\sqrt{1-(2x)^2}}$$

$$f'(x) = \frac{2(x+1)}{1+(x^2+2x)^2} + \frac{2x}{\sqrt{1-4x^2}}$$

$$f'(0) = \frac{2(0+1)}{1+0^2} + \frac{2\cdot 0}{\sqrt{1-0}} \Rightarrow f'(0) = 2$$

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LOGARİTMİK FONKSİYONLARIN TÜREVİ

1 $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \cdot \ln a} = \frac{1}{x} \cdot \log_a e$

$f(x) = \log_a (u(x)) \Rightarrow f'(x) = \frac{u'(x)}{u(x) \cdot \ln a} = \frac{u'(x)}{u(x)} \cdot \log_a e$

ÖRNEK 22

$f(x) = \log_5 (x^2 + 4x) \Rightarrow$ **f'(2) = ?**

A) $\frac{1}{3} \cdot \log_5 e$ B) $\frac{2}{3} \cdot \log_5 e$ C) $\log_5 e$

D) $\frac{4}{3} \cdot \log_5 e$ E) $\frac{5}{3} \cdot \log_5 e$

ÇÖZÜM

Yanıt **B**

$f'(x) = \frac{2x+4}{x^2+4x} \log_5 e \Rightarrow f'(2) = \frac{8}{12} \cdot \log_5 e$

$\Rightarrow f'(2) = \frac{2}{3} \cdot \log_5 e$

2 $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

$f(x) = \ln(u(x)) \Rightarrow f'(x) = \frac{u'(x)}{u(x)}$

ÖRNEK 23

$f(x) = \ln(x^3 + 6x) \Rightarrow$ **f'(1) = ?**

A) 1 B) $\frac{8}{7}$ C) $\frac{9}{7}$ D) $\frac{11}{7}$ E) 2

ÇÖZÜM

Yanıt **C**

$f'(x) = \frac{3x^2+6}{x^3+6x} \Rightarrow f'(1) = \frac{3 \cdot 1 + 6}{1+6} = \frac{9}{7}$

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DERIVATIVE of LOGARITHMIC FUNCTIONS

1 $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \cdot \ln a} = \frac{1}{x} \cdot \log_a e$

$f(x) = \log_a (u(x)) \Rightarrow f'(x) = \frac{u'(x)}{u(x) \cdot \ln a} = \frac{u'(x)}{u(x)} \cdot \log_a e$

EXAMPLE 22

$f(x) = \log_5 (x^2 + 4x) \Rightarrow f'(2) = ?$

A) $\frac{1}{3} \cdot \log_5 e$ B) $\frac{2}{3} \cdot \log_5 e$ C) $\log_5 e$

D) $\frac{4}{3} \cdot \log_5 e$ E) $\frac{5}{3} \cdot \log_5 e$

SOLUTION

The answer is B

$f'(x) = \frac{2x+4}{x^2+4x} \log_5 e \Rightarrow f'(2) = \frac{8}{12} \cdot \log_5 e$

$\Rightarrow f'(2) = \frac{2}{3} \cdot \log_5 e$

2 $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

$f(x) = \ln(u(x)) \Rightarrow f'(x) = \frac{u'(x)}{u(x)}$

EXAMPLE 23

$f(x) = \ln(x^3 + 6x) \Rightarrow f'(1) = ?$

A) 1 B) $\frac{8}{7}$ C) $\frac{9}{7}$ D) $\frac{11}{7}$ E) 2

SOLUTION

The answer is C

$f'(x) = \frac{3x^2+6}{x^3+6x} \Rightarrow f'(1) = \frac{3 \cdot 1 + 6}{1+6} = \frac{9}{7}$

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ÜSTEL FONKSİYONLARIN TÜREVİ

1 $f(x) = e^x \Rightarrow f'(x) = e^x$

$f(x) = e^{u(x)} \Rightarrow f'(x) = u'(x) \cdot e^{u(x)}$

ÖRNEK 24

$f(x) = e^{x^2 + 3x} \Rightarrow f'(-1) = ?$

A) e B) e^{-2} C) e^3 D) $2 \cdot e^2$ E) $\frac{1}{e}$

ÇÖZÜM

Yanıt **B**

$f(x) = e^{x^2 + 3x} \Rightarrow f'(x) = (x^2 + 3x)' \cdot e^{x^2 + 3x}$

$= (2x + 3) \cdot e^{x^2 + 3x}$

$f'(-1) = (2 \cdot (-1) + 3) \cdot e^{(-1)^2 + 3 \cdot (-1)}$

$= 1 \cdot e^{-2} = e^{-2}$

2 $f(x) = a^x \Rightarrow f'(x) = a^x \cdot \ln a = \frac{1}{\log_a e} \cdot a^x$

$f(x) = a^{u(x)} \Rightarrow f'(x) = u'(x) \cdot a^{u(x)} \cdot \ln a = \frac{u'(x)}{\log_a e} a^{u(x)}$

ÖRNEK 25

$f(x) = 5^{x^2 + 4} \Rightarrow f'(x) = ?$

A) $2x \cdot 5^{x^2 + 4} \cdot \ln 5$

B) $2x \cdot 5^{x + 4} \cdot \ln 5$

C) $2x \cdot 5^{x^2 + 2} \cdot \ln 5$

D) $2x \cdot 5^{x + 4} \cdot \ln 5$

E) $2x \cdot 5^{x^2 - 2} \cdot \ln 5$

ÇÖZÜM

Yanıt **A**

$f'(x) = 2x \cdot 5^{x^2 + 4} \cdot \ln 5$

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DERIVATIVE of EXPONENTIAL FUNCTIONS

1 $f(x) = e^x \Rightarrow f'(x) = e^x$

$f(x) = e^{u(x)} \Rightarrow f'(x) = u'(x) \cdot e^{u(x)}$

EXAMPLE 24

$f(x) = e^{x^2 + 3x} \Rightarrow f'(-1) = ?$

A) e B) e^{-2} C) e^3 D) $2 \cdot e^2$ E) $\frac{1}{e}$

SOLUTION

The answer is B

$f(x) = e^{x^2 + 3x} \Rightarrow f'(x) = (x^2 + 3x)' \cdot e^{x^2 + 3x}$

$= (2x + 3) \cdot e^{x^2 + 3x}$

$f'(-1) = (2 \cdot (-1) + 3) \cdot e^{(-1)^2 + 3 \cdot (-1)}$

$= 1 \cdot e^{-2} = e^{-2}$

2 $f(x) = a^x \Rightarrow f'(x) = a^x \cdot \ln a = \frac{1}{\log_a e} \cdot a^x$

$f(x) = a^{u(x)} \Rightarrow f'(x) = u'(x) \cdot a^{u(x)} \cdot \ln a = \frac{u'(x)}{\log_a e} a^{u(x)}$

EXAMPLE 25

$f(x) = 5^{x^2 + 4} \Rightarrow f'(x) = ?$

A) $2x \cdot 5^{x^2 + 4} \cdot \ln 5$

B) $2x \cdot 5^{x + 4} \cdot \ln 5$

C) $2x \cdot 5^{x^2 + 2} \cdot \ln 5$

D) $2x \cdot 5^{x + 4} \cdot \ln 5$

E) $2x \cdot 5^{x^2 - 2} \cdot \ln 5$

SOLUTION

The answer is A

$f'(x) = 2x \cdot 5^{x^2 + 4} \cdot \ln 5$

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TERS FONKSİYONUN TÜREVİ

A, B ⊂ R ve f : A → B fonksiyonu birebir ve örtendir.

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$$

ÖRNEK 26

$f(x) = \sqrt[3]{\frac{x}{16}} \Rightarrow f(2)\cdot(f^{-1})'(1) = ?$

A) 24 B) 21 C) 18 D) 15 E) 12

ÇÖZÜM

Yanıt A

$$(f^{-1})'(x) = \frac{1}{\frac{1}{3}\sqrt[3]{\left(\frac{x}{16}\right)^2}}$$

$$\sqrt[3]{\frac{x}{16}} = 1 \Rightarrow \frac{x}{16} = 1$$

$$\left.\begin{array}{l} x = 16 \Rightarrow (f^{-1})'(1) = \dfrac{1}{\frac{1}{16}\cdot\frac{1}{3}\sqrt[3]{1}} = 48 \\ f(2) = \sqrt[3]{\dfrac{2}{16}} = \dfrac{1}{2} \end{array}\right\} \Rightarrow f(2)\cdot(f^{-1})'(1) = \frac{1}{2}\cdot 48 = 24$$

ÖRNEK 27

$f(x) = \sqrt[3]{x+a}$, $f^{-1}(1) + (f^{-1})'(1) = 2 \Rightarrow a = ?$

A) –1 B) –2 C) –3 D) –4 E) –5

ÇÖZÜM

Yanıt B

$$(f^{-1})'(x) = \frac{1}{\frac{1}{3}\sqrt[3]{(x+a)^{-2}}}$$

$$\sqrt[3]{x+a} = 1 \Rightarrow \sqrt[3]{(x+a)^2} = 1$$

$$(f^{-1})'(1) = 3$$

$$f^{-1}(1) + (f^{-1})'(1) = 2$$

$$f^{-1}(1) + 3 = 2 \Rightarrow f^{-1}(1) = -1$$

$$\sqrt[3]{1+a} = -1 \Rightarrow 1 + a = -1$$

$$a = -2$$

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DERIVATIVE of an INVERSE FUNCTION

Let $f : A \to B$ and $A, B \subset R$ be one to one and onto function.

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$$

EXAMPLE 26

$f(x) = \sqrt[3]{\frac{x}{16}} \Rightarrow f(2)\cdot(f^{-1})'(1) = ?$

A) 24 B) 21 C) 18 D) 15 E) 12

SOLUTION

The answer is A

$$(f^{-1})'(x) = \frac{1}{\frac{1}{3}\sqrt[3]{\left(\frac{x}{16}\right)^2}}$$

$$\sqrt[3]{\frac{x}{16}} = 1 \Rightarrow \frac{x}{16} = 1$$

$$\left.\begin{array}{l} x = 16 \Rightarrow (f^{-1})'(1) = \dfrac{1}{\frac{1}{16}\cdot\frac{1}{3}\sqrt[3]{1}} = 48 \\ f(2) = \sqrt[3]{\frac{2}{16}} = \frac{1}{2} \end{array}\right\} \Rightarrow f(2)\cdot(f^{-1})'(1) = \frac{1}{2}\cdot 48 = 24$$

EXAMPLE 27

$f(x) = \sqrt[3]{x+a}$, $f^{-1}(1) + (f^{-1})'(1) = 2 \Rightarrow a = ?$

A) –1 B) –2 C) –3 D) –4 E) –5

SOLUTION

The answer is C

$$(f^{-1})'(x) = \frac{1}{\frac{1}{3}\sqrt[3]{(x+a)^{-2}}}$$

$$\sqrt[3]{x+a} = 1 \Rightarrow \sqrt[3]{(x+a)^2} = 1$$

$$(f^{-1})'(1) = 3$$

$$f^{-1}(1) + (f^{-1})'(1) = 2$$

$$f^{-1}(1) + 3 = 2 \Rightarrow f^{-1}(1) = -1$$

$$\sqrt[3]{1+a} = -1 \Rightarrow 1 + a = -1$$

$$a = -2$$

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YÜKSEK BASAMAKTAN TÜREVLER

$y' = f'(x)$ 1. basamaktan türev

$y'' = f''(x)$ 2. basamaktan türev

$y''' = f'''(x)$ 3. basamaktan türev

.
.
.

$y^{(n)} = f^{(n)}(x)$: n. basamaktan türev

ÖRNEK 28

$y = (3x-2)^4 \Rightarrow \dfrac{d^2y}{dx^2} = ?$

A) $36(3x-2)^2$ B) $96(3x-2)^2$ C) $108(3x-2)^2$

D) $112(3x-2)^2$ E) $144(3x-2)^2$

ÇÖZÜM

Yanıt C

$$\frac{dy}{dx} = 4\cdot(3x-2)^3\cdot 3 = 12\cdot(3x-2)^3$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(12(3x-2)^3\right) = 36(3x-2)^2\cdot 3$$

$$= 108(3x-2)^2$$

ÖRNEK 29

$f(x) = \cos(3x+8) \Rightarrow f^{(100)}(x) = ?$

A) $3^{100}\cdot\sin(3x+8)$ B) $-3^{100}\cdot\sin(3x+8)$ C) $3^{100}\cdot\cos(3x+8)$

D) $-3^{100}\cdot\cos(3x+8)$ E) $100!\cdot\cos(3x+8)$

ÇÖZÜM

Yanıt C

$f'(x) = -\sin(3x+8)\cdot 3$

$f''(x) = -\cos(3x+8)\cdot 3^2$

$f'''(x) = \sin(3x+8)\cdot 3^3$

$f^{(4)}(x) = \cos(3x+8)\cdot 3^4$

$\Rightarrow f^{(100)}(x) = f(x)\cdot 3^{100} = 3^{100}\cdot\cos(3x+8)$

$100 \mid 4$
$0 \mid 25$

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HIGHER ORDER DERIVATIVES

$y' = f'(x)$ 1st order derivative

$y'' = f''(x)$ 2nd order derivative

$y''' = f'''(x)$ 3rd order derivative

.
.
.

$y^{(n)} = f^{(n)}(x)$: n th order derivative

EXAMPLE 28

$y = (3x-2)^4 \Rightarrow \dfrac{d^2y}{dx^2} = ?$

A) $36(3x-2)^2$ B) $96(3x-2)^2$ C) $108(3x-2)^2$

D) $112(3x-2)^2$ E) $144(3x-2)^2$

SOLUTION

The answer is C

$$\frac{dy}{dx} = 4\cdot(3x-2)^3\cdot 3 = 12\cdot(3x-2)^3$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(12(3x-2)^3\right) = 36(3x-2)^2\cdot 3$$

$$= 108(3x-2)^2$$

EXAMPLE 29

$f(x) = \cos(3x+8) \Rightarrow f^{(100)}(x) = ?$

A) $3^{100}\cdot\sin(3x+8)$ B) $-3^{100}\cdot\sin(3x+8)$ C) $3^{100}\cdot\cos(3x+8)$

D) $-3^{100}\cdot\cos(3x+8)$ E) $100!\cdot\cos(3x+8)$

SOLUTION

The answer is C

$f'(x) = -\sin(3x+8)\cdot 3$

$f''(x) = -\cos(3x+8)\cdot 3^2$

$f'''(x) = \sin(3x+8)\cdot 3^3$

$f^{(4)}(x) = \cos(3x+8)\cdot 3^4$

$\Rightarrow f^{(100)}(x) = f(x)\cdot 3^{100} = 3^{100}\cdot\cos(3x+8)$

$100 \div 4 = 25$, remainder 0

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PARAMETRİK FONKSİYONLARIN TÜREVİ

$$\left.\begin{array}{l} y = f(x) \\ x = u(t) \\ y = v(t) \end{array}\right\} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{v'(t)}{u'(t)}$$

NOT

$x = f(t),\ y = g(t) \Rightarrow$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{y''(t)\cdot x'(t) - x''(t)\cdot y'(t)}{(x'(t))^3}$$

ÖRNEK 30

$$\left.\begin{array}{l} x = \sqrt{t^2+3t} \\ y = (t+5)^2 \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = ?$$

A) $\frac{36}{5}$ B) $\frac{42}{5}$ C) $\frac{44}{5}$ D) $\frac{48}{5}$ E) $\frac{52}{5}$

ÇÖZÜM

Yanıt **D**

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = 2(t+5)\cdot\frac{1}{\frac{1}{2}(t^2+3t)^{-\frac{1}{2}}(2t+3)}$$

$$\left.\frac{dy}{dx}\right|_{t=1} = 2\cdot 6\cdot\frac{2\cdot(1+3)^{\frac{1}{2}}}{2+3} = \frac{48}{5}$$

ÖRNEK 31

$$\left.\begin{array}{l} y = t^3 - t \\ x = 2t^2 + 1 \end{array}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=1} = ?$$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM

Yanıt **C**

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = (3t^2-1)\cdot\frac{1}{4t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx} = \frac{6t\cdot 4t - (3t^2-1)\cdot 4}{16t^2}\cdot\frac{1}{4t}$$

$$= \frac{24t^2 - 12t^2 + 4}{16t^2}\cdot\frac{1}{4t} = \frac{12t^2+4}{64t^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{12\cdot 1^2 + 4}{64\cdot 1^3} = \frac{16}{64} = \frac{1}{4}$$

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NOTE

$x = f(t),\ y = g(t) \Rightarrow$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{y''(t)\cdot x'(t) - x''(t)\cdot y'(t)}{(x'(t))^3}$$

DERIVATIVE of PARAMETRIC FUNCTIONS

$$\left.\begin{array}{l} y = f(x) \\ x = u(t) \\ y = v(t) \end{array}\right\} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{v'(t)}{u'(t)}$$

EXAMPLE 30

$$\left.\begin{array}{l} x = \sqrt{t^2+3t} \\ y = (t+5)^2 \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = ?$$

A) $\frac{36}{5}$ B) $\frac{42}{5}$ C) $\frac{44}{5}$ D) $\frac{48}{5}$ E) $\frac{52}{5}$

SOLUTION

The answer is **D**

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = 2(t+5)\cdot\frac{1}{\frac{1}{2}(t^2+3t)^{-\frac{1}{2}}(2t+3)}$$

$$\left.\frac{dy}{dx}\right|_{t=1} = 2\cdot 6\cdot\frac{2\cdot(1+3)^{\frac{1}{2}}}{2+3} = \frac{48}{5}$$

EXAMPLE 31

$$\left.\begin{array}{l} y = t^3 - t \\ x = 2t^2 + 1 \end{array}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=1} = ?$$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

SOLUTION

The answer is **C**

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = (3t^2-1)\cdot\frac{1}{4t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx} = \frac{6t\cdot 4t - (3t^2-1)\cdot 4}{16t^2}\cdot\frac{1}{4t}$$

$$= \frac{24t^2 - 12t^2 + 4}{16t^2}\cdot\frac{1}{4t} = \frac{12t^2+4}{64t^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{12\cdot 1^2 + 4}{64\cdot 1^3} = \frac{16}{64} = \frac{1}{4}$$

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KAPALI FONKSİYONLARIN TÜREVİ

F(x,y) = 0 biçiminde tanımlanan fonksiyona *kapalı fonksiyon* denir.

$$F(x,y)=0 \Rightarrow \frac{dy}{dx}=-\frac{F'_x(x,y)}{F'_y(x,y)}$$

F'_x : x' e göre türev, $y \in R$

F'_y : y' ye göre türev, $x \in R$

ÖRNEK 32

$F(x, y) = x^3y - 2xy^2 + 3x^2 - 5xy + y^2 - 10 = 0$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(-1,\,1)} = ?$$

A) $\frac{1}{5}$ B) $\frac{3}{5}$ C) 1 D) $\frac{7}{5}$ E) $\frac{9}{5}$

ÇÖZÜM

Yanıt **C**

$$F_x = 3x^2y - 2y^2 + 6x - 5y$$

$$F_y = x^3 - 4xy - 5x + 2y$$

$$\left.\frac{dy}{dx}\right|_{(-1,\,1)} = -\frac{F_x}{F_y} = -\left.\frac{3x^2y - 2y^2 + 6x - 5y}{x^3 - 4xy - 5x + 2y}\right|_{(-1,\,1)} = \frac{-3+2+6+5}{-1+9+2} = \frac{10}{10} = 1$$

ÖRNEK 33

$F(x, y) = 3x^2y - xy^3 + 2xy - 4 = 0$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,\,1)} = ?$$

A) $-\frac{17}{2}$ B) $-\frac{15}{2}$ C) $-\frac{13}{2}$ D) $-\frac{9}{2}$ E) $-\frac{7}{2}$

ÇÖZÜM

Yanıt **E**

$$F_x = 6xy - y^3 + 2y$$

$$F_y = 3x^2 - 3xy^2 + 2x$$

$$\left.\frac{dy}{dx}\right|_{(1,\,1)} = -\left.\frac{6xy - y^3 + 2y}{3x^2 - 3xy^2 + 2x}\right|_{(1,\,1)} = -\frac{6-1+2}{3-3+2} = -\frac{7}{2}$$

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DERIVATIVE of IMPLICIT FUNCTIONS

Implicit function is a function that is formed by F(x,y) = 0.

$$F(x,y)=0 \Rightarrow \frac{dy}{dx}=-\frac{F'_x(x,y)}{F'_y(x,y)}$$

F'_x : derivative of F with respect to x

F'_y : derivative of F with respect to y

EXAMPLE 32

$F(x, y) = x^3y - 2xy^2 + 3x^2 - 5xy + y^2 - 10 = 0$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(-1,\,1)} = ?$$

A) $\frac{1}{5}$ B) $\frac{3}{5}$ C) 1 D) $\frac{7}{5}$ E) $\frac{9}{5}$

SOLUTION

The answer is C

$$F_x = 3x^2y - 2y^2 + 6x - 5y$$

$$F_y = x^3 - 4xy - 5x + 2y$$

$$\left.\frac{dy}{dx}\right|_{(-1,\,1)} = -\frac{F_x}{F_y} = -\left.\frac{3x^2y - 2y^2 + 6x - 5y}{x^3 - 4xy - 5x + 2y}\right|_{(-1,\,1)} = \frac{-3+2+6+5}{-1+9+2} = \frac{10}{10} = 1$$

EXAMPLE 33

$F(x, y) = 3x^2y - xy^3 + 2xy - 4 = 0$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,\,1)} = ?$$

A) $-\frac{17}{2}$ B) $-\frac{15}{2}$ C) $-\frac{13}{2}$ D) $-\frac{9}{2}$ E) $-\frac{7}{2}$

SOLUTION

The answer is E

$$F_x = 6xy - y^3 + 2y$$

$$F_y = 3x^2 - 3xy^2 + 2x$$

$$\left.\frac{dy}{dx}\right|_{(1,\,1)} = -\left.\frac{6xy - y^3 + 2y}{3x^2 - 3xy^2 + 2x}\right|_{(1,\,1)} = -\frac{6-1+2}{3-3+2} = -\frac{7}{2}$$

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1. $f(x)=\frac{5x-2}{4x+1} \Rightarrow f'(2)=?$

A) $\frac{11}{81}$

B) $\frac{13}{81}$

C) $\frac{15}{81}$

D) $\frac{17}{81}$

E) $\frac{21}{81}$

2. $f(x)=x^3-x^2+5x-7 \Rightarrow \lim_{x\to 1}\frac{f(x)-f(1)}{2x-2}=?$

A) 3

B) 4

C) 5

D) 6

E) 7

3. $f(x)=\frac{2\sin^2 x}{1-\cos x} \Rightarrow \lim_{h\to 0}\frac{f(h)-f(0)}{h}=?$

A) 0

B) 2

C) 4

D) 6

E) 8

4. $f(x)=\arctan x+(\arccos x)^2 \Rightarrow f'(0)=?$

A) $1+\pi$

B) $1-\pi$

C) $2+\pi$

D) $2-\pi$

E) $3+\pi$

5. $y=\ln\left(\frac{x^2}{x^2+1}\right) \Rightarrow (x^2+1)\cdot\frac{dy}{dx}=?$

A) $2x^2+1$

B) $2x^2+2$

C) $\frac{2}{x^2}$

D) $\frac{2}{x}$

E) $4x^2+2$

6. $f(x)=\ln(\operatorname{cosec}x+\cot x) \Rightarrow f'(x)=?$

A) $\frac{\cot x+\sin x}{\sin x-1}$

B) $\frac{\sin x}{\sin x-1}$

C) $\frac{\cos x}{\cos x-1}$

D) $\sec x$

E) $-\operatorname{cosec}x$

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7. $f(x)=\dfrac{x}{x^2+3} \Rightarrow f'(x)=?$

A) $\dfrac{3-x^2}{(x+3)^2}$

B) $\dfrac{3+x^2}{(x^2-3)^2}$

C) $\dfrac{3+x^2}{(x^2-3)^2}$

D) $\dfrac{3+x^2}{(x^2+3)^2}$

E) $\dfrac{3-x^2}{(x^2+3)^2}$

8. $f(x)=e^{\sin x}+\cos^2 x \Rightarrow f'\left(\dfrac{\pi}{2}\right)=?$

A) –1

B) 0

C) 1

D) 2

E) e

9. $f(x)=4x^2+2$,

$g(x)=x^3-3 \Rightarrow (gof)'(x)=?$

A) $96x\cdot(2x^2+1)^2$

B) $48x\cdot(2x^2+1)^2$

C) $24x\cdot(2x^2+1)^2$

D) $12x\cdot(2x^2+1)^2$

E) $6x\cdot(2x^2+1)^2$

10. $f(x)=\sqrt{3x} \Rightarrow (f^{-1})'(3)=?$

A) 2

B) 3

C) 4

D) 5

E) 6

11. $\left.\begin{array}{l} y=5u^2 \\ u=\sqrt{t^2+1} \\ t=2x \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{x=0}=?$

A) –2

B) –1

C) 0

D) 2

E) 3

12. $\arctan\dfrac{x}{y}+\ln\sqrt{x^2+y^2}=0 \Rightarrow \dfrac{dy}{dx}=?$

A) $\dfrac{x-y}{x+y}$ B) $\dfrac{x+y}{x-y}$ C) $\dfrac{2x+y}{2x-y}$

D) $\dfrac{y-2x}{2x+y}$ E) $\dfrac{x+y}{x-2y}$

YANITLAR / ANSWERS

1. B	2. A	3. A	4. B	5. D	6. E	7. E	8. B
9. A	10. A	11. C	12. B				

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1. $f(x)=\dfrac{5x-2}{4x+1}$

$\Rightarrow f'(x)=\dfrac{(5x-2)'(4x+1)-(5x-2)(4x+1)'}{(4x+1)^2}$

$=\dfrac{5(4x+1)-(5x-2)\cdot 4}{(4x+1)^2}$

$=\dfrac{13}{(4x+1)^2}$

$\Rightarrow f'(2)=\dfrac{13}{(4\cdot 2+1)^2}=\dfrac{13}{81}$

Yanıt / The Answer is B

2. $\lim\limits_{x\to 1}\dfrac{f(x)-f(1)}{2x-2}=\lim\limits_{x\to 1}\dfrac{f(x)-f(1)}{2(x-1)}=\dfrac{f'(1)}{2}$

$f(x)=x^3-x^2+5x-7$

$f'(x)=3x^2-2x+5$

$\Rightarrow f'(1)=3\cdot 1^2-2\cdot 1+5=6$

$\Rightarrow \dfrac{f'(1)}{2}=\dfrac{6}{2}=3$

Yanıt / The Answer is A

3. $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=f'(0)$

$f(x)=\dfrac{2\sin^2 x}{1-\cos x}=\dfrac{2(1-\cos x)(1+\cos x)}{1-\cos x}$

$f'(x)=-2\sin x$

$f'(0)=-2\sin 0=0$

Yanıt / The Answer is A

4. $f(x)=\arctan x+(\arccos x)^2$

$f'(x)=\dfrac{1}{1+x^2}+2\cdot(\arccos x)\cdot\left(-\dfrac{1}{\sqrt{1-x^2}}\right)$

$f'(0)=\dfrac{1}{1+0}+2\cdot(\arccos 0)\cdot\left(-\dfrac{1}{\sqrt{1-0}}\right)$

$=1+2\cdot\dfrac{\pi}{2}\cdot(-1)=1-\pi$

Yanıt / The Answer is B

5. $\dfrac{dy}{dx}=\dfrac{\left(\dfrac{x^2}{x^2+1}\right)'}{\dfrac{x^2}{x^2+1}}=\dfrac{2x(x^2+1)-x^2\cdot 2x}{(x^2+1)^2}\cdot\dfrac{x^2}{x^2+1}$

$=\dfrac{2x(x^2+1-x^2)}{(x^2+1)^2}\cdot\dfrac{x^2+1}{x^2}=\dfrac{2}{x(x^2+1)}$

$(x^2+1)\cdot\dfrac{dy}{dx}=(x^2+1)\cdot\dfrac{2}{x(x^2+1)}=\dfrac{2}{x}$

Yanıt / The Answer is D

6. $f(x)=\ln(\operatorname{cosec}x+\cot x)$

$f(x)=\ln\left(\dfrac{1}{\sin x}+\dfrac{\cos x}{\sin x}\right)=\ln\left(\dfrac{1+\cos x}{\sin x}\right)$

$f'(x)=\dfrac{\left(\dfrac{1+\cos x}{\sin x}\right)'}{\dfrac{1+\cos x}{\sin x}}$

$=\dfrac{\dfrac{-\sin x\cdot\sin x-\cos x\cdot(1+\cos x)}{(\sin x)^2}}{\dfrac{1+\cos x}{\sin x}}$

$=\dfrac{-\sin^2 x-\cos x-\cos^2 x}{\sin^2 x}\cdot\dfrac{\sin x}{1+\cos x}$

$=\dfrac{(-1-\cos x)\cdot\sin x}{\sin^2 x\cdot(1+\cos x)}$

$=\dfrac{-(1+\cos x)\cdot\sin x}{\sin^2 x\cdot(1+\cos x)}=-\dfrac{1}{\sin x}=-\operatorname{cosec}x$

Yanıt / The Answer is E

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7. $f(x)=\frac{x}{x^2+3} \Rightarrow f'(x)=\frac{1(x^2+3)-2x(x)}{(x^2+3)^2}$

$=\frac{x^2+3-2x^2}{(x^2+3)^2}=\frac{3-x^2}{(x^2+3)^2}$

Yanıt / The Answer is E

8. $f(x)=e^{\sin x}+\cos^2 x$

$f'(x)=e^{\sin x}\cdot\cos x+2\cos x(-\sin x)$

$f'\left(\frac{\pi}{2}\right)=e^{\sin\frac{\pi}{2}}\cos\frac{\pi}{2}-2\cos\frac{\pi}{2}\sin\frac{\pi}{2}$

$=0-0$

$=0$

Yanıt / The Answer is B

9. $(gof)(x)=(4x^2+2)^3-3$

$(gof)'(x)=3(4x^2+2)^2\cdot 8x$

$=12\,(2x^2+1)^2\cdot 8x$

$=96x\cdot(2x^2+1)^2$

Yanıt / The Answer is A

10. $(f^{-1})'(x)=\frac{1}{f'(x)}=\frac{1}{\frac{3}{2\sqrt{3x}}}=\frac{2\sqrt{3x}}{3}$

$\sqrt{3x}=3 \Rightarrow (f^{-1})'(3)=\frac{2\cdot 3}{3}=2$

Yanıt / The Answer is A

11. $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dt}\cdot\frac{dt}{dx}$

$=10u\cdot\frac{2t}{2\sqrt{t^2+1}}\cdot 2$

$\left.\begin{array}{l} x=0\Rightarrow t=0 \\ t=0\Rightarrow u=1 \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0}=\frac{40\cdot 1\cdot 0}{2\sqrt{0^2+1}}=0$

Yanıt / The Answer is C

12. $F_x=\frac{\frac{1}{y}}{1+\frac{x^2}{y^2}}+\frac{x}{x^2+y^2}=\frac{x+y}{x^2+y^2}$

$F_y=\frac{-\frac{x}{y^2}}{1+\frac{x^2}{y^2}}+\frac{y}{x^2+y^2}=\frac{y-x}{x^2+y^2}$

$\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{\frac{x+y}{x^2+y^2}}{\frac{y-x}{x^2+y^2}}=\frac{x+y}{x-y}$

Yanıt / The Answer is B

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1. $f(x) = x\sqrt{x} - x\sqrt{1-x} \Rightarrow f'\left(\frac{3}{4}\right) = ?$

A) $\frac{3\sqrt{3}+1}{4}$

B) $\frac{3\sqrt{3}+4}{4}$

C) $\frac{3\sqrt{3}+2}{4}$

D) $3\sqrt{3}-1$

E) $\frac{5\sqrt{3}+1}{4}$

2. $y = (x^2+1)\sqrt{x} \Rightarrow \frac{dy}{dx} = y' = ?$

A) $\frac{x^2+1}{2\sqrt{x}}$

B) $\frac{2x^2+1}{2\sqrt{x}}$

C) $\frac{5x^2+1}{2\sqrt{x}}$

D) $\frac{3x^2+2}{2\sqrt{x}}$

E) $\frac{6x^2+1}{2\sqrt{x}}$

3. $f(x) = \tan(\sqrt[3]{x}) \Rightarrow f'(\pi^3) = ?$

A) π^2

B) $2\pi^2$

C) $3\pi^2$

D) $\frac{1}{3\pi^2}$

E) $\frac{1}{2\pi^2}$

4. $f(x) = \text{arccot}(x^2+2x) - (\arcsin x)^2 \Rightarrow f'(\frac{1}{2}) = ?$

A) $\frac{432+82\sqrt{3}\,\pi}{369}$

B) $-\frac{432+82\sqrt{3}\,\pi}{369}$

C) $\frac{232+82\sqrt{3}\,\pi}{369}$

D) $-\frac{232+82\sqrt{3}\,\pi}{369}$

E) $-\frac{232+82\sqrt{2}\,\pi}{369}$

5. $a > 2$

$f(x) = a \cdot \ln(ax-3)^2$

$\frac{d}{dx} f(x)\Big|_{x=2} = 8 \Rightarrow a = ?$

A) 1

B) 3

C) 4

D) 5

E) 6

6. $y = (2x+8)^6$, $\frac{d^3y}{dx^3} = a\cdot(2x+8)^3 \Rightarrow a = ?$

A) 880

B) 900

C) 920

D) 940

E) 960

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7. $y = (5-x)^5$, $\dfrac{d^3y}{dx^3} = a\cdot(5-x)^2 \Rightarrow a = ?$

A) 60

B) 40

C) 0

D) –40

E) –60

8. $\left.\begin{array}{l} y = t^3 - t^2 + 2 \\ x = \sqrt{t^2+3} \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=1} = ?$

A) 2

B) 4

C) 6

D) 8

E) 10

9. $F(x, y) = \cos 2x - \sin y + 3xy - 1 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(0,\,0)} = ?$

A) 0

B) 1

C) $\dfrac{3}{2}$

D) 2

E) $\dfrac{5}{2}$

10. $f(x, y) = x^2 + y^2 - 9 = 0 \Rightarrow f''(x, y) = ?$

A) $\dfrac{(x+y)^2}{y^3}$

B) $-\dfrac{(x+y)^2}{y^3}$

C) $\dfrac{x^2-y^2}{y^3}$

D) $\dfrac{x^2+y^2}{y^3}$

E) $-\dfrac{x^2+y^2}{y^3}$

11. $F(x, y) = x^4\cdot y^3 + 2x^3 + 4xy = 0 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(1,0)} = ?$

A) $-\dfrac{1}{2}$

B) $-\dfrac{3}{2}$

C) -1

D) -2

E) $-\dfrac{5}{2}$

12. $\left.\begin{array}{l} x = 3t^2 + 3t \\ y = t^3 - 3t \end{array}\right\} \Rightarrow \left.\dfrac{d^2y}{dx^2}\right|_{t=1} = ?$

A) $\dfrac{1}{7}$ B) $\dfrac{2}{27}$ C) $\dfrac{3}{7}$ D) $\dfrac{4}{7}$ E) $\dfrac{5}{7}$

YANITLAR / ANSWERS

1. A	2. C	3. D	4. B	5. E	6. E	7. E	8. A
9. A	10. E	11. B	12. B				

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1. $f(x)=x\sqrt{x}-x\sqrt{1-x}=\sqrt{x^3}-\sqrt{x^2-x^3}$

$$f'(x)=\frac{3}{2}x^{\frac{1}{2}}-\frac{2x-3x^2}{2\sqrt{x^2-x^3}}$$

$$f'\left(\frac{3}{4}\right)=\frac{3}{2}\cdot\sqrt{\frac{3}{4}}-\frac{\frac{3}{2}-\frac{27}{16}}{2\sqrt{\frac{9}{16}-\frac{27}{64}}}$$

$$=\frac{3\sqrt{3}}{4}+\frac{\frac{3}{16}}{2\cdot\frac{3}{8}}=\frac{3\sqrt{3}+1}{4}$$

Yanıt / The Answer is A

2. $y'=(x^2+1)'\sqrt{x}+(x^2+1)(\sqrt{x})'$

$$=2x\sqrt{x}+\frac{x^2+1}{2\sqrt{x}}$$

$$=\frac{5x^2+1}{2\sqrt{x}}$$

Yanıt / The Answer is C

3. $f(x)=\tan\sqrt[3]{x}\Rightarrow f'(x)=(1+\tan^2\sqrt[3]{x})\cdot\frac{1}{3}x^{-\frac{2}{3}}$

$$\Rightarrow f'(\pi^3)=(1+\tan^2\pi)\cdot\frac{1}{3}\cdot\frac{1}{\pi^2}=\frac{1}{3\pi^2}$$

Yanıt / The Answer is D

4. $$f'(x)=\frac{-(2x+2)}{1+(x^2+2x)^2}-\frac{1}{\sqrt{1-x^2}}\cdot 2\arcsin x$$

$$f'\left(\frac{1}{2}\right)=\frac{-\left(2\cdot\frac{1}{2}+2\right)}{1+\left(\frac{1}{4}+2\cdot\frac{1}{2}\right)^2}-\frac{1}{\sqrt{1-\frac{1}{4}}}\cdot 2\arcsin\frac{1}{2}$$

$$f'\left(\frac{1}{2}\right)=\frac{-3}{1+\frac{25}{16}}-\frac{2}{\sqrt{3}}\cdot 2\cdot\frac{\pi}{6}$$

$$=\frac{-48}{41}-\frac{2\sqrt{3}\pi}{9}=-\frac{432+82\sqrt{3}\pi}{369}$$

Yanıt / The Answer is B

5. $$f'(x)=a\cdot\frac{2(ax-3)\cdot a}{(ax-3)^2}$$

$$\Rightarrow f'(2)=\frac{a^2\cdot 2(2a-3)}{(2a-3)^2}=\frac{2a^2}{2a-3}=8$$

$2a^2=16a-24$

$a^2-8a+12=0$

$(a-6)(a-2)=0$

$a=6,\ a=2$

Yanıt / The Answer is E

6. $y=(2x+8)^6$

$y'=6\cdot(2x+8)^5\cdot 2$

$y''=24\cdot(2x+8)^4\cdot 5$

$y'''=120\cdot(2x+8)^3\cdot 4\cdot 2$

$$\frac{d^3y}{dx^3}=960(2x+8)^3=a(2x+8)^3$$

$a=960$

Yanıt / The Answer is E

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7. $y=(5-x)^5$

$y'=5\cdot(5-x)^4\cdot(-1)$

$y''=20\cdot(5-x)^3$

$y'''=60\cdot(5-x)^2\cdot(-1)$

$\frac{d^3y}{dx^3}=-60(5-x)^2=a(5-x)^2 \Rightarrow a=-60$

Yanıt / The Answer is E

8. $\left.\frac{dy}{dx}\right|_{t=1}=\left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3t^2-2t}{\frac{2t}{2\sqrt{t^2+3}}}\right|_{t=1}=\frac{1}{\frac{1}{2}}=2$

Yanıt / The Answer is A

9. $F_x=-\sin 2x\cdot 2+3y$

$F_y=-\cos y+3x$

$\left.\frac{dy}{dx}\right|_{(0,0)}=-\left.\frac{-\sin 2x\cdot 2+3y}{-\cos y+3x}\right|_{(0,0)}=\frac{0}{1}=0$

Yanıt / The Answer is A

10. $y'=f'(x)=\frac{-2x}{2y}=\frac{-x}{y}$

$y''=f''(x)=\frac{-1\cdot y-xy'}{y^2}$

$=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}=\frac{-y^2-x^2}{y^3}$

$=-\frac{x^2+y^2}{y^3}$

Yanıt / The Answer is E

11. $F_x=4x^3y^3+6x^2+4y$

$F_y=3x^4y^2+4x$

$\left.\frac{dy}{dx}\right|_{(1,0)}=-\frac{4\cdot1^3\cdot0^3+6\cdot1^2+4\cdot0}{3\cdot1^4\cdot0^2+4\cdot1}=-\frac{6}{4}=-\frac{3}{2}$

Yanıt / The Answer is B

12. $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3t^2-3}{6t+3}=\frac{t^2-1}{2t+1}$

$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$

$\frac{d^2y}{dx^2}=\frac{2t(2t+1)-2(t^2-1)}{(2t+1)^2}\cdot\frac{1}{6t+3}$

$t=1 \Rightarrow \frac{d^2y}{dx^2}=\frac{2\cdot3-2\cdot0}{(3)^2}\cdot\frac{1}{9}$

$\frac{d^2y}{dx^2}=\frac{6}{81}=\frac{2}{27}$

Yanıt / The Answer is B

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YÖS Soruları / YÖS Questions

1. $f(x) = |x^2 - 1| + |x - 1| + x + 1 \Rightarrow f'\left(\frac{1}{2}\right) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

[ANKARA ÜNİV. YÖS 2013]

2. $f(x) = (\cos 3x)^3 \Rightarrow f'(\frac{\pi}{3}) = ?$

A) –1 B) 0 C) 1 D) 2 E) 3

[İSTANBUL ÜNİV. YÖS 2013]

3. $f(x) = e^{2\sin x}$

$\Rightarrow$ **f'(x) = ?**

A) $2\cos x e^{2\sin x}$ B) $2\sin x e^{2\sin x}$

C) e^{2x-1} D) e^{x+2}

E) $e^{2\sin x}$

[İSTANBUL ÜNİV. YÖS 2013]

4. $f(2x + 1) = x^3 - 6x + 1$

f'(1) = ?

A) –3 B) –2 C) 0 D) 1 E) 5

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

5. $f(x) = e^x \cdot \sin x$

f'(0) = ?

A) –2 B) –1 C) 0 D) 1 E) e

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

6. $f(x) = \ln^2(x)$

f'(e) = ?

A) 1 B) e C) $\frac{2}{e}$ D) $\frac{1}{e^2}$ E) e^2

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

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7. $f(x)=\frac{x}{\ln x} \Rightarrow f'(e)=?$

A) $-e$ B) $\frac{1}{e}$ C) 0

D) 1 E) e

[MARMARA ÜNİV. YÖS 2013]

8. $f(x)=\frac{mx+4}{x-1}$, $f'(2)=-5 \Rightarrow m=?$

A) -3 B) 0 C) 1

D) 2 E) 4

[MARMARA ÜNİV. YÖS 2013]

9. $x > 0$

$f(x) = x^2 + 1$

$\Rightarrow (f^{-1})'(2) = ?$

A) 1 B) 2 C) $\frac{1}{4}$

D) $\frac{1}{2}$ E) 5

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

10. $f(x) = \ln^2(xe^x)$

$\Rightarrow f'(1) = ?$

A) $-2e$ B) 3e C) 4

D) e^2 E) 2

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

11. $y=\frac{\sin x}{x^2} \Rightarrow y'\left(\frac{\pi}{2}\right)=?$

A) $2^5\pi^2$ B) $2^4\pi^3$ C) $-2^4\pi^2$

D) $-2^4\pi^{-3}$ E) $-2^4\pi^4$

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

12. k sabit sayı (k, constant)

$\sqrt{xy}=k \Rightarrow \left.\frac{dy}{dx}\right|_{x=2,\,y=2} = ?$

A) -2 B) -1 C) 0

D) 1 E) 2

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

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13. $y = f(x)$,

$\ln(x^2 + y^2) = x$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(0,1)} = ?$

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

[MARMARA ÜNİV. YÖS 2012]

14. $f(x) = x\ln(x^2)$

$\Rightarrow f'(x) = ?$

A) $2(1 + \ln x^2)$ B) $2 + (\ln x)^2$ C) $2 + \ln x$

D) $2 + \ln x^2$ E) $2\ln x^2$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

15. $f(x) = \tan(\sin 2x) \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) -2 B) -1 C) 0

D) 1 E) 2

[EGE ÜNİV. YÖS 2012]

16. $f(x) = \log_{10} e^x \Rightarrow f'(x) = ?$

A) $\ln e^x$ B) $\log x$ C) $\log_{10} e$

D) $\log x^2$ E) $\log_e x$

(ANKARA ÜNİV. YÖS 2011)

17. $f(x) = \log_9 e^x \Rightarrow f'(x) = ?$

A) $\log e$ B) $\ln e^x$ C) $\log_9 e$

D) $\log_e 9$ E) $\ln x$

(ANKARA ÜNİV. YÖS 2011)

18. $f(x) = \sqrt{\frac{2x-1}{x}} \Rightarrow f(2) \cdot f'(2) = ?$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$

D) $\frac{1}{8}$ E) $\frac{1}{12}$

(ANADOLU ÜNİV. YÖS 2011)

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19. $f(x) = \ln x^x$

$f'(x) = ?$

A) $x \ln x$ B) $\ln x$ C) $x \ln e$

D) $\ln(ex)$ E) x^x

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

20. $3xy + y^2 - 5 = 0$

$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=-1}} = ?$

A) -3 B) -1 C) 1

D) 2 E) 3

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

21. $f(x) = \ln(e^{2x}\, x)$

$f'(1) = ?$

A) -2 B) -1 C) 1

D) 2 E) 3

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

22. $f(x) = \frac{1}{\sqrt[3]{(2-x)^5}}$

$f'(1) = ?$

A) $\frac{5}{6}$ B) $\frac{5}{3}$ C) $\frac{3}{5}$

D) -1 E) 1

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

23. $f(x) = \ln(2x^2 \cdot \ln x^2) \Rightarrow f'(e) = ?$

A) $\frac{e}{2}$ B) $\frac{2}{e}$ C) $\frac{3}{e}$

D) $\frac{e+1}{e}$ E) $\frac{e+1}{3}$

[YÖS 2010]

24. $f(x) = \sin(\cos 2x) \Rightarrow f'(\frac{\pi}{4}) = ?$

A) -2 B) -1 C) 0 D) 2 E) 4

[YÖS 2010]

YANITLAR / ANSWERS

1. B	2. B	3. A	4. A	5. D	6. C	7. C	8. C
9. D	10. C	11. D	12. B	13. B	14. D	15. A	16. C
17. C	18. D	19. D	20. E	21. E	22. B	23. C	24. A

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1. $f(x) = -x^2 + 1 - x + 1 + x + 1$

$f(x) = -x^2 + 3$

$f'(x) = -2x$

$f'\left(\frac{1}{2}\right) = -2 \cdot \frac{1}{2} = -1$

Yanıt / The Answer is B

2. $f(x) = (\cos 3x)^3$

$= -3 \cdot (\cos 3x)^2 \cdot \sin 3x \cdot 3$

$= 3 \cdot (\cos 3 \cdot \frac{\pi}{3}) \cdot \underbrace{\sin 3 \cdot \frac{\pi}{3}}_{0} \cdot 3 = 0$

Yanıt / The Answer is B

3. $f(x) = e^{2\sin x} \Rightarrow f'(x) = 2\cos x \cdot e^{2\sin x}$

Yanıt / The Answer is A

4. $2 \cdot f'(2x + 1) = 3x^2 - 6$

$x = 0 \Rightarrow 2 \cdot f'(1) = -6$

$f'(1) = -3$

Yanıt / The Answer is A

5. $f'(x) = e^x \cdot \sin x + e^x \cdot \cos x$

$f'(0) = e^0 \cdot \sin 0 + e^0 \cdot \cos 0$

$= 0 + 1$

$= 1$

Yanıt / The Answer is D

6. $f'(x) = 2\ln(x) \cdot \frac{1}{x}$

$f'(e) = 2\ln(e) \cdot \frac{1}{e}$

$= \frac{2}{e}$

Yanıt / The Answer is C

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7. $f(x)=\frac{x}{\ln x} \Rightarrow f'(x)=\frac{x'\cdot \ln x-(\ln x)'\cdot x}{(\ln x)^2}$

$$=\frac{\ln x-\frac{1}{x}\cdot x}{(\ln x)^2}$$

$\Rightarrow f'(e)=\frac{\ln e-1}{(\ln e)^2}=\frac{1-1}{1}=\frac{0}{1}=0$

Yanıt / The Answer is C

8. $f(x)=\frac{mx+4}{x-1}$, $f'(2)=-5$

$f'(x)=\frac{m\cdot(x-1)-(mx+4)}{(x-1)^2}$

$f'(2)=\frac{m-2m-4}{1}$

$-m-4=-5$

$-m=-1$

$m=1$

Yanıt / The Answer is C

9. $f(x)=x^2+1 \Rightarrow f^{-1}(x)=\sqrt{x-1}$

$(f^{-1})'(x)=\frac{1}{2\sqrt{x-1}}$

$(f^{-1})'(2)=\frac{1}{2\sqrt{2-1}}=\frac{1}{2}$

Yanıt / The Answer is D

10. $f(x) = \ln^2(xe^x)$

$\Rightarrow f'(x)=\frac{2\ln(xe^x)\cdot(e^x+xe^x)}{xe^x}$

$\Rightarrow f'(1)=\frac{2\ln(e)\cdot(2e)}{e}=4$

Yanıt / The Answer is C

11. $y=\frac{\sin x}{x^2} \Rightarrow y'=\frac{\cos x\cdot x^2-2x\sin x}{x^4}$

$$y'\left(\frac{\pi}{2}\right)=\frac{\cos\frac{\pi}{2}\cdot\left(\frac{\pi}{2}\right)^2-2\cdot\frac{\pi}{2}\sin\frac{\pi}{2}}{\left(\frac{\pi}{2}\right)^4}$$

$$=\frac{-\pi}{\frac{\pi^4}{16}}=-\frac{16}{\pi^3}=-2^4\pi^{-3}$$

Yanıt / The Answer is D

12. $\sqrt{xy}=k \Rightarrow x^2y^2-k^2=0$

$\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{2xy^2}{2x^2y}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2,\,y=2}=-1$

Yanıt / The Answer is B

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13. $\ln(x^2+y^2)=x$

$$\ln(x^2+y^2)-x=0 \Rightarrow \frac{dy}{dx}=-\frac{\frac{2x}{x^2+y^2}-1}{\frac{2y}{x^2+y^2}}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)}=-\frac{\frac{2\cdot 0}{0^2+1^2}-1}{\frac{2\cdot 1}{0^2+1^2}}=\frac{1}{2}$$

Yanıt / The Answer is B

14. $f(x)=x\cdot\ln(x^2) \Rightarrow f'(x)=1\cdot\ln(x^2)+\frac{2x}{x^2}\cdot x$

$$=\ln(x^2)+2$$

Yanıt / The Answer is D

15. $f(x)=\tan(\sin 2x) \Rightarrow f'(x)=(\sin 2x)'\cdot(1+\tan^2(\sin 2x))$

$$=2\cdot\cos 2x\cdot(1+\tan^2(\sin 2x))$$

$$f'\left(\frac{\pi}{2}\right)=2\cdot\cos 2\cdot\frac{\pi}{2}\cdot\left(1+\tan^2\left(\sin 2\cdot\frac{\pi}{2}\right)\right)$$

$$=2\cdot(-1)\cdot(1+0)=-2$$

Yanıt / The Answer is A

16. $f(x)=x\cdot\log_{10} e \Rightarrow f'(x)=\log_{10} e$

Yanıt / The Answer is C

17. $f(x)=\log_9 e^x = x\cdot\log_9 e$

$\Rightarrow f'(x)=\log_9 e$

Yanıt / The Answer is C

18. $f(x)=\sqrt{\frac{2x-1}{x}} \Rightarrow f'(x)=\frac{1}{2\sqrt{\frac{2x-1}{x}}}\cdot\frac{1}{x^2}$

$$\Rightarrow f'(2)=\frac{1}{2\sqrt{\frac{2\cdot 2-1}{2}}}\cdot\frac{1}{(2)^2}=\frac{\sqrt{6}}{24}$$

$$f(2)=\sqrt{\frac{3}{2}}=\frac{\sqrt{6}}{2}$$

$$f(2)\cdot f'(2)=\frac{\sqrt{6}}{2}\cdot\frac{\sqrt{6}}{24}=\frac{1}{8}$$

Yanıt / The Answer is D

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19. $f(x) = \ln x^x = x \ln x$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

$= \ln x + \ln e$

$= \ln(x \cdot e)$

Yanıt / The Answer is D

20. $F(x, y) = 3xy + y^2 - 5 = 0$

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = -\frac{3y}{3x+2y} = -\frac{-3}{3-2} = 3$$

Yanıt / The Answer is E

21. $f(x) = \ln(e^{2x} \cdot x) = \ln e^{2x} + \ln x$

$= 2x + \ln x$

$f'(x) = 2 + \frac{1}{x} \Rightarrow f'(1) = 2 + 1 = 3$

Yanıt / The Answer is E

22. $f(x) = (2-x)^{-\frac{5}{3}}$

$f'(x) = \frac{5}{3}(2-x)^{-\frac{8}{3}}$

$f'(1) = \frac{5}{3}$

Yanıt / The Answer is B

23. $f(x) = \ln(2x^2 \cdot \ln x^2)$

$$f'(x) = \frac{(2x^2 \cdot \ln x^2)'}{2x^2 \cdot \ln x^2}$$

$$= \frac{4x \cdot \ln x^2 + \frac{2x}{x^2} \cdot 2x^2}{2x^2 \cdot \ln x^2}$$

$$f'(e) = \frac{4e \cdot \ln e^2 + 4e}{2e^2 \cdot \ln e^2}$$

$$f'(e) = \frac{12e}{4e^2} = \frac{3}{e}$$

Yanıt / The Answer is C

24. $f(x) = \sin(\cos 2x)$

$f'(x) = (\cos 2x)' \cdot \cos(\cos 2x)$

$= -2 \cdot \sin 2x \cdot \cos(\cos 2x)$

$$f'\left(\frac{\pi}{4}\right) = -2 \cdot \sin\frac{\pi}{2} \cdot \cos\left(\cos\frac{\pi}{2}\right)$$

$= -2 \cdot 1 \cdot 1 = -2$

Yanıt / The Answer is A

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1. $f(x)=\frac{ax^2+1}{x^2+1}$, $\frac{d}{dx}f(x)\Big|_{x=2}=4 \Rightarrow a=?$

Yanıt / The Answer is **26**

2. $y=\frac{x}{(x-1)^2} \Rightarrow (x-1)^4\cdot\frac{dy}{dx}=?$

Yanıt / The Answer is $1-x^2$

3. $f(x)=3x^2-5x+7 \Rightarrow \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=?$

Yanıt / The Answer is **13**

4. $f(x)=\cot 4x \Rightarrow f'(x)=?$

Yanıt / The Answer is $-4\text{cosec}^2\,4x$

5. $f(x)=\arcsin\sqrt{x} \Rightarrow f'\left(\frac{1}{2}\right)=?$

Yanıt / The Answer is **1**

6. $f(x)=\arctan\left(\frac{1}{x}\right) \Rightarrow f'(2)=?$

Yanıt / The Answer is $-\frac{1}{5}$

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7. $f(x) = \log_2 x \Rightarrow \left.\frac{d}{dx} f(x)\right|_{x=1} = ?$

Yanıt / The Answer is $\log_2 e$

8. $f(x) = \ln(\cos x + \sin x) \Rightarrow f'(x) = ?$

Yanıt / The Answer is $\frac{\cos x - \sin x}{\cos x + \sin x}$

9. $y = x^{\ln 3} \Rightarrow \frac{dy}{dx} = ?$

Yanıt / The Answer is $\ln 3 \cdot x^{\ln \frac{3}{e}}$

10. $f(x) = e^{\sin(\ln x)} \Rightarrow f'(1) = ?$

Yanıt / The Answer is 1

11. $y = (1 - x^2)^3 \Rightarrow \left.\frac{d^2 y}{dx^2}\right|_{x=1} = ?$

Yanıt / The Answer is 0

12. $e^{-x^3} \frac{d^2}{dx^2}(x^2 e^{x^3}) = ?$

Yanıt / The Answer is $9x^6 + 18x^3 + 2$

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13. $f(x) = x \cdot e^x \Rightarrow f''(x) - f'(x) = ?$

Yanıt / The Answer is e^x

14. $y = f(x)$, $\left.\begin{matrix} x = 6t - 4 \\ y = t^3 + 8 \end{matrix}\right\} \Rightarrow f'(8) = ?$

Yanıt / The Answer is **2**

15. $\left.\begin{matrix} x = t^3 - 2t^2 + 3t \\ y = t^3 - 2t^2 \end{matrix}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = ?$

Yanıt / The Answer is $-\frac{1}{2}$

16. $f(x) = x^3 + 4x^2 - 2x + 6 \Rightarrow f'''(x) = ?$

Yanıt / The Answer is **6**

17. $x^2 - xy - y^2 + 5 = 0 \Rightarrow \frac{dy}{dx} = ?$

Yanıt / The Answer is $\frac{2x - y}{x + 2y}$

18. $x^2 - xy + y^2 - 3 = 0 \Rightarrow \frac{dy}{dx} = ?$

Yanıt / The Answer is $\frac{2x - y}{x - 2y}$

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1. $f(x) = (x^2 + 2)\cdot(x^3 + x + 1) \Rightarrow f'(1) = ?$

A) 18

B) 16

C) 14

D) 12

E) 10

2. $f(x) = \sqrt[5]{x^2 + 2x + 3} \Rightarrow f'(x) = ?$

A) $\frac{2(x-1)}{5\cdot\sqrt[5]{(x^2+2x+3)^4}}$

B) $\frac{2(x+1)}{5\cdot\sqrt[5]{(x^2+2x+3)^4}}$

C) $\frac{-2(x+1)}{5\cdot\sqrt[5]{(x^2+2x+3)^4}}$

D) $\frac{2(x-1)}{4\cdot\sqrt[5]{(x^2+2x+3)^4}}$

E) $\frac{2(x+1)}{4\cdot\sqrt[5]{(x^2+2x+3)^4}}$

3. $f(x) = (x^3 - x^2)^3 \Rightarrow$

$\lim_{h\to 0} \frac{f(1+h) - f(h)}{h} = ?$

A) –2

B) 0

C) 2

D) 4

E) 6

4. $f(x) = x^2 - 3x + 1 \Rightarrow$

$\lim_{x\to 1} \frac{f(x) - f(1)}{1-x} = ?$

A) –1

B) 0

C) 1

D) 2

E) 3

5. $f(x) = x^3 - 4|x| + 2x \Rightarrow f'(2) = ?$

A) 8

B) 10

C) 12

D) 15

E) 24

6. $f(2x - 1) = 2x^2 - 6x + 4 \Rightarrow \frac{df(x)}{dx} = ?$

A) 2x + 1

B) 2x – 1

C) x – 2

D) 2x – 2

E) x – 3

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7. $f(x)=\frac{x^3-2x^2+1}{3x^2} \Rightarrow f'(-1)=?$

A) 4
B) 3
C) 2
D) 1
E) 0

8. $f(x) = (x^3 - x)\cdot(x^2 + 2x)^3 \Rightarrow f'(1) = ?$

A) 42
B) 44
C) 48
D) 50
E) 54

9. $f(x) = \arcsin\left(x^2+\frac{1}{4}\right) \Rightarrow f'(0) = ?$

A) -1
B) 0
C) $\frac{1}{2}$
D) 1
E) $\frac{3}{2}$

10. $f(x) = \arctan x^2 \Rightarrow f'(-1) = ?$

A) -3
B) -2
C) -1
D) 0
E) 1

11. $f(x) = \ln(ax + 6)$

$\left.\frac{d}{dx}f(x)\right|_{x=1} = \frac{a}{5} \Rightarrow a = ?$

A) -2
B) -1
C) 0
D) 1
E) 2

12. $f(x) = 4^x \Rightarrow f'(2) = ?$

A) 4ln2
B) 6ln2
C) 8ln2
D) 12ln4
E) 16ln4

YANITLAR / ANSWERS							
1. A	2. B	3. B	4. C	5. B	6. C	7. D	8. E
9. B	10. C	11. B	12. E				

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1. $\left.\begin{array}{l} y = \sin\left(\dfrac{2x-1}{x+1}\right) \\ x = \dfrac{\pi+2}{4-\pi} \end{array}\right\} \Rightarrow \dfrac{dy}{dx} = ?$

A) 0
B) 1
C) 3
D) 5
E) 7

2. $f(x) = |x^3 - 2x^2 + 3| \Rightarrow f'(3) = ?$

A) 6
B) 9
C) 10
D) 12
E) 15

3. $f(x) = \dfrac{x^2+ax}{3x-1}$, $f'(1) = -\dfrac{1}{2} \Rightarrow a = ?$

A) 2
B) 3
C) 5
D) 8
E) 12

4. $f(x) = \arctan(3x^2 - x) \Rightarrow f'(1) = ?$

A) $\dfrac{1}{9}$
B) $\dfrac{1}{7}$
C) $\dfrac{1}{5}$
D) $\dfrac{1}{3}$
E) 1

5. $y = \log(2x+1) \Rightarrow \dfrac{dy}{dx} = ?$

A) $\dfrac{2}{2x+1}$
B) $\dfrac{2}{2x+1} \cdot \ln 10$
C) $\dfrac{2\log e}{2x+1}$
D) $\dfrac{2}{(2x+1)^2}$
E) $\dfrac{\log e}{2x+1}$

6. $f(x) = \ln\left[\dfrac{(8x^3-3x^2+5)^3}{2x-1}\right] \Rightarrow f'(1) = ?$

A) $\dfrac{11}{5}$
B) $\dfrac{12}{5}$
C) $\dfrac{14}{5}$
D) $\dfrac{17}{5}$
E) $\dfrac{18}{5}$

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7. $f(x) = 5^{\cos x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) $-\ln 5$

B) $-2\ln 5$

C) $-3\ln 5$

D) $\ln 25$

E) $\ln 125$

8. $f(x) = 2^{\ln 3x} \Rightarrow f'\left(\frac{1}{3}\right) = ?$

A) $\ln 2$

B) $2\ln 2$

C) $3\ln 2$

D) $4\ln 2$

E) $5\ln 2$

9. $f(x) = (4)^{3^{2x}} \Rightarrow f'(2) = ?$

A) $2\cdot 3^{3}\cdot 4^{72}\cdot \ln 3\cdot \ln 4$

B) $2\cdot 3^{4}\cdot 4^{81}\cdot \ln 3\cdot \ln 4$

C) $2\cdot 3^{5}\cdot 4^{80}\cdot \ln 3\cdot \ln 4$

D) $2\cdot 3^{2}\cdot 4^{70}\cdot \ln 3\cdot \ln 4$

E) $2\cdot 3^{6}\cdot 4^{90}\cdot \ln 3\cdot \ln 4$

10. $f(x) = \frac{(x-1)^3}{4} \Rightarrow (f^{-1})(16)\cdot(f^{-1})'(16) = ?$

A) $\frac{5}{4}$

B) $\frac{5}{8}$

C) $\frac{5}{12}$

D) $\frac{5}{16}$

E) $\frac{5}{48}$

11. $y^2 = e^{x+y} \Rightarrow \frac{d^2y}{dx^2} = ?$

A) $\frac{2y}{(2-y)^2}$

B) $\frac{2y}{(2+y)^2}$

C) $\frac{y}{(2-y)^2}$

D) $\frac{2y}{(2+y)^2}$

E) $\frac{y}{(2+y)^3}$

12. $y = (2x-5)^5 \Rightarrow \frac{d^2y}{dx^2} = ?$

A) $20(2x-5)^3$

B) $40(2x-5)^3$

C) $60(2x-5)^3$

D) $80(2x-5)^3$

E) $90(2x-5)^3$

YANITLAR / ANSWERS							
1. A	2. E	3. B	4. E	5. C	6. D	7. A	8. C
9. B	10. C	11. A	12. D				

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1. $f(x) = \sqrt[5]{-x^3 + 2x} \Rightarrow f'(-1) = ?$

A) -5

B) $-\frac{1}{5}$

C) 1

D) $\frac{1}{5}$

E) 5

2. $y = \frac{2x+1}{x-2} \Rightarrow \left.\frac{dy}{dx}\right|_{x=3} = ?$

A) -5

B) -4

C) -3

D) 3

E) 4

3. $f(x) = \ln \frac{x-a}{x+a} \Rightarrow f'(x) = ?$

A) $\frac{2a+2x}{x^2-a^2}$

B) $\frac{2x^2}{a^2-x^2}$

C) $\frac{2x^2}{x^2-a^2}$

D) $\frac{2a}{a^2-x^2}$

E) $\frac{2a}{x^2-a^2}$

4. $f(x) = \sin x,\ g(x) = 2^x \Rightarrow (fog)'(2) = ?$

A) $\ln 2 \cdot \cos 16°$

B) $\ln 4 \cdot \cos 4°$

C) $\ln 8 \cdot \cos 4°$

D) $\ln 16 \cdot \cos 4°$

E) $\ln 16 \cdot \cos 16°$

5. $\left.\begin{matrix} x = \ln t \\ y = \sin t \end{matrix}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=\pi} = ?$

A) $-\pi$

B) -2π

C) 0

D) 1

E) 2

6. $y = \ln[(x^2+2)^2 \cdot (x^3+x-1)] \Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = ?$

A) $\frac{37}{9}$

B) $\frac{8}{3}$

C) $\frac{25}{9}$

D) 3

E) 4

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7. $y = \ln^2(2x+3) \Rightarrow \left.\frac{dy}{dx}\right|_{x=3} = ?$

A) $\frac{\ln 3}{9}$

B) $\frac{4\ln 3}{9}$

C) $\frac{8}{9}$

D) $\frac{16}{27}$

E) $\frac{8\ln 3}{9}$

8. $y = e^x \cdot x^{3x} \Rightarrow \frac{dy}{dx} = ?$

A) $e^x \cdot x^{3x} \cdot (4 + 3\ln x)$

B) $e^x \cdot x^{3x} \cdot (4 - 3\ln x)$

C) $e^x \cdot x^{3x} \cdot (4 + 2\ln x)$

D) $e^x \cdot x^{3x} \cdot (4 - 2\ln x)$

E) $e^x \cdot x^{3x} \cdot (4 - \ln x)$

9. $\left.\begin{matrix} y = e^x \\ x = \cos t \end{matrix}\right\} \Rightarrow \left.\frac{dy}{dt}\right|_{t=\frac{\pi}{3}} = ?$

A) $-\frac{\sqrt{3e}}{2}$

B) $-\frac{\sqrt{e}}{4}$

C) $-\frac{\sqrt{2e}}{3}$

D) $\sqrt{3e}$

E) $\sqrt{2e}$

10. $\left.\begin{matrix} y = e^t + 2t \\ x = e^{2t} \end{matrix}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=0} = ?$

A) $-\frac{3}{4}$

B) $-\frac{5}{4}$

C) $-\frac{7}{4}$

D) $\frac{1}{2}$

E) $\frac{3}{2}$

11. $xy = (x+y)^2 \Rightarrow \frac{dy}{dx} = ?$

A) $-\frac{2x+y}{x+2y}$

B) $\frac{x-y}{x+y}$

C) $\frac{y-2x}{2y-x}$

D) $\frac{2xy}{x+y}$

E) $\frac{x+y}{y-2x}$

12. $F(x, y) = 3x^4 - 5x^2y^2 + y^3 - 4x + 5 = 0$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,1)} = ?$

A) $\frac{2}{7}$ B) $\frac{1}{7}$ C) 0

D) $-\frac{1}{7}$ E) $-\frac{2}{7}$

YANITLAR / ANSWERS

1. B	2. A	3. E	4. D	5. A	6. C	7. E	8. A
9. A	10. B	11. A	12. E				

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1. $f(x) = 2x^3 - 3x^2 - 12x + 20 \Rightarrow f'(-1) + f''(1) = ?$

A) -6
B) -1
C) 0
D) 1
E) 6

2. $f(x) = x^3 + ax^2 + bx + 3$,
$f'(1) = -2$, $f''(2) = 0 \Rightarrow b = ?$

A) 2
B) 3
C) 5
D) 6
E) 7

3. $f(x) = \ln \cos x \Rightarrow f'\left(\frac{\pi}{3}\right) = ?$

A) $-2\sqrt{3}$
B) $-\sqrt{3}$
C) -1
D) $\sqrt{3}$
E) $2\sqrt{3}$

4. $f(x) = (x+1)^{\frac{1}{x}} \Rightarrow f'(1) = ?$

A) $\ln\frac{e}{4}$
B) $\ln 2e$
C) $2 - \ln 2$
D) 1
E) -1

5. $y = f(x)$, $\frac{x-y}{x-2y} = 2 \Rightarrow f'(x) = ?$

A) $\frac{2}{3}$
B) 1
C) $\frac{1}{3}$
D) $-\frac{1}{3}$
E) $-\frac{4}{3}$

6. $\left.\begin{matrix} x = t^2 - 2t \\ y = t^3 + t \end{matrix}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=0} = ?$

A) 2
B) $\frac{4}{3}$
C) 1
D) $\frac{3}{4}$
E) $\frac{1}{4}$

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7. $\left.\begin{array}{l} x = 3t^3 + 2t^2 - 1 \\ y = 8t^2 - 3t \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=1} = ?$

A) 1

B) $\frac{3}{2}$

C) 2

D) $\frac{5}{2}$

E) 3

8. $f(x) = x^{2x} \Rightarrow f'(2) = ?$

A) $8(\ln 2 + 1)$

B) $16(\ln 2 + 1)$

C) $32(\ln 2 + 1)$

D) $64(\ln 2 + 1)$

E) $128(\ln 2 + 1)$

9. $\left.\begin{array}{l} x = 2 \cdot \sin t \\ y = 3 \cdot \cos t \end{array}\right\} \Rightarrow \dfrac{d^2y}{dx^2} = ?$

A) $-\frac{3}{4\cos^3 t}$

B) $-\frac{3}{4\sin t}$

C) $\frac{3}{4\tan t}$

D) $\frac{3}{4\sin t}$

E) $\frac{3\sin t}{4\cos t}$

10. $\left.\begin{array}{l} x = t^2 + 3 \\ y = 3t^2 - 2t \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=1} = ?$

A) –2

B) –1

C) 0

D) 1

E) 2

11. $\sin^2 x + \cos^2 y - 2x = 0 \Rightarrow \dfrac{dy}{dx} = ?$

A) $\tan 2x$

B) $\frac{\sin 2x + 1}{\sin y}$

C) $\frac{\sin 2x - 2}{\sin 2y}$

D) $\frac{\sin 2x - 1}{\cos 2y}$

E) $\frac{\sin x - 2}{\cos 2y}$

12. $\sin(xy) = x^2 + y^3 \Rightarrow \dfrac{dy}{dx} = ?$

A) $\frac{2x - y \cdot \cos(xy)}{x \cdot \cos(xy) - 3y^2}$

B) $\frac{2x + y \cdot \sin(xy}{x \cdot \cos(xy) - 3y^2}$

C) $\frac{2x + y \cdot \cos(xy)}{x \cdot \sin(xy) - 3y^2}$

D) $\frac{2x - y \cdot \cos(xy)}{x \cdot \cos(xy) + 3y^2}$

E) $\frac{2x + y \cdot \cos(xy)}{x \cdot \cos(xy) + 3y^2}$

YANITLAR / ANSWERS

1. E	2. E	3. B	4. A	5. C	6. E	7. A	8. C
9. A	10. E	11. C	12. A				

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BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

L' Hospital, Guillaume François Antoine, Marquis de [1661 – 1704]

Fransız matematikçi. 1696'da diferansiyel calculus ile ilgili ilk yazısını yazmıştır. Bu ve daha sonra analitik geometri ile ilgili yazmış olduğu kitap, 18. Yüzyılda yazılan yazılardandı. Ilki L'Hospital kuralı olarak bilinen kuralı içermektedir.

French mathematician who in 1696 produced the first textbook on differential calculus. This, and a subsequent book on analytical geometry, were standard texts for much of the eighteenth century. The first contains l' Hospital' s rule, known to be due to Jean Bernoulli, who is thought to have agreed to keep the Marquis l' Hopital informed of his discoveries in return for financial support.

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APPLICATIONS of DERIVATIVE

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- ✔ Türevin Geometrik Anlamı . . . . . 610 – 612
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- ✔ Çözümlü Test . . . . . 618 – 621
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TÜREVİN UYGULAMALARI
APPLICATIONS of DERIVATIVE

BÖLÜM 2
CHAPTER

L' HOSPITAL KURALI

$\lim_{x\to x_0} \frac{f(x)}{g(x)}$ limiti bulunurken $\frac{0}{0}, \frac{\infty}{\infty}$ şeklinde belirsizlik oluşuyorsa limit, pay ve paydanın ayrı ayrı türevleri alınarak bulunabilir.

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x_0)}{g'(x_0)}$$

Tekrar belirsizlik devam ediyorsa kural yeniden uygulanır.

ÖRNEK 1

$f(x) = \dfrac{\sqrt{x}-2\sqrt{2}}{\sqrt[3]{x}-2} \Rightarrow \lim_{x\to 8} f(x) = ?$

A) $\dfrac{\sqrt{2}}{4}$ B) $\dfrac{\sqrt{2}}{2}$ C) $\dfrac{3\sqrt{2}}{2}$

D) $\dfrac{1}{2}$ E) $\dfrac{3\sqrt{2}}{4}$

ÇÖZÜM

Yanıt C

$$\lim_{x\to 8} \frac{\sqrt{x}-2\sqrt{2}}{\sqrt[3]{x}-2} = \lim_{x\to 8} \frac{\sqrt{8}-2\sqrt{2}}{\sqrt[3]{8}-2} \quad (\frac{0}{0})$$

$$\lim_{x\to 8} \frac{\left(\sqrt{x}-2\sqrt{2}\right)'}{\left(\sqrt[3]{x}-2\right)'} = \lim_{x\to 8} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{3\cdot\sqrt[3]{x^2}}}$$

$$= \frac{1}{2\sqrt{8}} \cdot \frac{3\sqrt[3]{64}}{1}$$

$$= \frac{3\sqrt{2}}{2}$$

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L' HOSPITAL'S RULE

$\lim_{x\to x_0} \frac{f(x)}{g(x)}$ is equal to $\frac{0}{0}, \frac{\infty}{\infty}$ then the limit is found by taking the derivatives of the numerator and denominator seperately.

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x_0)}{g'(x_0)}$$

If the indeterminate forms continue, then apply the same rule one more time.

EXAMPLE 1

$$f(x) = \frac{\sqrt{x} - 2\sqrt{2}}{\sqrt[3]{x} - 2} \Rightarrow \lim_{x\to 8} f(x) = ?$$

A) $\frac{\sqrt{2}}{4}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{3\sqrt{2}}{2}$

D) $\frac{1}{2}$ E) $\frac{3\sqrt{2}}{4}$

SOLUTION

The answer is C

$$\lim_{x\to 8} \frac{\sqrt{x} - 2\sqrt{2}}{\sqrt[3]{x} - 2} = \lim_{x\to 8} \frac{\sqrt{8} - 2\sqrt{2}}{\sqrt[3]{8} - 2} \quad (\frac{0}{0})$$

$$\lim_{x\to 8} \frac{\left(\sqrt{x} - 2\sqrt{2}\right)'}{\left(\sqrt[3]{x} - 2\right)'} = \lim_{x\to 8} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{3\cdot\sqrt[3]{x^2}}}$$

$$= \frac{1}{2\sqrt{8}} \cdot \frac{3\sqrt[3]{64}}{1}$$

$$= \frac{3\sqrt{2}}{2}$$

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ÖRNEK 2

$$\lim_{x\to e}\frac{e^{\ln x}-e}{\ln(\ln x)}=?$$

A) $\frac{e}{2}$ B) $\frac{e^2}{2}$ C) $\frac{\sqrt{e}}{2}$ D) e^2 E) e

ÇÖZÜM

Yanıt **E**

$$\lim_{x\to e}\frac{e^{\ln x}-e}{\ln(\ln x)}=\frac{e-e}{\ln(\ln e)}=\frac{0}{\ln 1}=\frac{0}{0}$$

$$\lim_{x\to e}\frac{e^{\ln x}-e}{\ln(\ln x)}=\lim_{x\to e}\frac{\frac{1}{x}e^{\ln x}}{\frac{1}{x}\frac{1}{\ln x}}=\frac{1}{\frac{1}{e}\cdot 1}=e$$

ÖRNEK 3

$$\lim_{x\to -1}\frac{\sin\pi x}{x+1}=?$$

A) $-\pi$ B) $-\frac{\pi}{2}$ C) 0 D) $\frac{\pi}{2}$ E) π

ÇÖZÜM

Yanıt **A**

$$\lim_{x\to -1}\frac{\sin(\pi x)}{x+1}=\frac{0}{0}$$

$$\lim_{x\to -1}\frac{\sin(\pi x)}{x+1}=\lim_{x\to -1}\frac{\cos\pi x\cdot\pi}{1}=\pi\cdot\cos\pi=-\pi$$

PEKİŞTİRME

$$\lim_{x\to 1}\frac{\cos\frac{\pi}{2}x}{x-1}=?$$

YANIT : $-\pi/2$

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EXAMPLE 2

$$\lim_{x\to e}\frac{e^{\ln x}-e}{\ln(\ln x)}=?$$

A) $\frac{e}{2}$ B) $\frac{e^2}{2}$ C) $\frac{\sqrt{e}}{2}$ D) e^2 E) e

SOLUTION

The answer is E

$$\lim_{x\to e}\frac{e^{\ln x}-e}{\ln(\ln x)}=\frac{e-e}{\ln(\ln e)}=\frac{0}{\ln 1}=\frac{0}{0}$$

$$\lim_{x\to e}\frac{e^{\ln x}-e}{\ln(\ln x)}=\lim_{x\to e}\frac{\frac{1}{x}e^{\ln x}}{\frac{1}{x}\frac{1}{\ln x}}=\frac{1}{\frac{1}{e}\cdot 1}=e$$

REINFORCE

$$\lim_{x\to 1}\frac{\cos\frac{\pi}{2}x}{x-1}=?$$

ANSWER : $-\pi/2$

EXAMPLE 3

$$\lim_{x\to -1}\frac{\sin\pi x}{x+1}=?$$

A) $-\pi$ B) $-\frac{\pi}{2}$ C) 0 D) $\frac{\pi}{2}$ E) π

SOLUTION

The answer is A

$$\lim_{x\to -1}\frac{\sin(\pi x)}{x+1}=\frac{0}{0}$$

$$\lim_{x\to -1}\frac{\sin(\pi x)}{x+1}=\lim_{x\to -1}\frac{\cos\pi x\cdot\pi}{1}=\pi\cdot\cos\pi=-\pi$$

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ÖRNEK 4

$$\lim_{x\to\pi}\frac{1+\cos x}{1-\sin\frac{x}{2}}=?$$

A) 2 B) 3 C) 4 D) $\frac{3}{4}$ E) $\frac{9}{2}$

ÇÖZÜM

Yanıt C

$$\lim_{x\to\pi}\frac{1+\cos x}{1-\sin\frac{x}{2}}=\frac{1+\cos\pi}{1-\sin\frac{\pi}{2}}=\frac{0}{0}$$

$$\lim_{x\to\pi}\frac{1+\cos x}{1-\sin\frac{x}{2}}=\lim_{x\to\pi}\frac{-\sin x}{-\frac{1}{2}\cos\frac{x}{2}}=\frac{0}{0}$$

$$\lim_{x\to\pi}\frac{-\cos x}{\frac{1}{4}\sin\frac{x}{2}}=\frac{-\cos\pi}{\frac{1}{4}\sin\frac{\pi}{2}}$$

$$=\frac{-(-1)}{\frac{1}{4}\cdot 1}=4$$

ÖRNEK 5

$$\lim_{x\to 0}\frac{\ln(1+4x)}{\sin 5x}=?$$

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) 1

ÇÖZÜM

Yanıt D

$$\lim_{x\to 0}\frac{\ln(1+4x)}{\sin 5x}=\frac{\ln(1+4\cdot 0)}{\sin 5\cdot 0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\ln(1+4x)}{\sin 5x}=\lim_{x\to 0}\frac{\frac{4}{1+4x}}{5\cos 5x}=\frac{4}{5}$$

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EXAMPLE 4

$$\lim_{x\to\pi}\frac{1+\cos x}{1-\sin\frac{x}{2}}=?$$

A) 2 B) 3 C) 4 D) $\frac{3}{4}$ E) $\frac{9}{2}$

SOLUTION

The answer is C

$$\lim_{x\to\pi}\frac{1+\cos x}{1-\sin\frac{x}{2}}=\frac{1+\cos\pi}{1-\sin\frac{\pi}{2}}=\frac{0}{0}$$

$$\lim_{x\to\pi}\frac{1+\cos x}{1-\sin\frac{x}{2}}=\lim_{x\to\pi}\frac{-\sin x}{-\frac{1}{2}\cos\frac{x}{2}}=\frac{0}{0}$$

$$\lim_{x\to\pi}\frac{-\cos x}{\frac{1}{4}\sin\frac{x}{2}}=\frac{-\cos\pi}{\frac{1}{4}\sin\frac{\pi}{2}}$$

$$=\frac{-(-1)}{\frac{1}{4}\cdot 1}=4$$

EXAMPLE 5

$$\lim_{x\to 0}\frac{\ln(1+4x)}{\sin 5x}=?$$

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) 1

SOLUTION

The answer is D

$$\lim_{x\to 0}\frac{\ln(1+4x)}{\sin 5x}=\frac{\ln(1+4\cdot 0)}{\sin 5\cdot 0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\ln(1+4x)}{\sin 5x}=\frac{\frac{4}{1+4x}}{5\cos 5x}=\frac{4}{5}$$

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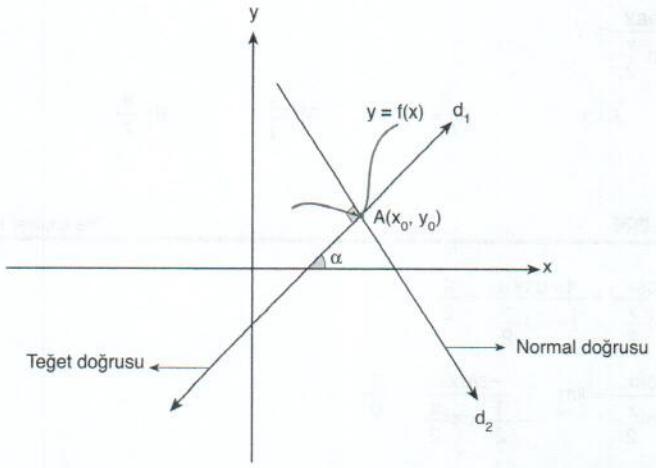
TÜREVİN GEOMETRİK ANLAMI



y = f(x) fonksiyonuna A (x_0, y_0) noktasından çizilen teğetin eğimi aşağıdaki gibidir.

$$\left.\begin{array}{l} m_{d_1} = f'(x_0) \\ m_{d_1} = \tan\alpha \end{array}\right\} \Rightarrow m_{d_1} = f'(x_0) = \tan\alpha$$

NOT

Normal doğrusunun eğimi, $m_{d_1} \cdot m_{d_2} = -1$ denkleminden bulunur.

ÖRNEK 6

$f(x) = x^3 + ax^2 + x$

$d : y = x - 3 \Rightarrow a = ?$

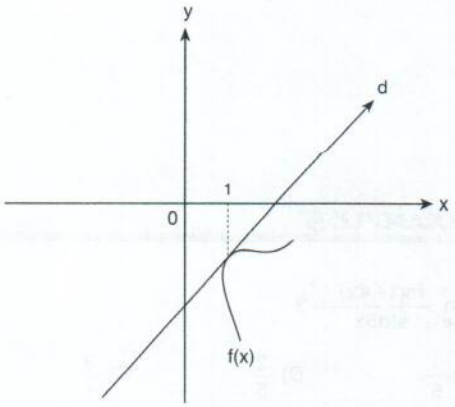
A) $-\frac{5}{2}$

B) $-\frac{3}{2}$

C) $-\frac{1}{2}$

D) $\frac{1}{2}$

E) $\frac{3}{2}$



ÇÖZÜM

Yanıt B

$f'(1) = m_d$

$f'(1) = 3 \cdot 1^2 + 2a \cdot 1 + 1 = 2a + 4$

$y = x - 3 \Rightarrow m_d = 1$

$2a + 4 = 1$

$2a = -3$

$a = -\frac{3}{2}$

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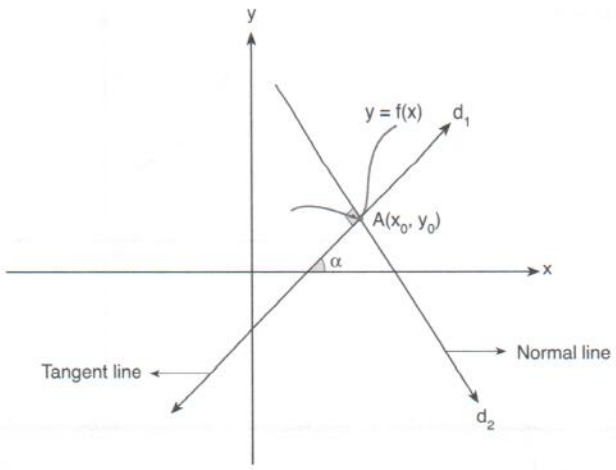
NOTE

Slope of normal line is found following equation.

$$m_{d_1} \cdot m_{d_2} = -1$$

GEOMETRIC MEANING of DERIVATIVE



The function y = f(x) has a tangent at A (x_0, y_0) whose slope is

$$\left.\begin{array}{l} m_{d_1} = f'(x_0) \\ m_{d_1} = \tan\alpha \end{array}\right\} \Rightarrow m_{d_1} = f'(x_0) = \tan\alpha$$

EXAMPLE 6

$f(x) = x^3 + ax^2 + x$

$d : y = x - 3 \Rightarrow a = ?$

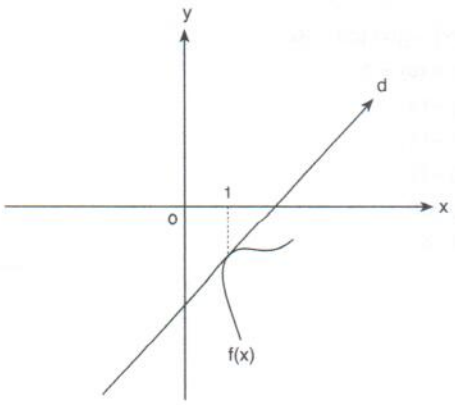
A) $-\frac{5}{2}$

B) $-\frac{3}{2}$

C) $-\frac{1}{2}$

D) $\frac{1}{2}$

E) $\frac{3}{2}$



SOLUTION

The answer is **B**

$f'(1) = m_d$

$f'(1) = 3 \cdot 1^2 + 2a \cdot 1 + 1 = 2a + 4$

$y = x - 3 \Rightarrow m_d = 1$

$2a + 4 = 1$

$2a = -3$

$a = -\frac{3}{2}$

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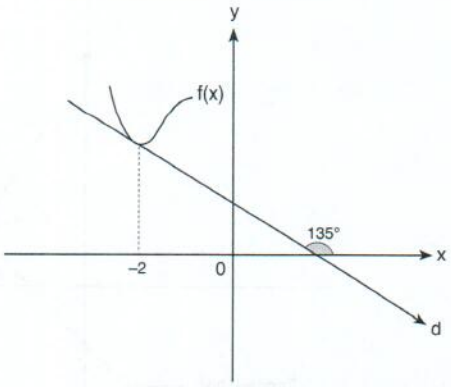
ÖRNEK 7

$f(x) = 2x^3 + x^2 - ax$

$\Rightarrow$ **a = ?**

A) –27
B) –23
C) –17
D) –13
E) 21



ÇÖZÜM

Yanıt **E**

$f'(-2) = \tan 135° = -1$

$f'(-2) = 6\cdot(-2)^2 + 2\cdot(-2) - a = -1$

$24 - 4 - a = -1$

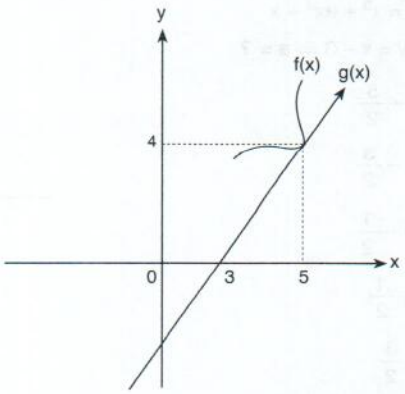
$a = 21$

ÖRNEK 8

$h(x) = g(x)\cdot f(x) - 3x^2$

$\Rightarrow$ **h'(5) = ?**

A) –16
B) –14
C) –12
D) –10
E) –8



ÇÖZÜM

Yanıt **B**

$h(x) = g(x)\cdot f(x) - 3x^2$

$h'(x) = g'(x)\cdot f(x) + g(x)\cdot f'(x) - 6x$

$h'(5) = g'(5)\cdot f(5) + g(5)\cdot f'(5) - 30$

g : (5, 4) (3, 0)

$\frac{y-0}{0-4} = \frac{x-3}{3-5} \Rightarrow y = 2x - 6$

$h'(5) = 2\cdot 4 + 4\cdot\frac{4}{2} - 30 = 16 - 30 = -14$

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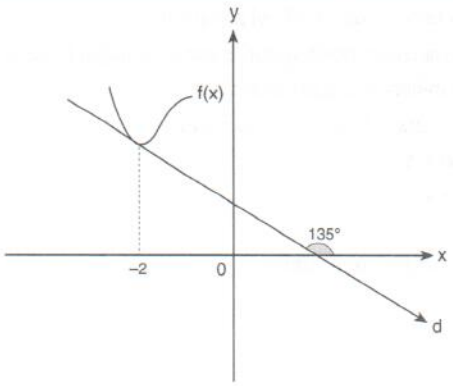
EXAMPLE 7

$f(x) = 2x^3 + x^2 - ax$

$\Rightarrow$ **a = ?**

A) –27
B) –23
C) –17
D) –13
E) 21



SOLUTION

The answer is E

$f'(-2) = \tan 135° = -1$

$f'(-2) = 6\cdot(-2)^2 + 2\cdot(-2) - a = -1$

$24 - 4 - a = -1$

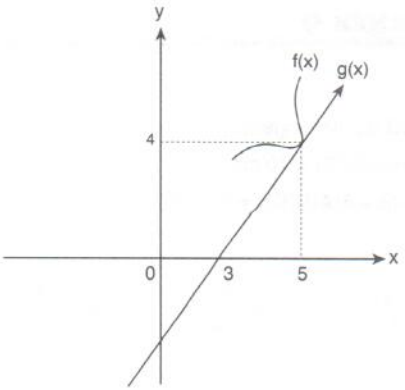
$a = 21$

EXAMPLE 8

$h(x) = g(x)\cdot f(x) - 3x^2$

$\Rightarrow$ **h'(5) = ?**

A) –16
B) –14
C) –12
D) –10
E) –8



SOLUTION

The answer is B

$h(x) = g(x)\cdot f(x) - 3x^2$

$h'(x) = g'(x)\cdot f(x) + g(x)\cdot f'(x) - 6x$

$h'(5) = g'(5)\cdot f(5) + g(5)\cdot f'(5) - 30$

g : (5, 4) (3, 0)

$\frac{y-0}{0-4} = \frac{x-3}{3-5} \Rightarrow y = 2x - 6$

$h'(5) = 2\cdot 4 + 4\cdot\frac{4}{2} - 30 = 16 - 30 = -14$

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Maksimum ve Minimum

Bir fonksiyonun 1. türevi alınıp sıfıra eşitlenirse, bulunan x değeri, fonksiyonun maksimum veya minimum noktası olur. Bulunan x değeri fonksiyonda yazılırsa, fonksiyonun maksimum veya minimum değeri elde edilir.

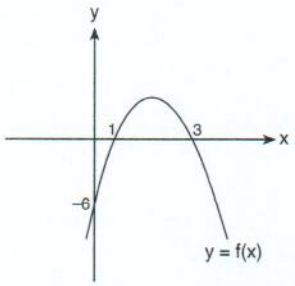
$y = -2(x-1)(x-3) = -2x^2 + 8x - 6$

$f'(x) = 0$

$-4x + 8 = 0$

$x = 2$

2, maksimum noktadır.

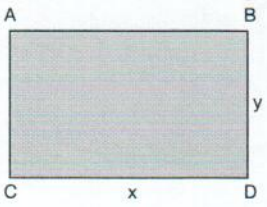


ÖRNEK 9

ABCD bir dikdörtgendir.

Çevre(ABCD) = 10 cm

⇒ **maks(A(ABCD)) = ? cm²**



A) $\frac{25}{4}$ B) $\frac{27}{4}$ C) $\frac{29}{4}$ D) $\frac{31}{4}$ E) $\frac{33}{4}$

ÇÖZÜM

Yanıt **A**

$2x + 2y = 10$

$x + y = 5 \Rightarrow y = 5 - x$

$A(ABCD) = x \cdot y = x(5-x) = 5x - x^2$

$A' = 5 - 2x = 0$

$x = \frac{5}{2} \Rightarrow y = \frac{5}{2}$

$\max(A(ABCD)) = \frac{5}{2} \cdot \frac{5}{2} = \frac{25}{4}$ cm²

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Maximum and Minimum

If we take first derivative of a function and equalizing zero, finding x is maximum and minimum point of the function. If finding x is written to the function, we obtain maximum and minimum value of function.

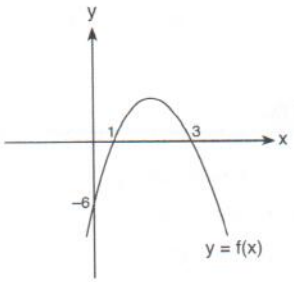
$y = -2(x-1)(x-3) = -2x^2 + 8x - 6$

$f'(x) = 0$

$-4x + 8 = 0$

$x = 2$

2 is maximum point.

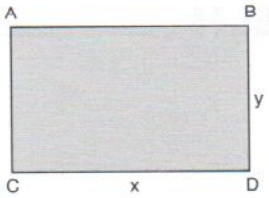


EXAMPLE 9

ABCD is a rectangle.

The perimeter(ABCD) = 10 cm

$\Rightarrow$ **max(A(ABCD)) = ? cm^2**



A) $\frac{25}{4}$ B) $\frac{27}{4}$ C) $\frac{29}{4}$ D) $\frac{31}{4}$ E) $\frac{33}{4}$

SOLUTION

The answer is **A**

$2x + 2y = 10$

$x + y = 5 \Rightarrow y = 5 - x$

$A(ABCD) = x \cdot y = x(5-x) = 5x - x^2$

$A' = 5 - 2x = 0$

$x = \frac{5}{2} \Rightarrow y = \frac{5}{2}$

$\max(A(ABCD)) = \frac{5}{2} \cdot \frac{5}{2} = \frac{25}{4}$ cm^2

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ÖRNEK 10

$x^2 + (k - 1)x + 5 = 0,$

ÇK = $\{x_1, x_2\}$ $\Rightarrow$ $\min(x_1^2 + x_2^2) = ?$

A) –10 B) –8 C) –4 D) 8 E) 10

ÇÖZÜM

Yanıt **A**

$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 \cdot x_2$

$= (1 - k)^2 - 2 \cdot 5$

$= -9 - 2k + k^2$

$(x_1^2 + x_2^2)' = -2 + 2k = 0$

$k = 1$

$\Rightarrow \min(x_1^2 + x_2^2) = -9 - 2 \cdot 1 + 1^2 = -11 + 1 = -10$

ÖRNEK 11

a = ?

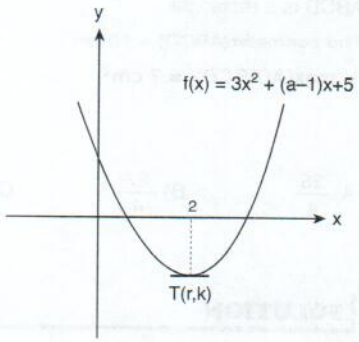
A) –1

B) –3

C) –5

D) –9

E) –11



ÇÖZÜM

Yanıt **E**

$f(x) = 3x^2 + (a - 1)x + 5$

$f'(2) = 0$

$6 \cdot 2 + a - 1 = 0$

$a = -11$

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EXAMPLE 10

$x^2 + (k-1)x + 5 = 0,$

$SS = \{x_1, x_2\} \Rightarrow \min(x_1^2 + x_2^2) = ?$

A) –10 B) –8 C) –4 D) 8 E) 10

SOLUTION

The answer is A

$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 \cdot x_2$

$= (1-k)^2 - 2 \cdot 5$

$= -9 - 2k + k^2$

$(x_1^2 + x_2^2)' = -2 + 2k = 0$

$k = 1$

$\Rightarrow \min(x_1^2 + x_2^2) = -9 - 2 \cdot 1 + 1^2 = -11 + 1 = -10$

EXAMPLE 11

a = ?

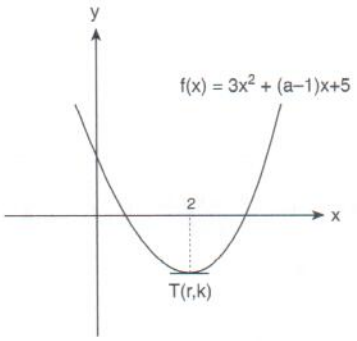
A) –1

B) –3

C) –5

D) –9

E) –11



SOLUTION

The answer is E

$f(x) = 3x^2 + (a-1)x + 5$

$f'(2) = 0$

$6 \cdot 2 + a - 1 = 0$

$a = -11$

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1. $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2+5}-3}{x^2-4} = ?$

A) $\dfrac{1}{2}$

B) $\dfrac{1}{3}$

C) $\dfrac{1}{4}$

D) $\dfrac{1}{6}$

E) $\dfrac{1}{8}$

2. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\sin x-1}{\cos x} = ?$

A) -1

B) 0

C) $\dfrac{1}{2}$

D) 1

E) $\dfrac{3}{2}$

3. $\lim\limits_{x\to 2} \dfrac{x^2+x+a}{x-2} = b \in R \Rightarrow a = ?$

A) -4

B) -6

C) -8

D) -10

E) -12

4. $\lim\limits_{x\to 1} \dfrac{\ln x}{x^2-1} = ?$

A) 0

B) $\dfrac{1}{2}$

C) 1

D) $\dfrac{3}{2}$

E) 2

5. $\lim\limits_{x\to 2} [(x-2)\cdot \ln(x-2)] = ?$

A) 0

B) 1

C) 2

D) 3

E) 4

6. $\lim\limits_{x\to 0} \dfrac{\sqrt[4]{2x}}{x} = ?$

A) ∞

B) $-\infty$

C) -1

D) 1

E) 0

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7. $\lim\limits_{x\to 2} \dfrac{3-\sqrt{5x-1}}{\sqrt{x+2}-x} = ?$

A) $\dfrac{5}{9}$

B) $\dfrac{2}{3}$

C) $\dfrac{8}{9}$

D) 1

E) $\dfrac{10}{9}$

8. $\lim\limits_{x\to \pi} \dfrac{\pi\cos 2x - x}{1+\cos x} = ?$

A) $-\infty$

B) $+\infty$

C) 0

D) 1

E) -1

9. $\lim\limits_{x\to \frac{\pi}{3}} \dfrac{1-2\cos x}{\sin\left(x-\frac{\pi}{3}\right)} = ?$

A) $\dfrac{\sqrt{2}}{2}$

B) 1

C) $\sqrt{2}$

D) $\dfrac{3}{2}$

E) $\sqrt{3}$

10. $\lim\limits_{x\to -2} \left(4-x^2\right)\cdot \tan\left(\dfrac{\pi x}{4}\right) = ?$

A) $\dfrac{8}{\pi}$

B) $\dfrac{16}{\pi}$

C) 0

D) $-\dfrac{16}{\pi}$

E) $-\dfrac{8}{\pi}$

11. $g(x) = \dfrac{f(x)}{x^2+2x} \Rightarrow g'(1) = ?$

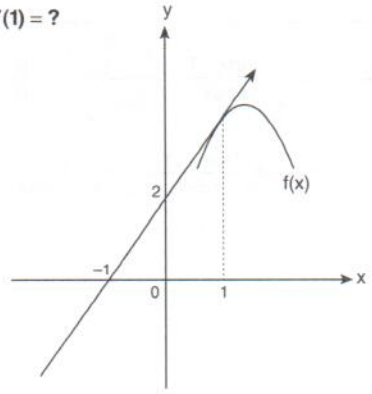
A) $-\dfrac{20}{9}$

B) $-\dfrac{16}{9}$

C) $-\dfrac{10}{9}$

D) $\dfrac{10}{9}$

E) $\dfrac{16}{9}$



12. $f(x) = \dfrac{2x+3}{x^2-ax+1-a} \Rightarrow a = ?$

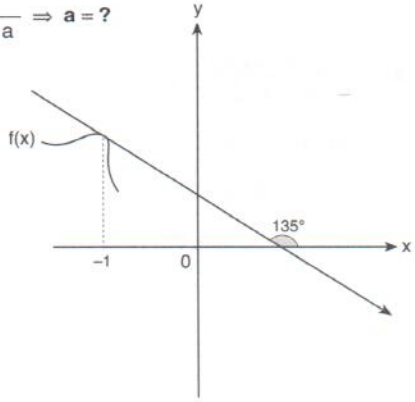
A) -10

B) -8

C) -6

D) -4

E) -2



YANITLAR / ANSWERS							
1. D	2. B	3. B	4. B	5. A	6. A	7. E	8. A
9. E	10. D	11. C	12. A				

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1. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-3}{x^2-4}=\dfrac{\sqrt{2^2+5}-3}{2^2-4}=\dfrac{0}{0}$

$$\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x^2-4}=\lim_{x\to 2}\frac{\frac{2x}{2\sqrt{x^2+5}}}{2x}$$

$$=\frac{1}{2\sqrt{4+5}}=\frac{1}{6}$$

Yanıt / The Answer is D

2. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin x-1}{\cos x}=\dfrac{\sin\frac{\pi}{2}-1}{\cos\frac{\pi}{2}}=\dfrac{0}{0}$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\cos x}=\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{-\sin x}=0$$

Yanıt / The Answer is B

3. $\lim\limits_{x\to 2}\dfrac{x^2+x+a}{x-2}=b\in R$

$$\Rightarrow x^2+x+a=0$$

$$4+2+a=0$$

$$a=-6$$

Yanıt / The Answer is B

4. $\lim\limits_{x\to 1}\dfrac{\ln x}{x^2-1}=\dfrac{\ln 1}{1^2-1}=\dfrac{0}{0}$

$$\Rightarrow \lim_{x\to 1}\frac{\ln x}{x^2-1}=\frac{\frac{1}{x}}{2x}=\frac{1}{2x^2}=\frac{1}{2}$$

Yanıt / The Answer is B

5. $\lim\limits_{x\to 2}\left[(x-2)\cdot\ln(x-2)\right]=0\cdot\ln 0=0\cdot\infty$

$$\lim_{x\to 2}\left[(x-2)\cdot\ln(x-2)\right]=\lim_{x\to 2}\frac{\ln(x-2)}{\frac{1}{x-2}}=\frac{\infty}{\infty}$$

$$\Rightarrow \lim_{x\to 2}\frac{\ln(x-2)}{\frac{1}{x-2}}=\lim_{x\to 2}\frac{\frac{1}{x-2}}{\frac{-1}{(x-2)^2}}$$

$$=\lim_{x\to 2}-(x-2)=0$$

Yanıt / The Answer is A

6. $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{2x}}{x}=\dfrac{0}{0}$

$$\lim_{x\to 0}\frac{2\cdot\frac{1}{4}(2x)^{-\frac{3}{4}}}{1}=\lim_{x\to 0}\frac{1}{2(2x)^{\frac{3}{4}}}=\frac{1}{0}=\infty$$

Yanıt / The Answer is A

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7. $\lim\limits_{x\to 2} \dfrac{3-\sqrt{5x-1}}{\sqrt{x+2}-x} = \dfrac{3-3}{2-2} = \dfrac{0}{0}$

$\Rightarrow \lim\limits_{x\to 2} \dfrac{3-\sqrt{5x-1}}{\sqrt{x+2}-x} = \lim\limits_{x\to 2} \dfrac{\dfrac{-5}{2\sqrt{5x-1}}}{\dfrac{1}{2\sqrt{x+2}}-1}$

$= \lim\limits_{x\to 2} \dfrac{-\dfrac{5}{6}}{-\dfrac{3}{4}} = \dfrac{10}{9}$

Yanıt / The Answer is E

8. $\lim\limits_{x\to \pi} \dfrac{\pi\cos 2x - x}{1+\cos x} = \dfrac{\pi\cdot\cos 2\pi - \pi}{1+\cos\pi} = \dfrac{0}{0}$

$\Rightarrow \lim\limits_{x\to \pi} \dfrac{\pi\cos 2x - x}{1+\cos x} = \lim\limits_{x\to \pi} \dfrac{-2\pi\cdot\sin 2x - 1}{-\sin x}$

$= -\dfrac{1}{0} = -\infty$

Yanıt / The Answer is A

9. $\lim\limits_{x\to \frac{\pi}{3}} \dfrac{1-2\cos x}{\sin\left(x-\frac{\pi}{3}\right)} = \dfrac{0}{0}$

$\Rightarrow \lim\limits_{x\to \frac{\pi}{3}} \dfrac{1-2\cos x}{\sin\left(x-\frac{\pi}{3}\right)} = \dfrac{2\sin x}{\cos\left(x-\frac{\pi}{3}\right)}$

$= \dfrac{\sqrt{3}}{1} = \sqrt{3}$

Yanıt / The Answer is E

10. $\lim\limits_{x\to -2} \left(4-x^2\right)\cdot\tan\left(\dfrac{\pi x}{4}\right) = 0\cdot\infty$

$\Rightarrow \lim\limits_{x\to -2} \dfrac{4-x^2}{\dfrac{1}{\tan\frac{\pi x}{4}}} = \dfrac{0}{0}$

$\Rightarrow \lim\limits_{x\to -2} \dfrac{4-x^2}{\dfrac{1}{\tan\frac{\pi x}{4}}} = \lim\limits_{x\to -2} \dfrac{4-x^2}{\cot\frac{\pi}{4}x}$

$= \lim\limits_{x\to -2} \dfrac{-2x}{-\left(1+\cot^2\frac{\pi}{4}x\right)\cdot\frac{\pi}{4}} = \dfrac{4}{\frac{-\pi}{4}} = -\dfrac{16}{\pi}$

Yanıt / The Answer is D

11. $g(x) = \dfrac{f(x)}{x^2+2x} \Rightarrow g'(x) = \dfrac{f'(x)(x^2+2x) - f(x)(2x+2)}{(x^2+2x)^2}$

$g'(1) = \dfrac{2\cdot 3 - 4\cdot 4}{9} = -\dfrac{10}{9}$

Yanıt / The Answer is C

12. $f'(-1) = \tan 135° = -1$

$f(x) = \dfrac{2x+3}{x^2-ax+1-a}$

$\Rightarrow f'(x) = \dfrac{2(x^2-ax+1-a) - (2x+3)(2x-a)}{(x^2-ax+1-a)^2}$

$f'(-1) = \dfrac{4+1(2+a)}{4} = -1$

$6+a = -4$

$a = -10$

Yanıt / The Answer is A

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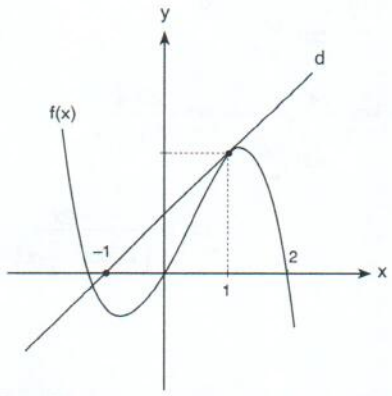
1.



d doğrusu, f(x) fonksiyonuna (1, 3) noktasında teğettir.

f′(1) + f(2) = ?

A) 1 B) $\frac{3}{2}$ C) 2

D) $\frac{5}{2}$ E) 3

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

---

2. $\lim\limits_{x \to 0^+} \frac{\ln \tan 2x}{\ln \tan 3x} = ?$

A) 1 B) $\frac{3}{2}$ C) $\frac{2}{3}$

D) 0 E) 2

[ANKARA ÜNİV. YÖS 2011]

---

3. $\lim\limits_{x \to \infty} \frac{2\sqrt[3]{x}}{\ln 3x} = ?$

A) $+\infty$ B) 0 C) $-\infty$

D) 1 E) –1

[ANKARA ÜNİV. YÖS 2011]

---

4. $\lim\limits_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{1 - \tan x} = ?$

A) 1 B) 0 C) $\sqrt{2}$

D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{3}}{2}$

[ANADOLU ÜNİV. YÖS 2011]

---

5. $\lim\limits_{x \to \infty} x \ln\left(1 + \frac{1}{x}\right) = ?$

A) ∞ B) –1 C) 0

D) 1 E) 2

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

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6. $\lim_{x\to 0} \frac{x+x\cos x}{\sin x \cdot \cos x} = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

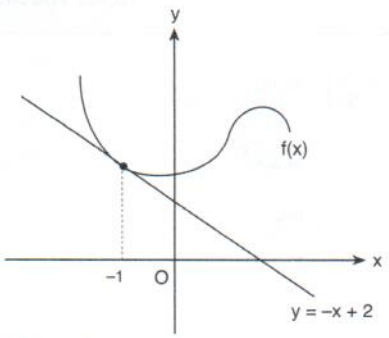
[YÖS 2010]

7. $\lim_{x\to \infty} \frac{2\sqrt{x}}{\ln x} = ?$

A) $-\infty$ B) 0 C) 1 D) 2 E) ∞

[YÖS 2010]

8.



$g(x) = x \cdot f(x) \Rightarrow g'(-1) = ?$

A) –3 B) –2 C) 2 D) 3 E) 4

[YÖS 2009]

9. $\lim_{x\to 5} \frac{\sqrt{x+11}-4}{\sqrt{x-1}-2} = ?$

A) $\frac{9}{5}$ B) $\frac{3}{5}$ C) $\frac{1}{5}$ D) $\frac{1}{2}$ E) $\frac{5}{2}$

[YÖS 2008]

10. $\lim_{x\to 0} \frac{\sqrt[3]{x+27}-3}{x} = ?$

A) $-\frac{1}{24}$ B) $-\frac{1}{9}$ C) $-\frac{1}{3}$ D) $\frac{1}{27}$ E) $\frac{1}{3}$

[YÖS 2007]

YANITLAR / ANSWERS							
1.B	2. A	3. A	4. D	5. D	6. E	7. E	8. E
9. D	10. D						

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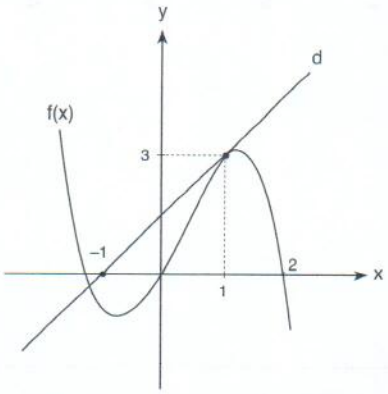
1.



$f'(x) = m_d \mid f(2) = 0$

$f'(1) = \frac{3}{2} \mid f'(1) + f(2) = \frac{3}{2} + 0 = \frac{3}{2}$

Yanıt / The Answer is B

2. $\lim\limits_{x \to 0^+} \frac{\ln \tan 2x}{\ln \tan 3x} = \lim\limits_{x \to 0^+} \frac{\frac{\tan 2x}{(1+\tan^2 2x)\cdot 2}}{\frac{\tan 3x}{(1+\tan^2 3x)\cdot 3}}$

$= \lim\limits_{x \to 0^+} \frac{3}{2} \frac{\tan 2x}{\tan 3x} \cdot \frac{\cos^2 3x}{\cos^2 2x}$

$= 1$

Yanıt / The Answer is A

3. $\lim\limits_{x \to \infty} \frac{2\sqrt[3]{x}}{\ln 3x} = \frac{\frac{2}{3} x^{\frac{-2}{3}}}{\frac{3}{3x}}$

$= \lim\limits_{x \to \infty} \frac{2}{3} x^{\frac{1}{3}}$

$= \infty$

Yanıt / The Answer is A

4. $\lim\limits_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{1 - \tan x} = \frac{-\sin x - \cos x}{-(1+\tan^2 x)} = \frac{\sqrt{2}}{2}$

Yanıt / The Answer is D

5. $\lim\limits_{x \to \infty} x \ln\left(1 + \frac{1}{x}\right) = \ln \lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

$= \ln e^{\frac{1}{x} x}$

$= \ln e$

$= 1$

Yanıt / The Answer is D

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6. $\lim\limits_{x\to 0} \dfrac{x+x\cos x}{\sin x\cdot\cos x} = \dfrac{0}{0}$

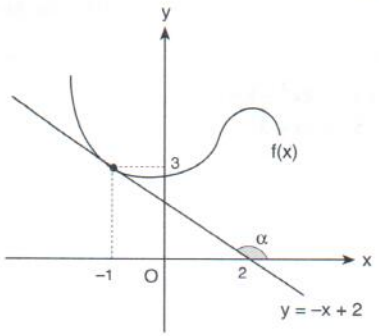
$\Rightarrow \lim\limits_{x\to 0} \dfrac{1+\cos x - x\cdot\sin x}{\cos^2 x - \sin^2 x} = \dfrac{1+1-0}{1-0} = 2$

Yanıt / The Answer is E

7. $\lim\limits_{x\to\infty} \dfrac{2\sqrt{x}}{\ln x} = \dfrac{\infty}{\infty} \Rightarrow \lim\limits_{x\to\infty} \dfrac{2\cdot\dfrac{1}{2\sqrt{x}}}{\dfrac{1}{x}} = \lim\limits_{x\to\infty} \sqrt{x} = \infty$

Yanıt / The Answer is E

8.



$\tan(180° - \alpha) = \dfrac{3}{3} = 1 \Rightarrow \tan\alpha = -1$

$g(x) = x \cdot f(x)$

$g'(x) = f(x) + x\cdot f'(x)$

$g'(-1) = f(-1) - f'(-1)$

$f(-1) = 3,\ f'(-1) = \tan\alpha = -1$

$\Rightarrow g'(-1) = 3 - (-1) = 4$

Yanıt / The Answer is E

9. $\lim\limits_{x\to 5} \dfrac{\sqrt{x+11}-4}{\sqrt{x-1}-2} = \dfrac{4-4}{2-2} = \dfrac{0}{0}$

$\Rightarrow \lim\limits_{x\to 5} \dfrac{\sqrt{x+11}-4}{\sqrt{x-1}-2} = \lim\limits_{x\to 5} \dfrac{\dfrac{1}{2\sqrt{x+11}}}{\dfrac{1}{2\sqrt{x-1}}}$

$= \dfrac{\dfrac{1}{2\cdot\sqrt{16}}}{\dfrac{1}{2\sqrt{4}}} = \dfrac{\dfrac{1}{8}}{\dfrac{1}{4}} = \dfrac{1}{2}$

Yanıt / The Answer is D

10. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{x+27}-3}{x} = \dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{\dfrac{1}{\sqrt[3]{(x+27)^2}}}{1} = \dfrac{1}{3\cdot\sqrt[3]{27^2}} = \dfrac{1}{27}$

Yanıt / The Answer is D

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1. $\lim_{x \to 1} \frac{x^4 - x^2 + 2x - 2}{x^3 + x - 2} = ?$

Yanıt / The Answer is **1**

2. $\lim_{x \to 3} \frac{\sqrt{2x-2} - 2}{x^2 - 2x - 3} = ?$

Yanıt / The Answer is $\frac{1}{8}$

3. $\lim_{x \to 2} \frac{\sqrt{4x+1} - 3}{x^3 - 2x - 4} = ?$

Yanıt / The Answer is $\frac{1}{15}$

4. $\lim_{x \to -1} \frac{x^4 - 3x^3 + x^2 - 5}{x^3 + 2x + 3} = ?$

Yanıt / The Answer is **–3**

5. $\lim_{x \to 1} \frac{x^3 - 2x^2 + 2x - 1}{x^4 - x^3 + 3x^2 + x - 4} = ?$

Yanıt / The Answer is $\frac{1}{8}$

6. $\lim_{x \to -1} \frac{x^4 + x^3 - 2x^2 + x + 3}{x^3 + 2x + 3} = ?$

Yanıt / The Answer is $\frac{4}{5}$

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7. $\lim_{x \to -1} \frac{x^3 + 4x^2 + 5x + 2}{3x^2 + 6x + 3} = ?$

Yanıt / The Answer is $\frac{1}{3}$

8. $\lim_{b \to 2} \frac{\sin(\pi \cdot b)}{4 - b^2} = ?$

Yanıt / The Answer is $-\frac{\pi}{4}$

9. $\lim_{x \to 1} \tan\left(\frac{\pi}{2}x\right) \cdot (x - 1) = ?$

Yanıt / The Answer is $-\frac{2}{\pi}$

10. $\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{\sqrt{x+16} - 4} = ?$

Yanıt / The Answer is $\frac{4}{3}$

11. $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{1 - \tan^2 x} = ?$

Yanıt / The Answer is $\frac{1}{4}$

12. $\lim_{x \to 0} \frac{5e^x + 5e^{-x} - 10}{4x^2} = ?$

Yanıt / The Answer is $\frac{5}{4}$

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13. $\lim\limits_{x\to 0} \dfrac{\ln(3x+1)}{2x} = ?$

Yanıt / The Answer is $\dfrac{3}{2}$

14. $\lim\limits_{x\to 2} \dfrac{\sqrt{x+2}-x}{3-\sqrt{4x+1}} = ?$

Yanıt / The Answer is $\dfrac{9}{8}$

15. $\lim\limits_{x\to 1} \sec\left(\dfrac{\pi}{2}x\right)\cdot\left(\arctan x - \dfrac{\pi}{4}\right) = ?$

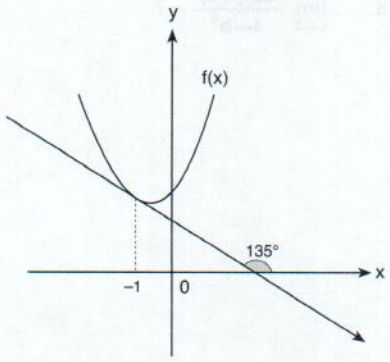
Yanıt / The Answer is $-\dfrac{1}{\pi}$

16. $\lim\limits_{x\to 0} x\cdot \operatorname{cosec}^2\sqrt{2}\,x = ?$

Yanıt / The Answer is ∞

17. $f(x) = x^2 + ax + b$

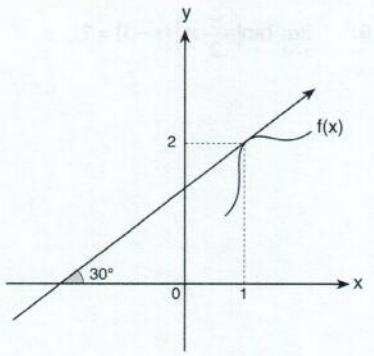
$\Rightarrow a = ?$



Yanıt / The Answer is **1**

18. $h(x) = \dfrac{1}{f(x)}$

$\Rightarrow h'(1) = ?$



Yanıt / The Answer is $\dfrac{-\sqrt{3}}{12}$

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1. $\lim_{x \to 0} \frac{\tan 9x^2}{2x^2} = ?$

A) $\frac{3}{2}$

B) $\frac{5}{2}$

C) $\frac{7}{2}$

D) $\frac{9}{2}$

E) $\frac{11}{2}$

2. $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right) = ?$

A) 0

B) 1

C) 2

D) 3

E) 4

3. $\lim_{x \to 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x-2} = ?$

A) $\frac{1}{2}$

B) $\frac{1}{3 \cdot \sqrt[3]{6}}$

C) $\frac{1}{3 \cdot \sqrt[3]{4}}$

D) $\frac{1}{3 \cdot \sqrt[3]{2}}$

E) $\frac{1}{6}$

4. $\lim_{x \to 2} \frac{x - \sqrt{2x}}{\sqrt{2x+5} - 3} = ?$

A) $\frac{1}{2}$

B) 1

C) $\frac{3}{2}$

D) 2

E) $\frac{5}{2}$

5. $\lim_{x \to 1} \frac{1 - \sqrt[p]{x}}{1 - \sqrt[q]{x}} = ?$

A) $-\frac{p}{q}$

B) $-\frac{q}{p}$

C) $\frac{p}{q}$

D) $\frac{q}{p}$

E) $p \cdot q$

6. $\lim_{x \to 9} \frac{\sqrt{x} - 3}{\sqrt[4]{x} - \sqrt{3}} = ?$

A) $2\sqrt{3}$

B) $\sqrt{15}$

C) 4

D) $3\sqrt{2}$

E) $3\sqrt{3}$

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7. $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{x+3}-\sqrt{3x+1}}{x^2-3x+2}\right)=?$

A) 1

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) 0

E) -1

8. $\lim\limits_{x\to m}\dfrac{\sin m-\sin x}{\cos x-\cos m}=?$

A) cotm

B) –cosm

C) $-\frac{1}{\cos m}$

D) $-\frac{1}{\sin m}$

E) –sinm

9. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\sin(x-\frac{\pi}{4})}{\cos(x-\frac{\pi}{4})}=?$

A) $-\frac{1}{3}$

B) 0

C) $\frac{1}{4}$

D) $\frac{1}{2}$

E) $\frac{\sqrt{2}}{2}$

10. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\sin x-\cos x}{\cot x-1}=?$

A) $-2\sqrt{2}$

B) $-\sqrt{2}$

C) $-\frac{\sqrt{2}}{2}$

D) $\frac{\sqrt{2}}{2}$

E) $\sqrt{2}$

11. $f(x)=ax^3$,
$d: y=3x-6 \Rightarrow a=?$

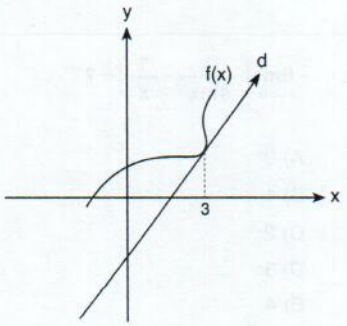
A) $\frac{1}{9}$

B) $\frac{1}{6}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

E) 1



12. $f(x)=x^2-mx+5$
$\Rightarrow m=?$

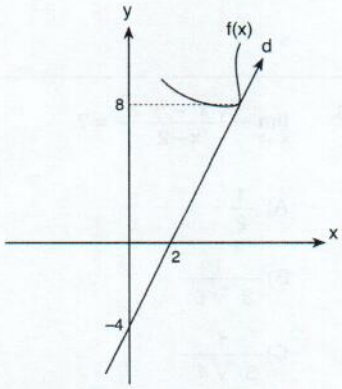
A) –5

B) –2

C) 2

D) 5

E) 10



YANITLAR / ANSWERS

1. D	2. A	3. C	4. C	5. D	6. A	7. B	8. A
9. B	10. C	11. A	12. E				

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1. $\lim\limits_{\alpha\to 1} \dfrac{\sqrt[4]{\alpha}-1}{\sqrt[3]{\alpha}-1} = ?$

A) $-\dfrac{4}{3}$

B) $-\dfrac{3}{4}$

C) 1

D) $\dfrac{3}{4}$

E) $\dfrac{4}{3}$

2. $\lim\limits_{x\to c} \dfrac{x\sqrt{x}-c\sqrt{c}}{\sqrt{x}-\sqrt{c}} = ?$

A) $\dfrac{c}{3}$

B) $\dfrac{c}{2}$

C) c

D) 2c

E) 3c

3. $\lim\limits_{x\to 2} \dfrac{\sqrt[3]{x+6}-2}{x-2} = ?$

A) $-\dfrac{1}{4}$

B) $-\dfrac{1}{8}$

C) 0

D) $\dfrac{1}{12}$

E) $\dfrac{1}{4}$

4. $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2-3x+6}-\sqrt{x^2-2x+4}}{4x-8} = ?$

A) $-\dfrac{1}{4}$

B) $-\dfrac{1}{8}$

C) $-\dfrac{1}{16}$

D) $\dfrac{1}{8}$

E) $\dfrac{1}{16}$

5. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{x+27}-3}{\sqrt{x+64}-8} = ?$

A) $-\dfrac{27}{22}$

B) $-\dfrac{32}{27}$

C) $\dfrac{27}{32}$

D) $\dfrac{32}{27}$

E) $\dfrac{16}{27}$

6. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x} = ?$

A) $\dfrac{1}{4}$

B) $\dfrac{1}{2}$

C) $\dfrac{3}{4}$

D) 1

E) 2

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7. $\lim_{x\to 3} \frac{x^2-9}{\sqrt{x+6}-3} = ?$

A) 4
B) 9
C) 16
D) 36
E) 39

8. $\lim_{x\to \frac{\pi}{2}} \frac{\sin 2x - \cos x}{x - \frac{\pi}{2}} = ?$

A) –1
B) –2
C) –3
D) 1
E) 2

9. $k \in R$,

$\lim_{x\to 1} \left(\frac{x+m}{\sqrt{3+x}-2} \right) = k \Rightarrow m = ?$

A) –1
B) –2
C) –3
D) –4
E) –5

10. $\lim_{x\to \frac{\pi}{2}} \left(\frac{1+\cos 2x}{1-\sin x} \right) = ?$

A) 0
B) 2
C) –2
D) 4
E) –4

11. $\lim_{x\to 0} \left(\frac{x^2+\sin^2 x}{\tan^2 x} \right) = ?$

A) 3
B) $\frac{3}{2}$
C) 2
D) 1
E) $\frac{5}{2}$

12. $k \in R$,

$\lim_{x\to 2} \left(\frac{x^3-ax^2+3x-2}{x^2-4} \right) = k \Rightarrow a = ?$

A) 3 B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) 2 E) $-\frac{3}{2}$

YANITLAR / ANSWERS							
1. D	2. E	3. D	4. C	5. E	6. B	7. D	8. A
9. A	10. D	11. C	12. A				

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ÜNİTE 7

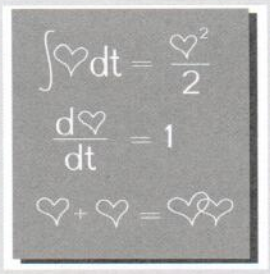
UNIT 7

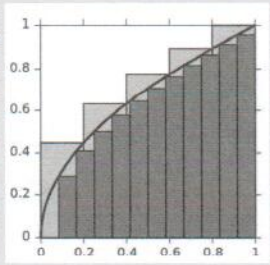
Bu ünitede, belirsiz integral, belirli integral ve integral uygulamaları ayrıntılı anlatılmıştır. Ilk olarak, belirsiz integral ve integral alma yöntemleri, daha sonra belirli integral konusu anlatılmıştır. Son olarak, integral uygulamaları anlatılmıştır.

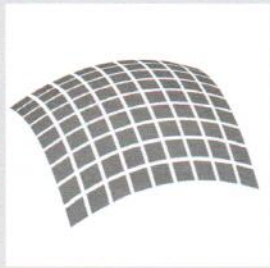
Bu ünite; 122 çözümlü örnek, 60 alıştırma ve 68 test sorusundan oluşmaktadır.

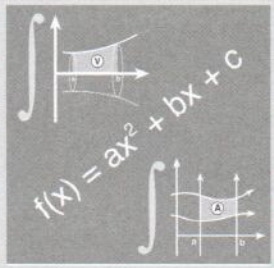
In this unit, indefinite integral, definite integral and applications of the integral are explained in detailed. Firstly, indefinite integral and integral methods are explained. Later, definite integral is explained. Subsequently, applications of integral are explained.

This unit includes; 122 examples, 60 exercises, and 68 test questions.









- **BELİRSİZ İNTEGRAL** ..... 635 – 692
- **BELİRLİ İNTEGRAL** ..... 693 – 730
- **İNTEGRAL UYGULAMALARI** ..... 731 – 756

- **INDEFINITE INTEGRAL** ..... 635 – 692
- **DEFINITE INTEGRAL** ..... 693 – 730
- **APPLICATIONS of INTEGRAL** ..... 731 – 756

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BÖLÜM 1 CHAPTER

TARİHSEL NOT / HISTORICAL NOTE

Lebesque, Henri – Leon [1875 – 1941]

Fransız matematikçi. Integral teorisini inşa etmiştir. Önceki Fransız matematikçilerin inşa ettiği ölçü kavramını, Lebesque(Löbek) integrali olarak geliştirmiştir. Sürekli fonsiyonlar için kullanılan Riemann integraline genelleştirilmiştir.

French mathematician who revolutionized the theory of integration. Building on the work of earlier French mathematicians on the notion of measure, he developed the Lebesque integral, It generalizes the Riemann integral, which can deal only with continuous functions and other functions not too unlike them.

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INDEFINITE INTEGRAL

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- ✔ İntegral Alma Yöntemleri . . . . . 646 – 662
  - ■ Değişken Değiştirme Yöntemi
  - ■ Basit Kesirlere Ayırma Yöntemi
  - ■ Kısmi İntegrasyon Yöntemi
- ✔ $\int \sin^a x\,dx$, $\int \cos^a x\,dx$, $\int \sin^a x \cdot \cos^b x\,dx$ in İntegrali . . . . . 664 – 666
- ✔ $\int \cos ax \cdot \sin bx\,dx$, $\int \sin ax \cdot \sin bx\,dx$, $\int \cos ax \cdot \cos bx$ in İntegrali 668
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- ✔ YÖS Soruları ve Çözümleri . . . . . 678 – 681
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  - ■ Integration by Parts
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BELİRSİZ İNTEGRAL
INDEFINITE INTEGRAL

BÖLÜM 1
CHAPTER

Tanım: F(x) fonksiyonunun türevi f(x) ve diferansiyeli d(F(x)) = f(x) dx olmak üzere, f(x) + C yi bulma işlemine *belirsiz integral alma işlemi* denir.

$$F'(x) = f(x) \Leftrightarrow \int f(x)\,dx = F(x) + C$$

Belirsiz İntegralin Özellikleri

1. $\frac{d}{dx}\int f(x)\,dx = \left(\int f(x)\,dx\right)' = f(x)$

2. $\int k \cdot f(x)\,dx = k \cdot \int f(x)dx \quad (k \in R)$

3. $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

ÖRNEK 1

$\int f(x) \cdot x\,dx = 5x^3 - 3x^2 + 8 \Rightarrow$ **f(2) = ?**

A) 24 B) 27 C) 29 D) 30 E) 32

ÇÖZÜM

Yanıt **A**

$$\frac{d}{dx}\int f(x) \cdot x\,dx = \frac{d}{dx}(5x^3 - 3x^2 + 8)$$

$$f(x) \cdot x = 15x^2 - 6x$$

$$f(x) = 15x - 6$$

$$f(2) = 15 \cdot 2 - 6$$

$$= 24$$

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Definition: Let the derivative of F(x) is f(x) and the differantion is dF(x) = f(x)dx.
The operation of finding all solutions of f(x) + C is called *indefinite integration.*

$$F'(x) = f(x) \Leftrightarrow \int f(x)\,dx = F(x) + C$$

Properties of Indefinite Integral

1. $\frac{d}{dx}\int f(x)\,dx = \left(\int f(x)\,dx\right)' = f(x)$
2. $\int k \cdot f(x)\,dx = k \cdot \int f(x)dx \ (k \in R)$
3. $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

EXAMPLE 1

$\int f(x) \cdot x\,dx = 5x^3 - 3x^2 + 8 \Rightarrow$ **f(2) = ?**

A) 24 B) 27 C) 29 D) 30 E) 32

SOLUTION

The answer is **A**

$$\frac{d}{dx}\int f(x) \cdot x\,dx = \frac{d}{dx}(5x^3 - 3x^2 + 8)$$

$$f(x) \cdot x = 15x^2 - 6x$$

$$f(x) = 15x - 6$$

$$f(2) = 15 \cdot 2 - 6$$

$$= 24$$

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Temel İntegral Formülleri

1. $\int a\,dx = a\int dx = a\cdot x + C \quad (C \in R)$

Etkinlikler

a. $\int 2\,dx = 2x + C$

b. $\int k\,dx = kx + C$

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (C \in R)$

Etkinlikler

a. $\int x^2\,dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} + C$

b. $\int 4x^3\,dx = 4\int x^3 dx = 4\cdot\frac{x^{3+1}}{3+1} = x^4 + C$

c. $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{3}x^{\frac{3}{2}} + C$

ÖRNEK 2

$\int \left(\sqrt{x} + \sqrt[4]{x^3}\right) dx = ?$

A) $\frac{2}{3}x^{\frac{3}{2}} + \frac{4}{7}x^{\frac{4}{7}} + C$ B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{4}{7}x^{\frac{7}{4}} + C$ C) $\frac{2}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{7}{4}} + C$

D) $\frac{2}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{4}{7}} + C$ E) $\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{7}x^{\frac{7}{4}} + C$

ÇÖZÜM

Yanıt **B**

$\int \left(\sqrt{x} + \sqrt[4]{x^3}\right) dx = \int \sqrt{x}\,dx + \int \sqrt[4]{x^3}\,dx$

$= \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{7}x^{\frac{7}{4}} + C$

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Basic Integration Formulas

1. $\int a\,dx = a\int dx = a\cdot x + C \quad (C \in R)$

Activities

a. $\int 2\,dx = 2x + C$

b. $\int k\,dx = kx + C$

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (C \in R)$

Activities

a. $\int x^2\,dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} + C$

b. $\int 4x^3\,dx = 4\int x^3 dx = 4\cdot\frac{x^{3+1}}{3+1} = x^4 + C$

c. $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{3}x^{\frac{3}{2}} + C$

EXAMPLE 2

$\int \left(\sqrt{x} + \sqrt[4]{x^3}\right) dx = ?$

A) $\frac{2}{3}x^{\frac{3}{2}} + \frac{4}{7}x^{\frac{4}{7}} + C$ B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{4}{7}x^{\frac{7}{4}} + C$ C) $\frac{2}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{7}{4}} + C$

D) $\frac{2}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{4}{7}} + C$ E) $\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{7}x^{\frac{7}{4}} + C$

SOLUTION

The answer is **B**

$$\int \left(\sqrt{x} + \sqrt[4]{x^3}\right) dx = \int \sqrt{x}\,dx + \int \sqrt[4]{x^3}\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{7}x^{\frac{7}{4}} + C$$

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ÖRNEK 3

$f''(x) = 36x^2 - 24x + 4$,

$f'(0) = 3$, $f(0) = 2 \Rightarrow$ **f(−1) = ?**

A) 4 B) 6 C) 7 D) 8 E) 9

ÇÖZÜM

Yanıt **D**

$$\int f''(x)\,dx = \int (36x^2 - 24x + 4)\,dx = \int 36x^2 dx - \int 24x\,dx + \int 4\,dx$$

$$= 36\frac{x^3}{3} - 24\frac{x^2}{2} + 4x + C_1$$

$$\Rightarrow f'(x) = 12x^3 - 12x^2 + 4x + C_1$$

$$f'(0) = 12\cdot 0^3 - 12\cdot 0^2 + 4\cdot 0 + C_1 = 3 \Rightarrow C_1 = 3$$

$$\int f'(x)\,dx = \int (12x^3 - 12x^2 + 4x + 3)\,dx$$

$$f(x) = 12\cdot\frac{x^4}{4} - 12\cdot\frac{x^3}{3} + 4\cdot\frac{x^2}{2} + 3x + C_2 = 2 \Rightarrow C_2 = 2$$

$$f(x) = 3x^4 - 4x^3 + 2x^2 + 3x + 2$$

$$f(-1) = 3\cdot(-1)^4 - 4\cdot(-1)^3 + 2\cdot(-1)^2 + 3\cdot(-1) + 2 = 8$$

3. $\int \frac{1}{x}\,dx = \ln|x| + C$, $(C \in R)$

ÖRNEK 4

$$\int \frac{(x-3)^2}{x^3}\,dx = ?$$

A) $\ln x + 6x + 3x^3 + C$ B) $\ln x + \frac{6}{x} - \frac{9}{2x^2} + C$ C) $\ln x + \frac{3}{x} - \frac{9}{x^2} + C$

D) $\ln x - \frac{6}{x^2} - \frac{9}{2x^3} + C$ E) $\ln x + \frac{6}{x^2} + \frac{9}{2x^3} + C$

ÇÖZÜM

Yanıt **B**

$$\int \frac{(x-3)^2}{x^3}\,dx = \int \frac{x^2 - 6x + 9}{x^3}\,dx = \int \frac{x^2}{x^3}\,dx - \int \frac{6x}{x^3}\,dx + \int \frac{9}{x^3}\,dx$$

$$= \ln|x| - 6\frac{x^{-1}}{-1} + \frac{9x^{-2}}{-2} + C$$

$$= \ln|x| + \frac{6}{x} - \frac{9}{2x^2} + C$$

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EXAMPLE 3

$f''(x) = 36x^2 - 24x + 4$,

$f'(0) = 3,\ f(0) = 2 \Rightarrow$ **f(–1) = ?**

A) 4 B) 6 C) 7 D) 8 E) 9

SOLUTION

The answer is **D**

$$\int f''(x)\,dx = \int (36x^2 - 24x + 4)\,dx = \int 36x^2 dx - \int 24x\,dx + \int 4\,dx$$

$$= 36\frac{x^3}{3} - 24\frac{x^2}{2} + 4x + C_1$$

$$\Rightarrow f'(x) = 12x^3 - 12x^2 + 4x + C_1$$

$$f'(0) = 12\cdot 0^3 - 12\cdot 0^2 + 4\cdot 0 + C_1 = 3 \Rightarrow C_1 = 3$$

$$\int f'(x)\,dx = \int (12x^3 - 12x^2 + 4x + 3)\,dx$$

$$f(x) = 12\cdot\frac{x^4}{4} - 12\cdot\frac{x^3}{3} + 4\cdot\frac{x^2}{2} + 3x + C_2 = 2 \Rightarrow C_2 = 2$$

$$f(x) = 3x^4 - 4x^3 + 2x^2 + 3x + 2$$

$$f(-1) = 3\cdot(-1)^4 - 4\cdot(-1)^3 + 2\cdot(-1)^2 + 3\cdot(-1) + 2 = 8$$

3. $\int \frac{1}{x}\,dx = \ln|x| + C,\ (C \in R)$

EXAMPLE 4

$$\int \frac{(x-3)^2}{x^3}\,dx = ?$$

A) $\ln x + 6x + 3x^3 + C$ B) $\ln x + \frac{6}{x} - \frac{9}{2x^2} + C$ C) $\ln x + \frac{3}{x} - \frac{9}{x^2} + C$

D) $\ln x - \frac{6}{x^2} - \frac{9}{2x^3} + C$ E) $\ln x + \frac{6}{x^2} + \frac{9}{2x^3} + C$

SOLUTION

The answer is **B**

$$\int \frac{(x-3)^2}{x^3}\,dx = \int \frac{x^2-6x+9}{x^3}\,dx = \int \frac{x^2}{x^3}\,dx - \int \frac{6x}{x^3}\,dx + \int \frac{9}{x^3}\,dx$$

$$= \ln|x| - 6\frac{x^{-1}}{-1} + \frac{9x^{-2}}{-2} + C$$

$$= \ln|x| + \frac{6}{x} - \frac{9}{2x^2} + C$$

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4. $\int e^x\,dx = e^x + C \;,\; (C \in R)$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C = a^x \log_a e + C \;,\; (C \in R)$

ÖRNEK 5

$\int (e^x + 2^x \ln 2)\,dx = ?$

A) $e^x - 2^x + C$ B) $e^x + 2^x + C$ C) $e^x - \frac{2^x}{\ln 2} + C$

D) $e^x + 2^x \cdot \ln 2 + C$ E) $e^x - 2^x \cdot \ln 2 + C$

ÇÖZÜM

Yanıt B

$$\left.\begin{array}{l} \int e^x\,dx = e^x + C \\ \int 2^x \ln 2\,dx = \frac{2^x}{\ln 2} \cdot \ln 2 + C = 2^x + C \end{array}\right\} \Rightarrow \int (e^x + 2^x \ln 2)\,dx = e^x + 2^x + C$$

PEKİŞTİRME

$\int 3^x \ln 3\,dx = ?$

YANIT : 3^x

6. $\int \sin x\,dx = -\cos x + C \;,\; (C \in R)$

7. $\int \cos x\,dx = \sin x + C \;,\; (C \in R)$

8. $\int \frac{1}{\sin^2 x}\,dx = \int \operatorname{cosec}^2 x\,dx = \int (1 + \cot^2 x)\,dx = -\cot x + C \;,\; (C \in R)$

9. $\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx = \int (1 + \tan^2 x)\,dx = \tan x + C \;,\; (C \in R)$

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4. $\int e^x\,dx = e^x + C \;,\; (C \in R)$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C = a^x \log_a e + C \;,\; (C \in R)$

REINFORCE

$\int 3^x \ln 3\,dx = ?$

EXAMPLE 5

$\int (e^x + 2^x \ln 2)\,dx = ?$

A) $e^x - 2^x + C$ B) $e^x + 2^x + C$ C) $e^x - \frac{2^x}{\ln 2} + C$

D) $e^x + 2^x \cdot \ln 2 + C$ E) $e^x - 2^x \cdot \ln 2 + C$

ANSWER : 3^x

SOLUTION

The answer is **B**

$$\left.\begin{array}{l} \int e^x\,dx = e^x + C \\ \int 2^x \ln 2\,dx = \frac{2^x}{\ln 2} \cdot \ln 2 + C = 2^x + C \end{array}\right\} \Rightarrow \int (e^x + 2^x \ln 2)\,dx = e^x + 2^x + C$$

6. $\int \sin x\,dx = -\cos x + C \;,\; (C \in R)$

7. $\int \cos x\,dx = \sin x + C \;,\; (C \in R)$

8. $\int \frac{1}{\sin^2 x}\,dx = \int \operatorname{cosec}^2 x\,dx = \int (1 + \cot^2 x)\,dx = -\cot x + C \;,\; (C \in R)$

9. $\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx = \int (1 + \tan^2 x)\,dx = \tan x + C \;,\; (C \in R)$

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ÖRNEK 6

$\int \left(\frac{\sin^3 x + 1}{\sin^2 x} \right) dx = ?$

A) $\cos x + \cot x + C$ B) $-\cos x + \cot x + C$ C) $-\cos x - \cot x + C$

D) $-\cos x - \tan x + C$ E) $-\cos x + \tan x + C$

ÇÖZÜM

Yanıt C

$$\int \frac{\sin^3 x + 1}{\sin^2 x} dx = \int \frac{\sin^3 x}{\sin^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sin x \, dx + \int \frac{dx}{\sin^2 x}$$

$$= -\cos x - \cot x + C$$

10. $\int \frac{1}{1+x^2} dx = \arctan x + C \ , \ (C \in R)$

11. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C \ , \ (C \in R)$

12. $\int \frac{1}{\sqrt{1+x^2}} dx = \ln\left(x + \sqrt{1+x^2}\right) + C \ , \ (C \in R)$

ÖRNEK 7

$\int \frac{dx}{1+x^2} = ?$

A) $\arctan x + 2x + C$ B) $\arctan x + x + C$ C) $\arctan x + C$

D) $\arcsin x + C$ E) $\arccos x + C$

ÇÖZÜM

Yanıt C

$$\int \frac{dx}{1+x^2} = \arctan x + C$$

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EXAMPLE 6

$$\int \left(\frac{\sin^3 x + 1}{\sin^2 x} \right) dx = ?$$

A) cosx + cotx + C B) –cosx + cotx + C C) –cosx – cotx + C

D) –cosx – tanx + C E) –cosx + tanx + C

SOLUTION

The answer is C

$$\int \frac{\sin^3 x + 1}{\sin^2 x} dx = \int \frac{\sin^3 x}{\sin^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sin x \, dx + \int \frac{dx}{\sin^2 x}$$

$$= -\cos x - \cot x + C$$

10. $\int \frac{1}{1+x^2} dx = \arctan x + C$, $(C \in R)$

11. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$, $(C \in R)$

12. $\int \frac{1}{\sqrt{1+x^2}} dx = \ln\left(x + \sqrt{1+x^2}\right) + C$, $(C \in R)$

EXAMPLE 7

$$\int \frac{dx}{1+x^2} = ?$$

A) arctanx + 2x + C B) arctanx + x + C C) arctanx + C

D) arcsinx + C E) arccosx + C

SOLUTION

The answer is C

$$\int \frac{dx}{1+x^2} = \arctan x + C$$

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İNTEGRAL ALMA YÖNTEMLERİ

1. DEĞİŞKEN DEĞİŞTİRME YÖNTEMİ

$\int f(x)dx$ integralinde, $x = u(t)$ şeklinde bir değişken değiştirmesi yapıldığında,

$x = u(t) \Rightarrow dx = u'(t)\,dt$ olur.

$\int f(x)dx = \int f(u(t))\cdot u'(t)\cdot dt$ integrali hesaplandıktan sonra sonuç tekrar x değişkeni cinsinden ifade edilir.

NOTLAR

- ✓ $\int f^n(x)\cdot f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C$
- ✓ $\int \frac{f'(x)dx}{f(x)} = \ln|f(x)| + C$
- ✓ $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$
- ✓ $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$
- ✓ $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$
- ✓ $\int e^{ax+b}\,dx = \frac{1}{a}\cdot e^{ax+b} + C$

ÖRNEK 8

$\int (3x-2)\cdot(6x^2-8x+5)^3\,dx = ?$

A) $\ln|6x^2-8x+5| + C$

B) $\frac{(6x^2-8x+5)^2}{2} + C$

C) $2(6x^2-8x+5)^2 + C$

D) $\frac{(6x^2-8x+5)^4}{16} + C$

E) $4(6x^2-8x+5)^3 + C$

ÇÖZÜM

Yanıt **D**

$u = 6x^2-8x+5$

$du = (12x-8)\,dx = 4(3x-2)\,dx$

$(3x-2)\,dx = \frac{du}{4}$

$\Rightarrow \int (3x-2)(6x^2-8x+5)^3\,dx = \int u^3\,\frac{du}{4}$

$= \frac{1}{4}\cdot\frac{u^4}{4} + C$

$= \frac{1}{16}\cdot(6x^2-8x+5)^4 + C$

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NOTES

- ✓ $\int f^n(x)\cdot f'(x)dx=\frac{[f(x)]^{n+1}}{n+1}+C$
- ✓ $\int \frac{f'(x)dx}{f(x)}=\ln|f(x)|+C$
- ✓ $\int \sin(ax+b)\,dx=-\frac{1}{a}\cos(ax+b)+C$
- ✓ $\int \cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+C$
- ✓ $\int \frac{dx}{ax+b}=\frac{1}{a}\ln|ax+b|+C$
- ✓ $\int e^{ax+b}\,dx=\frac{1}{a}\cdot e^{ax+b}+C$

INTEGRATION METHODS

1. CHANGE of VARIABLE

When conversion x = u(t) is applied in $\int$ f(x) dx, dx is equal to u′(t)·dt. Then the resulting integral is as the following:

$\int$ f(x) dx = $\int$ f(u(t))·u′(t)·dt after calculating this integral, we again convert the result to find the answer in terms of x.

EXAMPLE 8

$\int (3x-2)\cdot(6x^2-8x+5)^3\,dx=?$

A) $\ln|6x^2-8x+5|+C$

B) $\frac{(6x^2-8x+5)^2}{2}+C$

C) $2(6x^2-8x+5)^2+C$

D) $\frac{(6x^2-8x+5)^4}{16}+C$

E) $4(6x^2-8x+5)^3+C$

SOLUTION

The answer is D

$$\left.\begin{array}{l} u=6x^2-8x+5 \\ du=(12x-8)\,dx \;=4(3x-2)\,dx \\ (3x-2)\,dx=\frac{du}{4} \end{array}\right\} \Rightarrow \int(3x-2)(6x^2-8x+5)^3\,dx=\int u^3\,\frac{du}{4}$$

$$=\frac{1}{4}\cdot\frac{u^4}{4}+C$$

$$=\frac{1}{16}\cdot(6x^2-8x+5)^4+C$$

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ÖRNEK 9

$\int \frac{dx}{9+x^2} = ?$

A) $-\frac{1}{2}\arctan\left(\frac{x}{3}\right)+C$ B) $\frac{1}{2}\arctan\left(\frac{x}{3}\right)+C$ C) $-\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

D) $\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$ E) $-\frac{1}{4}\arctan\left(\frac{x}{3}\right)+C$

ÇÖZÜM

Yanıt **D**

$$\int \frac{1}{9+x^2}dx = \frac{1}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}dx$$

$$\frac{x}{3} = t \Rightarrow dx = 3dt$$

$$\frac{1}{9}\int \frac{3}{1+t^2}dt = \frac{1}{3}\int \frac{1}{1+t^2}dt$$

$$\frac{1}{3}\arctan t + C = \frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$$

ÖRNEK 10

$\int \sin(3x-6)\,dx = ?$

A) $\frac{\sin(3x-6)}{3}+C$ B) $\frac{-\sin(3x-6)}{3}+C$ C) $\frac{\tan(3x-6)}{3}+C$

D) $\frac{-\cos(3x-6)}{3}+C$ E) $\frac{\cos(3x-6)}{3}+C$

ÇÖZÜM

Yanıt **D**

$$\left.\begin{array}{l} u = 3x-6 \\ du = 3\,dx \\ dx = \frac{du}{3} \end{array}\right\} \Rightarrow \int \sin u \cdot \frac{du}{3} = -\frac{1}{3}\cos u + C$$

$$= -\frac{1}{3}\cos(3x-6)+C$$

PEKİŞTİRME

$\int \cos(x+6)\,dx = ?$

YANIT : sin(x + 6) + C

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EXAMPLE 9

$\int \frac{dx}{9+x^2} = ?$

A) $-\frac{1}{2}\arctan\left(\frac{x}{3}\right)+C$ B) $\frac{1}{2}\arctan\left(\frac{x}{3}\right)+C$ C) $-\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

D) $\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$ E) $-\frac{1}{4}\arctan\left(\frac{x}{3}\right)+C$

SOLUTION

The answer is D

$$\int \frac{1}{9+x^2}dx = \frac{1}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}dx$$

$$\frac{x}{3} = t \Rightarrow dx = 3dt$$

$$\frac{1}{9}\int \frac{3}{1+t^2}dt = \frac{1}{3}\int \frac{1}{1+t^2}dt$$

$$\frac{1}{3}\arctan t + C = \frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$$

REINFORCE

$\int \cos(x+6)\,dx = ?$

ANSWER : sin(x + 6) + C

EXAMPLE 10

$\int \sin(3x-6)\,dx = ?$

A) $\frac{\sin(3x-6)}{3}+C$ B) $\frac{-\sin(3x-6)}{3}+C$ C) $\frac{\tan(3x-6)}{3}+C$

D) $\frac{-\cos(3x-6)}{3}+C$ E) $\frac{\cos(3x-6)}{3}+C$

SOLUTION

The answer is D

$$\left.\begin{array}{l} u = 3x-6 \\ du = 3\,dx \\ dx = \frac{du}{3} \end{array}\right\} \Rightarrow \int \sin u \cdot \frac{du}{3} = -\frac{1}{3}\cos u + C$$

$$= -\frac{1}{3}\cos(3x-6)+C$$

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ÖRNEK 11

$\int e^{3x}\,dx = ?$

A) $\frac{e^{4x}}{4}+C$ B) $\frac{e^{2x}}{2}+C$ C) $\frac{3}{e^{3x}}+C$

D) $\frac{e^{3x}}{3}+C$ E) $\frac{e^{3x}}{6}+C$

ÇÖZÜM

Yanıt **D**

$$\left.\begin{array}{l} u = 3x \\ du = 3\,dx \\ dx = \frac{du}{3} \end{array}\right\} \Rightarrow \int e^{3x}\cdot dx = \int e^{u}\frac{du}{3} = \frac{1}{3}\int e^{u}\,du$$

$$= \frac{1}{3}e^{u}+C$$

$$= \frac{1}{3}e^{3x}+C$$

PEKİŞTİRME

$\int e^{6x}\cdot 6\,dx = ?$

YANIT : $e^{6x}+C$

ÖRNEK 12

$\int \tan 3x\,dx = ?$

A) $\frac{\ln|\sin 3x|}{3}+C$ B) $\frac{\sin^2 3x}{3}+C$ C) $\frac{-\ln|\cos 3x|}{3}+C$

D) $\frac{\cos^2 3x}{3}+C$ E) $\frac{\tan^2 3x}{3}+C$

ÇÖZÜM

Yanıt **C**

$$\left.\begin{array}{l} u = \cos 3x \\ du = -3\sin 3x\,dx \\ \sin 3x\,dx = \frac{-du}{3} \end{array}\right\} \Rightarrow \int \frac{\sin 3x}{\cos 3x}dx = \int \frac{-du}{3u} = -\frac{1}{3}\int \frac{du}{u}$$

$$= -\frac{1}{3}\ln|u|+C$$

$$= -\frac{1}{3}\ln|\cos 3x|+C$$

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REINFORCE

$\int e^{6x} \cdot 6\,dx = ?$

ANSWER : $e^{6x} + C$

EXAMPLE 11

$\int e^{3x}\,dx = ?$

A) $\frac{e^{4x}}{4} + C$ B) $\frac{e^{2x}}{2} + C$ C) $\frac{3}{e^{3x}} + C$

D) $\frac{e^{3x}}{3} + C$ E) $\frac{e^{3x}}{6} + C$

SOLUTION The answer is **D**

$$\left.\begin{array}{l} u = 3x \\ du = 3\,dx \\ dx = \frac{du}{3} \end{array}\right\} \Rightarrow \int e^{3x} \cdot dx = \int e^{u} \frac{du}{3} = \frac{1}{3}\int e^{u}\,du$$

$$= \frac{1}{3} e^{u} + C$$

$$= \frac{1}{3} e^{3x} + C$$

EXAMPLE 12

$\int \tan 3x\,dx = ?$

A) $\frac{\ln|\sin 3x|}{3} + C$ B) $\frac{\sin^2 3x}{3} + C$ C) $\frac{-\ln|\cos 3x|}{3} + C$

D) $\frac{\cos^2 3x}{3} + C$ E) $\frac{\tan^2 3x}{3} + C$

SOLUTION The answer is **C**

$$\left.\begin{array}{l} u = \cos 3x \\ du = -3\sin 3x\,dx \\ \sin 3x\,dx = \frac{-du}{3} \end{array}\right\} \Rightarrow \int \frac{\sin 3x}{\cos 3x} dx = \int \frac{-du}{3u} = -\frac{1}{3}\int \frac{du}{u}$$

$$= -\frac{1}{3}\ln|u| + C$$

$$= -\frac{1}{3}\ln|\cos 3x| + C$$

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2. BASİT KESİRLERE AYIRMA YÖNTEMİ

$\int \frac{P(x)}{Q(x)}\,dx$ şeklinde verilen integral ifadesi hesaplanırken, aşağıdaki yöntemler kullanılır.

1. der [P(x)] ≥ der [Q(x)] ise, polinom bölmesi yapılarak integral alınır.

ÖRNEK 13

$\int \frac{x+2}{x+1}\,dx = ?$

A) $x + \ln|x+1| + C$ B) $x - \ln|x+1| + C$ C) $2x + \ln|x+1| + C$

D) $x + 2\cdot\ln|x+1| + C$ E) $x - 2\cdot\ln|x+1| + C$

ÇÖZÜM

Yanıt **A**

$$\int \frac{x+2}{x+1}dx = \int\left(1+\frac{1}{x+1}\right)dx = x+\ln|x+1|+C$$

ÖRNEK 14

$\int \frac{x^4+2x^2}{x^2+1}\,dx = ?$

A) $\frac{x^3}{3} - x - \arctan x + C$ B) $\frac{x^3}{3} + x + \arctan x + C$

C) $\frac{x^3}{3} + \frac{x^2}{2} - \arctan x + C$ D) $\frac{x^3}{3} + \frac{x^2}{2} + \arctan x + C$

E) $\frac{x^3}{3} + x - \arctan x + C$

ÇÖZÜM

Yanıt **E**

$$\begin{array}{r|l} x^4+2x^2 & x^2+1 \\ \hline -x^4 \mp x^2 & x^2+1 \\ \hline x^2 & \\ -x^2 \mp 1 & \\ \hline -1 & \end{array}$$

$$\int \frac{x^4+2x^2}{x^2+1}dx = \int\left(x^2+1-\frac{1}{x^2+1}\right)dx$$

$$= \int x^2\,dx + \int 1\cdot dx - \int \frac{1}{x^2+1}\,dx$$

$$= \frac{x^3}{3} + x - \arctan x + C$$

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2. DIVIDING into SIMPLE FRACTIONS

To calculate the integration $\int \frac{P(x)}{Q(x)}\,dx$, we can use methods following given.

1. **If der [P(x)] ≥ der [Q(x)], we use polynomial division**

EXAMPLE 13

$$\int \frac{x+2}{x+1}\,dx = ?$$

A) $x + \ln|x+1| + C$ B) $x - \ln|x+1| + C$ C) $2x + \ln|x+1| + C$

D) $x + 2\cdot\ln|x+1| + C$ E) $x - 2\cdot\ln|x+1| + C$

SOLUTION

The answer is A

$$\int \frac{x+2}{x+1}dx = \int\left(1+\frac{1}{x+1}\right)dx = x+\ln|x+1|+C$$

EXAMPLE 14

$$\int \frac{x^4+2x^2}{x^2+1}\,dx = ?$$

A) $\frac{x^3}{3} - x - \arctan x + C$ B) $\frac{x^3}{3} + x + \arctan x + C$

C) $\frac{x^3}{3} + \frac{x^2}{2} - \arctan x + C$ D) $\frac{x^3}{3} + \frac{x^2}{2} + \arctan x + C$

E) $\frac{x^3}{3} + x - \arctan x + C$

SOLUTION

The answer is E

$$\begin{array}{r|l} x^4+2x^2 & x^2+1 \\ \hline -x^4 \mp x^2 & x^2+1 \\ \hline x^2 & \\ -x^2 \mp 1 & \\ \hline -1 & \end{array}$$

$$\int \frac{x^4+2x^2}{x^2+1}\,dx = \int\left(x^2+1-\frac{1}{x^2+1}\right)dx$$

$$= \int x^2\,dx + \int 1\cdot dx - \int \frac{1}{x^2+1}\,dx$$

$$= \frac{x^3}{3} + x - \arctan x + C$$

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2. **der [P(x)] < der [Q(x)] ise,**

   a. Q(x) çarpanlara ayrılıyorsa ifade basit kesirlere ayrılarak integral alınır.

   b. Q(x) çarpanlarına ayrılmıyorsa Q(x) tam kare ifadeye tamamlanarak integral alınır.

ÖRNEK 15

$$\int \frac{x}{x^3-x^2+x-1}dx = ?$$

A) $-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$

B) $\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$

C) $-\frac{1}{2}\ln(x^2+1)+\frac{1}{4}\arctan x+\frac{1}{2}\ln|x-1|+C$

D) $-\frac{1}{4}\ln(x^2+1)+\frac{1}{4}\arctan x+\frac{1}{2}\ln|x-1|+C$

E) $\frac{1}{4}\ln(x^2+1)+\frac{1}{4}\arctan x+\frac{1}{4}\ln|x-1|+C$

ÇÖZÜM

Yanıt **A**

$$\frac{x}{x^3-x^2+x-1}=\frac{x}{(x^2+1)(x-1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$$

$$=\frac{Ax^2-Ax+Bx-B+Cx^2+C}{(x^2+1)(x-1)} \Rightarrow x=(A+C)x^2+(B-A)x+C-B$$

$$\left.\begin{matrix}A+C=0\\B-A=1\\C-B=0\end{matrix}\right\} \Rightarrow \left.\begin{matrix}A+C=0\\C-A=1\end{matrix}\right\} \Rightarrow C=\frac{1}{2}$$

$$C=\frac{1}{2},\quad A=-\frac{1}{2},\quad B=\frac{1}{2}$$

$$\int \frac{x}{x^3-x^2+x-1}dx=\frac{1}{2}\int\frac{-x+1}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=\frac{-1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x^2+1}dx+\frac{1}{2}\ln|x-1|$$

$$x^2+1=u \Rightarrow \frac{du}{2}=x\,dx$$

$$\int\frac{x\,dx}{x^3-x^2+x-1}=\frac{-1}{4}\int\frac{du}{u}+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$$

$$\int \frac{x}{x^3-x^2+x-1}dx=-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$$

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2. **If der [P(x)] < der [Q(x)],**

   a. If Q(x) is factoring, we use the method dividing into simple fractions.

   b. If Q(x) is not factoring, we use the perfect square expression.

EXAMPLE 15

$$\int \frac{x}{x^3-x^2+x-1}dx = ?$$

A) $-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$

B) $\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$

C) $-\frac{1}{2}\ln(x^2+1)+\frac{1}{4}\arctan x+\frac{1}{2}\ln|x-1|+C$

D) $-\frac{1}{4}\ln(x^2+1)+\frac{1}{4}\arctan x+\frac{1}{2}\ln|x-1|+C$

E) $\frac{1}{4}\ln(x^2+1)+\frac{1}{4}\arctan x+\frac{1}{4}\ln|x-1|+C$

SOLUTION

The answer is **A**

$$\frac{x}{x^3-x^2+x-1}=\frac{x}{(x^2+1)(x-1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$$

$$=\frac{Ax^2-Ax+Bx-B+Cx^2+C}{(x^2+1)(x-1)} \Rightarrow x=(A+C)x^2+(B-A)x+C-B$$

$$\left.\begin{matrix}A+C=0\\B-A=1\\C-B=0\end{matrix}\right\} \Rightarrow \left.\begin{matrix}A+C=0\\C-A=1\end{matrix}\right\} \Rightarrow C=\frac{1}{2}$$

$$C=\frac{1}{2},\quad A=-\frac{1}{2},\quad B=\frac{1}{2}$$

$$\int \frac{x}{x^3-x^2+x-1}dx=\frac{1}{2}\int\frac{-x+1}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=\frac{-1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x^2+1}dx+\frac{1}{2}\ln|x-1|$$

$$x^2+1=u \Rightarrow \frac{du}{2}=x\,dx$$

$$\int\frac{x\,dx}{x^3-x^2+x-1}=\frac{-1}{4}\int\frac{du}{u}+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$$

$$\int \frac{x}{x^3-x^2+x-1}dx=-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\arctan x+\frac{1}{2}\ln|x-1|+C$$

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ÖRNEK 16

$$\int \frac{dx}{x^2+8x+17} = ?$$

A) arctan(x + 1) + C

B) arctan(x + 2) + C

C) arctan(x + 3) + C

D) 3arctan(x + 3) + C

E) arctan(x + 4) + C

ÇÖZÜM

Yanıt **E**

$$\int \frac{dx}{x^2+8x+17} = \int \frac{dx}{(x+4)^2+1}$$

$$\left.\begin{matrix} u = x+4 \\ du = dx \end{matrix}\right\} \Rightarrow \int \frac{du}{u^2+1} = \arctan u + C = \arctan|x+4| + C$$

ÖRNEK 17

$$\int \frac{dx}{4x^2+12x+13} = ?$$

A) $\frac{1}{4}\arctan\frac{2x+3}{2}+C$

B) $\frac{1}{2}\arctan\frac{2x+3}{2}+C$

C) $\frac{1}{4}\arctan(2x+3)+C$

D) $\frac{1}{4}\arctan\frac{2x-3}{2}+C$

E) $\frac{1}{2}\arctan(2x+3)+C$

ÇÖZÜM

Yanıt **A**

$$\int \frac{dx}{4x^2+12x+13} = \int \frac{dx}{(2x+3)^2+4} = \frac{1}{4}\int \frac{dx}{\left(\frac{2x+3}{2}\right)^2+1}$$

$$\left.\begin{matrix} u = \frac{2x+3}{2} \\ du = dx \end{matrix}\right\} \Rightarrow \frac{1}{4}\int \frac{du}{u^2+1} = \frac{1}{4}\arctan u + C = \frac{1}{4}\arctan\frac{2x+3}{2} + C$$

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EXAMPLE 16

$$\int \frac{dx}{x^2+8x+17} = ?$$

A) arctan(x + 1) + C

B) arctan(x + 2) + C

C) arctan(x + 3) + C

D) 3arctan(x + 3) + C

E) arctan(x + 4) + C

SOLUTION

The answer is E

$$\int \frac{dx}{x^2+8x+17} = \int \frac{dx}{(x+4)^2+1}$$

$$\left.\begin{array}{l} u = x+4 \\ du = dx \end{array}\right\} \Rightarrow \int \frac{du}{u^2+1} = \arctan u + C = \arctan|x+4| + C$$

EXAMPLE 17

$$\int \frac{dx}{4x^2+12x+13} = ?$$

A) $\frac{1}{4}\arctan\frac{2x+3}{2}+C$

B) $\frac{1}{2}\arctan\frac{2x+3}{2}+C$

C) $\frac{1}{4}\arctan(2x+3)+C$

D) $\frac{1}{4}\arctan\frac{2x-3}{2}+C$

E) $\frac{1}{2}\arctan(2x+3)+C$

SOLUTION

The answer is A

$$\int \frac{dx}{4x^2+12x+13} = \int \frac{dx}{(2x+3)^2+4} = \frac{1}{4}\int \frac{dx}{\left(\frac{2x+3}{2}\right)^2+1}$$

$$\left.\begin{array}{l} u = \frac{2x+3}{2} \\ du = dx \end{array}\right\} \Rightarrow \frac{1}{4}\int \frac{du}{u^2+1} = \frac{1}{4}\arctan u + C = \frac{1}{4}\arctan\frac{2x+3}{2} + C$$

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3. KISMİ İNTEGRASYON YÖNTEMİ

u ile v, x değişkenine bağlı birer fonksiyon olmak üzere,
u ile dv çarpımının integrali $\int u\cdot dv = u\cdot v - \int v\cdot du$ dur.

NOT

Logaritma, Arc, Polinom, Trigonometrik, Üstel fonksiyonlarının herhangi ikisinin çarpımı biçimindeki fonksiyonların integrali alınırken verilen sıra önceliği olan fonksiyona u denir.

ÖRNEK 18

$\int \ln x\, dx = ?$

A) $x\cdot\ln x + C$ B) $x\cdot\ln x - x + C$ C) $\ln x - x + C$
D) $\ln x + x + C$ E) $x\cdot\ln x + x + C$

ÇÖZÜM

Yanıt **B**

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$$\int \ln x\, dx = x\cdot\ln x - \int x\cdot\frac{1}{x}\,dx$$
$$= x\cdot\ln x - x + C$$

ÖRNEK 19

$\int x\cdot\sin x\, dx = ?$

A) $x\cdot\sin x + \cos x + C$ B) $-x\cdot\sin x - \cos x + C$ C) $-x\cdot\cos x - \cos x + C$
D) $-x\cdot\cos x - \sin x + C$ E) $-x\cdot\cos x + \sin x + C$

NOT

$\int f(x)\cdot g(x)\,dx$ integralindeki çarpanlardan f(x) in sonlu sayıdaki türevi sıfır ve g(x) in integrali de kolay alınabiliyorsa integral aşağıdaki gibi hesaplanır.

f(x)	g(x)
+ f(x)	1. integral
− f′(x)	2. integral
+ f″(x)	3. integral
.	.
.	.
.	.

ÇÖZÜM

Yanıt **E**

Çözüm 1

$x = u \qquad \sin x\,dx = dv$

$dx = du \qquad -\cos x = v$

$$\int x\cdot\sin x\,dx = -x\cdot\cos x + \int \cos x\,dx$$
$$= -x\cdot\cos x + \sin x + C$$

Çözüm 2

Türev	İntegral
+x	sinx
−1	−cosx
+0	−sinx

$$\int x\cdot\sin x\,dx = x\cdot(-\cos x) + \sin x + C$$

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NOTE

*In the integration consists of **L**ogarithmic, **A**rc, **P**olynomial, **T**rigonometric, or **E**xponential functions; firstly u is given order expression.*

3. INTEGRATION by PARTS

Let u and v be a function x .

The integration of u and dv is $\int u \cdot dv = u \cdot v - \int v \cdot du$.

EXAMPLE 18

$\int \ln x \, dx = ?$

A) $x \cdot \ln x + C$ B) $x \cdot \ln x - x + C$ C) $\ln x - x + C$

D) $\ln x + x + C$ E) $x \cdot \ln x + x + C$

SOLUTION

The answer is **B**

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$$\int \ln x \, dx = x \cdot \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \cdot \ln x - x + C$$

NOTE

In integral $\int f(x) \cdot g(x) \, dx$, the finite derivative of f(x) is 0 and the integral of g(x) is taken easily, we calculate as follows.

f(x)	g(x)
+ f(x)	1st Integral
− f′(x)	2nd Integral
+ f″(x)	3rd Integral
.	.
.	.
.	.

EXAMPLE 19

$\int x \cdot \sin x \, dx = ?$

A) $x \cdot \sin x + \cos x + C$ B) $-x \cdot \sin x - \cos x + C$ C) $-x \cdot \cos x - \cos x + C$

D) $-x \cdot \cos x - \sin x + C$ E) $-x \cdot \cos x + \sin x + C$

SOLUTION

The answer is **E**

Solution 1

$x = u \qquad \sin x \, dx = dv$

$dx = du \qquad -\cos x = v$

$$\int x \cdot \sin x \, dx = -x \cdot \cos x + \int \cos x \, dx$$

$$= -x \cdot \cos x + \sin x + C$$

Solution 2

Derivative	Integral
+x	sinx
−1	−cosx
+0	−sinx

$$\int x \cdot \sin x \, dx = x \cdot (-\cos x) + \sin x + C$$

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ÖRNEK 20

$\int x \cdot e^x \, dx = ?$

A) $x \cdot e^x + C$ B) $x \cdot e^x - e^x + C$ C) $x \cdot e^x + e^x + C$

D) $x \cdot e^x - x + C$ E) $x \cdot e^x + x + C$

ÇÖZÜM

Yanıt **B**

Çözüm 1

$u = x \qquad dv = e^x \, dx$

$du = dx \qquad v = e^x$

$\int x \cdot e^x \, dx = x \cdot e^x - \int e^x \, dx$

$= x \cdot e^x - e^x + C$

Çözüm 2

Türev	İntegral
$+x$	e^x
-1	e^x
$+0$	e^x

$\int x \cdot e^x \, dx = x \cdot e^x - e^x + C$

ÖRNEK 21

$\int x^2 \cos x \, dx = ?$

A) $x^2\sin x - 2x\cos x + 2\sin x + C$ B) $x^2\sin x - 2x\cos x - 2\sin x + C$

C) $x^2\sin x + 2x\cos x - 2\sin x + C$ D) $x^2\sin x + 2x\cos x + 2\sin x + C$

E) $x^2\sin x + 2x\cos x + \sin x + C$

ÇÖZÜM

Yanıt **C**

Çözüm 1

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x dx \qquad v = \sin x$

$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$

$u = x$

$du = dx$

$dv = \sin x \, dx$ $\Rightarrow \int x \sin x \, dx = -x\cos x + \sin x$

$v = -\cos x$

$\int x^2 \cos x \, dx = x^2 \sin x + 2x\cos x - 2\sin x + C$

Çözüm 2

Türev	İntegral
$+x^2$	$\cos x$
$-2x$	$\sin x$
$+2$	$-\cos x$
-0	$-\sin x$

$\int x^2 \cos x \, dx = x^2 \sin x + 2x\cos x - 2\sin x + C$

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EXAMPLE 20

$\int x \cdot e^x \, dx = ?$

A) $x \cdot e^x + C$ B) $x \cdot e^x - e^x + C$ C) $x \cdot e^x + e^x + C$

D) $x \cdot e^x - x + C$ E) $x \cdot e^x + x + C$

SOLUTION

The answer is **B**

Solution 1

$u = x \qquad dv = e^x \, dx$

$du = dx \qquad v = e^x$

$\int x \cdot e^x \, dx = x \cdot e^x - \int e^x \, dx$

$= x \cdot e^x - e^x + C$

Solution 2

Derivative	Integral
$+x$	e^x
-1	e^x
$+0$	e^x

$\int x \cdot e^x \, dx = x \cdot e^x - e^x + C$

EXAMPLE 21

$\int x^2 \cos x \, dx = ?$

A) $x^2\sin x - 2x\cos x + 2\sin x + C$ B) $x^2\sin x - 2x\cos x - 2\sin x + C$

C) $x^2\sin x + 2x\cos x - 2\sin x + C$ D) $x^2\sin x + 2x\cos x + 2\sin x + C$

E) $x^2\sin x + 2x\cos x + \sin x + C$

SOLUTION

The answer is **C**

Solution 1

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x dx \qquad v = \sin x$

$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$

$u = x$

$du = dx$

$dv = \sin x \, dx$

$v = -\cos x$

$\Rightarrow \int x \sin x \, dx = -x\cos x + \sin x$

$\int x^2 \cos x \, dx = x^2 \sin x + 2x\cos x - 2\sin x + C$

Solution 2

Derivative	Integral
$+x^2$	$\cos x$
$-2x$	$\sin x$
$+2$	$-\cos x$
-0	$-\sin x$

$\int x^2 \cos x \, dx = x^2 \sin x + 2x\cos x - 2\sin x + C$

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ÖRNEK 22

$\int \arctan x\,dx = ?$

A) $\arctan x \cdot x - \frac{1}{2}\ln|1+x^2|+C$ B) $x\cdot\arctan x - \ln\sqrt{x^2+1}+C$

C) $x\cdot\tan x + \ln\sqrt{x^2+1}+C$ D) $x\cdot\tan x - \ln\sqrt{x^2+1}+C$

E) $\arctan x + \ln\sqrt{x^2+1}+C$

ÇÖZÜM

Yanıt **A**

$\arctan x = u \qquad dx = dv$

$\frac{1}{1+x^2}dx = du \qquad x = v$

$$\int \arctan x\,dx = \arctan x\cdot x - \int \frac{x}{1+x^2}dx \qquad \left[1+x^2=u\ ,\ 2xdx=du\ ,\ xdx=\frac{du}{2}\right]$$

$$= \arctan x\cdot x - \frac{1}{2}\int\frac{du}{u}$$

$$= \arctan x\cdot x - \frac{1}{2}\ln|1+x^2|+C$$

ÖRNEK 23

$\int (x^2+x)\cdot e^x\,dx = ?$

A) $e^x\cdot(x^2+x-2)+C$ B) $e^x\cdot(x^2-x+1)+C$ C) $e^x\cdot(x^2+2x-1)+C$

D) $e^x\cdot(x^2-x-2)+C$ E) $e^x\cdot(x^2+x+1)+C$

ÇÖZÜM

Yanıt **B**

Türev	İntegral
$+x^2+x$	e^x
$-(2x+1)$	e^x
$+2$	e^x
-0	e^x

$$\int (x^2+x)\cdot e^x\,dx = (x^2+x)e^x - (2x+1)e^x + 2e^x + C$$

$$= e^x(x^2+x-2x-1+2)+C$$

$$= e^x(x^2-x+1)+C$$

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EXAMPLE 22

$\int \arctan x\,dx = ?$

A) $\arctan x \cdot x - \frac{1}{2}\ln|1+x^2|+C$ B) $x\cdot\arctan x - \ln\sqrt{x^2+1}+C$

C) $x\cdot\tan x + \ln\sqrt{x^2+1}+C$ D) $x\cdot\tan x - \ln\sqrt{x^2+1}+C$

E) $\arctan x + \ln\sqrt{x^2+1}+C$

SOLUTION

The answer is A

$\arctan x = u \qquad dx = dv$

$\frac{1}{1+x^2}dx = du \qquad x = v$

$$\int \arctan x\,dx = \arctan x\cdot x - \int \frac{x}{1+x^2}dx \qquad \left[1+x^2=u\ ,\ 2xdx=du\ ,\ xdx=\frac{du}{2}\right]$$

$$= \arctan x\cdot x - \frac{1}{2}\int\frac{du}{u}$$

$$= \arctan x\cdot x - \frac{1}{2}\ln|1+x^2|+C$$

EXAMPLE 23

$\int (x^2+x)\cdot e^x\,dx = ?$

A) $e^x\cdot(x^2+x-2)+C$ B) $e^x\cdot(x^2-x+1)+C$ C) $e^x\cdot(x^2+2x-1)+C$

D) $e^x\cdot(x^2-x-2)+C$ E) $e^x\cdot(x^2+x+1)+C$

SOLUTION

The answer is B

Derivative	Integral
$+x^2+x$	e^x
$-(2x+1)$	e^x
$+2$	e^x
-0	e^x

$$\int (x^2+x)\cdot e^x\,dx = (x^2+x)e^x - (2x+1)e^x + 2e^x + C$$

$$= e^x(x^2+x-2x-1+2)+C$$

$$= e^x(x^2-x+1)+C$$

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$\int \sin^a x\,dx$, $\int \cos^a x\,dx$, $\int \sin^a x \cdot \cos^b x\,dx$'in İntegrali

İntegral	Kuvvetler	Özdeşlik
$\int \sin^a x\,dx$	a çift sayı	$\cos 2x = 1 - 2\sin^2 x$
$\int \sin^a x\,dx$	a tek sayı	$\cos^2 x = 1 - \sin^2 x$ ve $\sin^a x = \sin^{a-1} x \cdot \sin x$
$\int \cos^a x\,dx$	a çift sayı	$\cos 2x = 2\cos^2 x - 1$
$\int \cos^a x\,dx$	a tek sayı	$\cos^2 x = 1 - \sin^2 x$
$\int \sin^a x \cdot \cos^b x\,dx$	a ve b tek sayı (a < b)	$\sin^a x = \sin^{a-1} x \cdot \sin x$ ve $\cos^2 x = 1 - \sin^2 x$
$\int \sin^a x \cdot \cos^b x\,dx$	a ve b çift sayı	$\sin 2x = 2\sin x \cdot \cos x$, $\cos 2x = 1 - 2\sin^2 x$ $\cos 2x = 2\cos^2 x - 1$
$\int \sin^a x \cdot \cos^b x\,dx$	a ve b'den biri tek (a tek ise)	$\sin^a x = \sin^{a-1} x \cdot \sin x$

ÖRNEK 24

$\int \sin^3 x\,dx = ?$

A) $-\cos x + \frac{\cos^3 x}{3} + C$

B) $-\cos x - \frac{\cos^3 x}{3} + C$

C) $\cos x + \frac{\sin^3 x}{3} + C$

D) $\cos x - \sin^3 x + C$

E) $\sin x - \cos^3 x + C$

ÇÖZÜM

Yanıt **A**

$$\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx = \int (1 - \cos^2 x) \cdot \sin x\,dx$$

$$= \int \sin x\,dx - \int \cos^2 x \cdot \sin x\,dx \qquad [u = \cos x,\ du = -\sin x\,dx]$$

$$= -\cos x + \int u^2\,du$$

$$= -\cos x + \frac{u^3}{3} + C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

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Integrals of $\int \sin^a x\,dx$, $\int \cos^a x\,dx$, $\int \sin^a x \cdot \cos^b x\,dx$

Integral	Powers	Identity
$\int \sin^a x\,dx$	a is even number	$\cos 2x = 1 - 2\sin^2 x$
$\int \sin^a x\,dx$	a is odd number	$\cos^2 x = 1 - \sin^2 x$ and $\sin^a x = \sin^{a-1} x \cdot \sin x$
$\int \cos^a x\,dx$	a is even number	$\cos 2x = 2\cos^2 x - 1$
$\int \cos^a x\,dx$	a is odd number	$\cos^2 x = 1 - \sin^2 x$
$\int \sin^a x \cdot \cos^b x\,dx$	a and b are odd numbers (a < b)	$\sin^a x = \sin^{a-1} x \cdot \sin x$ and $\cos^2 x = 1 - \sin^2 x$
$\int \sin^a x \cdot \cos^b x\,dx$	a and b are even numbers	$\sin 2x = 2\sin x \cdot \cos x$, $\cos 2x = 1 - 2\sin^2 x$ $\cos 2x = 2\cos^2 x - 1$
$\int \sin^a x \cdot \cos^b x\,dx$	One of these is odd (a is odd)	$\sin^a x = \sin^{a-1} x \cdot \sin x$

EXAMPLE 24

$\int \sin^3 x\,dx = ?$

A) $-\cos x + \frac{\cos^3 x}{3} + C$

B) $-\cos x - \frac{\cos^3 x}{3} + C$

C) $\cos x + \frac{\sin^3 x}{3} + C$

D) $\cos x - \sin^3 x + C$

E) $\sin x - \cos^3 x + C$

SOLUTION

The answer is A

$$\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx = \int (1 - \cos^2 x) \cdot \sin x\,dx$$

$$= \int \sin x\,dx - \int \cos^2 x \cdot \sin x\,dx \qquad [u = \cos x,\ du = -\sin x\,dx]$$

$$= -\cos x + \int u^2\,du$$

$$= -\cos x + \frac{u^3}{3} + C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

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ÖRNEK 25

$\int \cos^2 x\,dx = ?$

A) $\frac{\sin 2x}{2} - \frac{x}{2} + C$ B) $\frac{\sin 2x}{2} + \frac{x}{2} + C$ C) $\frac{\sin 2x}{4} - \frac{x}{4} + C$

D) $\frac{\sin 2x}{4} - \frac{x}{2} + C$ E) $\frac{\sin 2x}{4} + \frac{x}{2} + C$

ÇÖZÜM

Yanıt **E**

$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$

$\int \cos^2 x\,dx = \int \frac{\cos 2x + 1}{2}\,dx = \int \frac{\cos 2x}{2}\,dx + \int \frac{1}{2}\,dx$

$= \frac{\sin 2x}{4} + \frac{x}{2} + C$

ÖRNEK 26

$\int \sin^2 x \cdot \cos^2 x\,dx = ?$

A) $\frac{x}{4} - \frac{\sin 4x}{32} + C$ B) $\frac{x}{8} - \frac{\sin 4x}{32} + C$ C) $\frac{x}{2} - \frac{\sin 4x}{32} + C$

D) $x - \frac{\sin 8x}{32} + C$ E) $\frac{x}{8} + \frac{\sin 4x}{32} + C$

ÇÖZÜM

Yanıt **B**

$\int \sin^2 x \cdot \cos^2 x\,dx = \int \left(\frac{\sin 2x}{2}\right)^2 dx$

$= \frac{1}{4}\int \sin^2 2x\,dx \qquad \left[\cos 4x = 1 - 2\sin^2 2x \Rightarrow \sin^2 2x = \frac{1 - \cos 4x}{2}\right]$

$= \frac{1}{4}\int \frac{1 - \cos 4x}{2}\,dx$

$= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C$

$= \frac{x}{8} - \frac{\sin 4x}{32} + C$

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EXAMPLE 25

$\int \cos^2 x\, dx = ?$

A) $\frac{\sin 2x}{2} - \frac{x}{2} + C$ B) $\frac{\sin 2x}{2} + \frac{x}{2} + C$ C) $\frac{\sin 2x}{4} - \frac{x}{4} + C$

D) $\frac{\sin 2x}{4} - \frac{x}{2} + C$ E) $\frac{\sin 2x}{4} + \frac{x}{2} + C$

SOLUTION

The answer is E

$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$

$\int \cos^2 x\, dx = \int \frac{\cos 2x + 1}{2}\, dx = \int \frac{\cos 2x}{2}\, dx + \int \frac{1}{2}\, dx$

$= \frac{\sin 2x}{4} + \frac{x}{2} + C$

EXAMPLE 26

$\int \sin^2 x \cdot \cos^2 x\, dx = ?$

A) $\frac{x}{4} - \frac{\sin 4x}{32} + C$ B) $\frac{x}{8} - \frac{\sin 4x}{32} + C$ C) $\frac{x}{2} - \frac{\sin 4x}{32} + C$

D) $x - \frac{\sin 8x}{32} + C$ E) $\frac{x}{8} + \frac{\sin 4x}{32} + C$

SOLUTION

The answer is B

$\int \sin^2 x \cdot \cos^2 x\, dx = \int \left(\frac{\sin 2x}{2}\right)^2 dx$

$= \frac{1}{4}\int \sin^2 2x\, dx \qquad \left[\cos 4x = 1 - 2\sin^2 2x \Rightarrow \sin^2 2x = \frac{1 - \cos 4x}{2}\right]$

$= \frac{1}{4}\int \frac{1 - \cos 4x}{2}\, dx$

$= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C$

$= \frac{x}{8} - \frac{\sin 4x}{32} + C$

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∫ cosax·sinbx dx, ∫ sinax·sinbx dx, ∫ cosax·cosbx dx'in İntegrali

Bu integralleri bulmak için ters dönüşüm formülleri kullanılır.

- $\sin x \cdot \cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$
- $\cos x \cdot \sin y = \frac{1}{2}[\sin(x+y)-\sin(x-y)]$
- $\cos x \cdot \cos y = \frac{1}{2}[\cos(x+y)+\cos(x-y)]$
- $\sin x \cdot \sin y = -\frac{1}{2}[\cos(x+y)-\cos(x-y)]$

ÖRNEK 27

$\int \sin 3x \cdot \cos x \, dx = ?$

A) $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$

B) $\frac{1}{8}\cos 4x + \frac{1}{4}\cos 2x + C$

C) $-\frac{1}{8}\cos 4x + \frac{1}{4}\cos 2x + C$

D) $-\frac{1}{8}\sin 4x - \frac{1}{4}\sin 2x + C$

E) $-\frac{1}{8}\cos 4x - \frac{1}{4}\sin 2x + C$

ÇÖZÜM

Yanıt **A**

$$\sin 3x \cdot \cos x = \frac{1}{2}[\sin(3x+x)+\sin(3x-x)]$$

$$= \frac{1}{2}(\sin 4x + \sin 2x)$$

$$\int \sin 3x \cdot \cos x \, dx = \frac{1}{2}\int (\sin 4x + \sin 2x)\, dx$$

$$= \frac{1}{2}\left(\frac{-\cos 4x}{4} - \frac{\cos 2x}{2}\right) + C$$

$$= -\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$$

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Integrals of $\int \cos ax \cdot \sin bx\,dx$, $\int \sin ax \cdot \sin bx\,dx$, $\int \cos ax \cdot \cos bx\,dx$

We can use inverse conversion formulas that finding the this integral.

- $\sin x \cdot \cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$
- $\cos x \cdot \sin y = \frac{1}{2}[\sin(x+y)-\sin(x-y)]$
- $\cos x \cdot \cos y = \frac{1}{2}[\cos(x+y)+\cos(x-y)]$
- $\sin x \cdot \sin y = -\frac{1}{2}[\cos(x+y)-\cos(x-y)]$

EXAMPLE 27

$\int \sin 3x \cdot \cos x\,dx = ?$

A) $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$

B) $\frac{1}{8}\cos 4x + \frac{1}{4}\cos 2x + C$

C) $-\frac{1}{8}\cos 4x + \frac{1}{4}\cos 2x + C$

D) $-\frac{1}{8}\sin 4x - \frac{1}{4}\sin 2x + C$

E) $-\frac{1}{8}\cos 4x - \frac{1}{4}\sin 2x + C$

SOLUTION

The answer is A

$$\sin 3x \cdot \cos x = \frac{1}{2}[\sin(3x+x)+\sin(3x-x)]$$

$$= \frac{1}{2}(\sin 4x + \sin 2x)$$

$$\int \sin 3x \cdot \cos x\,dx = \frac{1}{2}\int (\sin 4x + \sin 2x)\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 4x}{4} - \frac{\cos 2x}{2}\right) + C$$

$$= -\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$$

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1. $\int x^2 \cdot f(x)\,dx = x^5 + 8x^3 \Rightarrow f'(1) = ?$

A) 5
B) 8
C) 10
D) 12
E) 15

2. $\int \tan^2 x\,dx = ?$

A) $\tan x - x + C$
B) $\tan x + x + C$
C) $-\tan x + x + C$
D) $-\tan x - x + C$
E) $x \cdot \tan x + x + C$

3. $\frac{d}{dx}\int (3^x + \ln x)\,dx = ?$

A) $3^x - \ln x$
B) $3^x + \ln x$
C) $2^x - \ln x$
D) $2^x + \ln x$
E) $3x + \ln x$

4. $f'(x) = x^2 + 3x,\ f(1) = 5 \Rightarrow f(2) = ?$

A) $\frac{23}{2}$
B) $\frac{35}{3}$
C) $\frac{71}{6}$
D) 12
E) $\frac{73}{6}$

5. $\int (x^2+2)^4 \cdot x\,dx = ?$

A) $\frac{1}{4}(x^2+2)^5 + C$
B) $\frac{1}{6}(x^2+2)^5 + C$
C) $\frac{1}{8}(x^2+2)^5 + C$
D) $\frac{1}{10}(x^2+2)^5 + C$
E) $\frac{1}{12}(x^2+2)^5 + C$

6. $\int \frac{x}{x^2+4}\,dx = ?$

A) $\frac{1}{2}\ln|x^2+4| + C$
B) $\frac{1}{4}\ln|x^2+4| + C$
C) $\frac{1}{2}\ln|x^2-4| + C$
D) $\frac{1}{4}\ln|x^2-4| + C$
E) $\frac{1}{8}\ln|x^2+4| + C$

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7. $\int [f(x)]^2 \cdot f'(x)\,dx = ?$

A) $[f(x)]^3 + C$

B) $\frac{1}{2}[f(x)]^3 + C$

C) $\frac{1}{3}[f(x)]^3 + C$

D) $\frac{1}{4}[f(x)]^3 + C$

E) $\frac{1}{5}[f(x)]^3 + C$

8. $F(x) = \int (x^2-1)e^{x^3-3x}dx,\ F(\sqrt{3}) = 3 \Rightarrow F(x) = ?$

A) $e^{x^3-3x} + \frac{3}{4}$

B) $\frac{1}{3}e^{x^3-3x} + \frac{8}{3}$

C) $(x^2-1)e^{x^2-1} + 7$

D) $\frac{1}{2}e^{x^2-1} + \frac{9}{4}$

E) $\frac{1}{3}e^{x^3-3x} + \frac{11}{3}$

9. $f(x) = \frac{2x}{x^2+5}$, $f(1) = \frac{1}{3}$ ve(and) $\int d\left(\frac{2x}{x^2+5}\right) = \frac{3}{7}$

$\Rightarrow \sum x = ?$

A) 4

B) $\frac{13}{3}$

C) $\frac{14}{3}$

D) 5

E) $\frac{7}{3}$

10. $\int \frac{5x-3}{x^2-2x-3}dx = ?$

A) $\ln|x+1| + 3\ln|x-3| + C$

B) $\frac{1}{2}\ln|x+1| + 3\ln|x-3| + C$

C) $\frac{3}{2}\ln|x+1| + 3\ln|x-3| + C$

D) $2\cdot\ln|x+1| + 3\ln|x-3| + C$

E) $\frac{5}{2}\cdot\ln|x+1| + 3\ln|x-3| + C$

11. $\int \frac{2dx}{(2x-3)^6} = ?$

A) $\frac{-2}{5(2x-3)^5} + C$

B) $\frac{-1}{5(2x-3)^5} + C$

C) $\frac{1}{5(2x-3)^5} + C$

D) $\frac{2}{5(2x-3)^5} + C$

E) $\frac{3}{5(2x-3)^5} + C$

12. $\int x^2\cos(x^3+1)\,dx = ?$

A) $-\frac{1}{3}\sin(x^3+1) + C$

B) $-\frac{1}{2}\sin(x^3+1) + C$

C) $\frac{1}{3}\cos(x^3+1) + C$

D) $\frac{1}{3}\sin(x^3+1) + C$

E) $\frac{1}{2}\cos(x^3+1) + C$

YANITLAR / ANSWERS							
1. C	2. A	3. B	4. C	5. D	6. A	7. C	8. B
9. C	10. D	11. B	12. D				

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1. $\frac{d}{dx}\left(\int x^2 f(x)\,dx\right) = \frac{d}{dx}(x^5 + 8x^3)$

$x^2 f(x) = 5x^4 + 24x^2$

$f(x) = 5x^2 + 24$

$f'(x) = 10x$

$f'(1) = 10 \cdot 1 = 10$

Yanıt / The Answer is C

2. $\int \tan^2 x\,dx = \int (\tan^2 x + 1)\,dx - \int 1\,dx$

$= \tan x - x + C$

Yanıt / The Answer is A

3. $\frac{d}{dx}\int (3^x + \ln x)\,dx = 3^x + \ln x$

Yanıt / The Answer is B

4. $f'(x) = x^2 + 3x$

$\int f'(x)\,dx = \int (x^2 + 3x)dx$

$f(x) = \frac{x^3}{3} + \frac{3x^2}{2} + c$

$f(1) = \frac{1}{3} + \frac{3}{2} + c = 5$

$c = 5 - \frac{11}{6}$

$c = \frac{19}{6}$

$f(2) = \frac{2^3}{3} + \frac{3 \cdot 2^2}{2} + \frac{19}{6} = \frac{71}{6}$

Yanıt / The Answer is C

5. $u = x^2 + 2 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$\int \left(x^2 + 2\right)^4 x\,dx = \int u^4 \frac{du}{2}$

$= \frac{1}{2}\int u^4\,du = \frac{1}{10}u^5 + C$

$= \frac{1}{10}\left(x^2 + 2\right)^5 + C$

Yanıt / The Answer is D

6. $u = x^2 + 4 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$\int \frac{x}{x^2 + 4}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|x^2 + 4| + C$

Yanıt / The Answer is A

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7. $u = f(x) \Rightarrow du = f'(x)\,dx$

$$\int [f(x)]^2 \cdot f'(x)\,dx = \int u^2 du = \frac{u^3}{3} + C$$

$$= \frac{f^3(x)}{3} + C$$

Yanıt / The Answer is C

8. $u = x^3 - 3x \Rightarrow du = (3x^2 - 3)\,dx$

$$(x^2 - 1)\,dx = \frac{du}{3}$$

$$F(x) = \int e^u \frac{du}{3} = \frac{1}{3} e^{x^3 - 3x} + C$$

$$F(\sqrt{3}) = \frac{1}{3} e^0 + C = 3 \Rightarrow C = \frac{8}{3}$$

$$\Rightarrow F(x) = \frac{1}{3} e^{x^3 - 3x} + \frac{8}{3}$$

Yanıt / The Answer is B

9. $\int d\left(\frac{2x}{x^2+5}\right) = \frac{2x}{x^2+5}$

$$\frac{2x}{x^2+5} = \frac{3}{7}$$

$$14x = 3x^2 + 15$$

$$3x^2 - 14x + 15 = 0 \Rightarrow \sum x = \frac{14}{3}$$

Yanıt / The Answer is C

10. $$\frac{5x-3}{x^2-2x-3} = \frac{5x-3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$= \frac{(A+B)x + B - 3A}{(x+1)(x-3)}$$

$$(A+B)x + B - 3A = 5x - 3$$

$$\left.\begin{aligned} A + B &= 5 \\ B - 3A &= -3 \end{aligned}\right\} \Rightarrow 4A = 8 \Rightarrow A = 2$$

$$A = 2 \Rightarrow B = 3$$

$$\int \frac{5x-3}{x^2-2x-3}\,dx = 2\int \frac{dx}{x+1} + 3\int \frac{dx}{x-3}$$

$$= 2\ln|x+1| + 3\ln|x-3| + C$$

Yanıt / The Answer is D

11. $u = 2x - 3 \Rightarrow du = 2\,dx$

$$\int \frac{2dx}{(2x-3)^6} = \int \frac{du}{u^6}$$

$$= -\frac{1}{5} u^{-5} + C$$

$$= -\frac{1}{5(2x-3)^5} + C$$

Yanıt / The Answer is B

12. $u = x^3 + 1 \Rightarrow du = 3x^2 dx$

$$\int x^2 \cos(x^3 + 1)\,dx = \int \frac{du}{3} \cos u$$

$$= \frac{1}{3} \sin u + C$$

$$= \frac{1}{3} \sin(x^3 + 1) + C$$

Yanıt / The Answer is D

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1. $\int f(2x+1)\,dx = 3x^2 - 4x + 5 \Rightarrow f(5) = ?$

A) 4
B) 5
C) 6
D) 7
E) 8

2. $\int d\cdot(8x) = ?$

A) $8x + C$ B) $-x + C$ C) $4x^2 - 7x + C$

D) $\frac{(8x)^2}{2} + C$ E) $(8x)^2 + C$

3. $\int (2x-1)\cdot f(x)\,dx = x^3 + x^2 - 3x + C \Rightarrow f(3) = ?$

A) 4
B) 5
C) 6
D) 7
E) 8

4. $\int \frac{x+2}{\sqrt{16-x^2}}\,dx = ?$

A) $-5(16-x^2)^{\frac{5}{2}} + \arcsin(4-x) + C$

B) $-(16-x^2)^{\frac{1}{2}} + 2\arcsin\left(\frac{x}{4}\right) + C$

C) $-2(16-x^2)^{\frac{1}{2}} + \arcsin\left(\frac{x}{4}\right) + C$

D) $\frac{-(16-x^2)^{\frac{3}{2}}}{3} + \arcsin\left(\frac{x}{2}\right) + C$

E) $\frac{-(16-x^2)^{\frac{5}{2}}}{3} + \frac{\arcsin\left(\frac{x}{4}\right)}{2} + C$

5. $\int (x-3)^2\,dx = ?$

A) $\frac{x^3}{2} - 3x^2 + 9x + C$

B) $\frac{x^3}{3} - 3x^2 + 9x + C$

C) $\frac{x^3}{4} - 3x^2 + 9x + C$

D) $\frac{x^3}{3} + 3x^2 + 9x + C$

E) $\frac{x^3}{4} + 3x^2 + 9x + C$

6. $\int \frac{2x+1}{2x-1}\,dx = f(x), \quad f(1) = 1 \Rightarrow f(3) = ?$

A) 2 + ln 5
B) 3 + ln 5
C) 3 − ln 5
D) 4 + ln 5
E) 4 − ln 5

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7. $\int \frac{2x}{\sqrt{1-x^4}}\,dx = ?$

A) $\arcsin(x^2)+C$

B) $\frac{1}{2}\arcsin x^2+C$

C) $\arcsin x^4+C$

D) $\frac{1}{4}\arcsin x^4+C$

E) $\frac{1}{4}\arcsin\frac{x^4}{4}+C$

8. $\int \frac{x\sqrt[3]{x}+\sqrt[3]{x}}{\sqrt[3]{x}}\,dx = ?$

A) x^2+x+C

B) x^2-x+C

C) $\frac{x^2}{2}+x+C$

D) $\frac{x^2}{2}-x+C$

E) $\frac{x^2}{2}+2x+C$

9. $\int \frac{dx}{\sqrt{4-x^2}} = ?$

A) $\arctan\frac{x}{2}+C$

B) $\arctan\frac{x}{4}+C$

C) $\arcsin\frac{x}{4}+C$

D) $\arcsin\frac{x}{2}+C$

E) $\arcsin x+C$

10. $\int \frac{x^2+2x+8}{x+2}\,dx = ?$

A) $\frac{x^2}{2}-8\ln|x+2|+C$

B) $\frac{x^2}{2}+8\ln|x+2|+C$

C) $x^2-8\ln|x+2|+C$

D) $x^2+8\ln|x+2|+C$

E) $\frac{x^2}{2}-4\ln|x+2|+C$

11. $\int \tan^3 x\cdot\sec x\,dx = ?$

A) $\frac{1}{3}\tan^3 x-\tan x+C$

B) $\frac{1}{4}\tan^3 x+\tan x+C$

C) $\frac{1}{4}\sec^4 x+\tan x+C$

D) $\frac{1}{3}\sec^3 x-\sec x+C$

E) $\frac{1}{2}\sec^2 x+\sec x+C$

12. $\int \frac{dx}{x\cdot\sqrt{a^2+x^2}} = ?$

A) $\frac{x}{a+\sqrt{a^2-x^2}}+C$

B) $\frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right|+C$

C) $\frac{ax}{a+\sqrt{x^2+a^2}}+C$

D) $\frac{1}{a^2}\ln\left|\frac{2x}{a+\sqrt{x^2+a^2}}\right|+C$

E) $\frac{1}{a^2}\arcsin\left(\frac{x}{a+\sqrt{x^2+a^2}}\right)+C$

YANITLAR / ANSWERS

1. E	2. A	3. C	4. B	5. B	6. B	7. A	8. C
9. D	10. B	11. D	12. B				

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1. $\frac{d}{dx}\int f(2x+1)\,dx = \frac{d}{dx}(3x^2-4x+5)$

$f(2x+1) = 6x-4$

$\left.\begin{array}{l} 2x+1=5 \\ x=2 \end{array}\right\} \Rightarrow f(5) = 12-4 = 8$

Yanıt / The Answer is E

2. $\int d\cdot(8x) = 8x+C$

Yanıt / The Answer is A

3. $\frac{d}{dx}\int (2x-1)\cdot f(x)\,dx = \frac{d}{dx}(x^3+x^2-3x+C)$

$(2x-1)f(x) = 3x^2+2x-3$

$x=3 \Rightarrow (2\cdot 3-1)\cdot f(3) = 3\cdot 3^2+2\cdot 3-3$

$5\cdot f(3) = 27+6-3$

$f(3) = 6$

Yanıt / The Answer is C

4. $\int \frac{x+2}{\sqrt{16-x^2}}\,dx = \int \frac{x}{\sqrt{16-x^2}}\,dx + \int \frac{2dx}{\sqrt{16-x^2}}$

$\left.\begin{array}{l} u = 16-x^2 \\ du = -2x\,dx \\ x\,dx = -\frac{du}{2} \end{array}\right\} \Rightarrow \int \frac{-du}{2\sqrt{u}} + \frac{2}{4}\int \frac{dx}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

$\left.\begin{array}{l} t = \frac{x}{4} \\ dt = \frac{1}{4}dx \\ dx = 4dt \end{array}\right\} \Rightarrow = -\sqrt{u} + \frac{1}{2}\int \frac{4dt}{\sqrt{1-t^2}}$

$= -\sqrt{u} + 2\arcsin t + C$

$= -\sqrt{16-x^2} + 2\arcsin\frac{x}{4} + C$

Yanıt / The Answer is B

5. $\int (x-3)^2\,dx = \int x^2\,dx - 6\int x\,dx + 9dx$

$= \frac{x^3}{3} - 3x^2 + 9x + C$

Yanıt / The Answer is B

6. $f(x) = \int \frac{2x+1-2+2}{2x-1}\,dx = \int \frac{(2x-1)\,dx}{2x-1} + \int \frac{2dx}{2x-1}$

$= x + \ln|2x-1| + C$

$f(1) = 1+\ln 1+C = 1 \Rightarrow C = 0$

$f(3) = 3+\ln|6-1| = 3+\ln 5$

Yanıt / The Answer is B

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7. $\int \frac{2x}{\sqrt{1-x^4}}dx = \int \frac{2x}{\sqrt{1-(x^2)^2}}dx$

$$\left.\begin{array}{l} u = x^2 \\ du = 2xdx \end{array}\right\} \Rightarrow \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = \arcsin x^2 + C$$

Yanıt / The Answer is A

8. $\int \frac{x\sqrt[3]{x} + \sqrt[3]{x}}{\sqrt[3]{x}}dx = \int \frac{\sqrt[3]{x}\,(x+1)}{\sqrt[3]{x}}dx$

$$= \int (x+1)\,dx$$

$$= \frac{x^2}{2} + x + C$$

Yanıt / The Answer is C

9. $\int \frac{dx}{\sqrt{4-x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$

$$\left.\begin{array}{l} u = \frac{x}{2} \\ du = \frac{1}{2}dx \end{array}\right\} \Rightarrow \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = \arcsin\frac{x}{2} + C$$

Yanıt / The Answer is D

10.
$$\begin{array}{r|l} x^2+2x+8 & x+2 \\ \cline{2-2} -x^2 \mp 2x & x \\ \cline{1-1} 8 & \end{array}$$

$$\int \frac{x^2+2x+8}{x+2}dx = \int \left(x + \frac{8}{x+2}\right)dx$$

$$= \frac{x^2}{2} + 8\ln|x+2| + C$$

Yanıt / The Answer is B

11. $\int \tan^3 x \cdot \sec x\,dx = \int \frac{\sin^3 x}{\cos^4 x}dx = \int \frac{(1-\cos^2 x)\cdot \sin x}{\cos^4 x}dx$

$$\cos x = u \Rightarrow -\sin x\,dx = du$$

$$\sin x\,dx = -du$$

$$\int \frac{1-u^2}{u^4}du = -\left(\int \frac{1}{u^4}du - \int \frac{1}{u^2}du\right)$$

$$= +\frac{1}{3}u^{-3} - u^{-1}$$

$$= \frac{1}{3}\cdot\frac{1}{\cos^3 x} - \frac{1}{\cos x}$$

$$= \frac{1}{3}\sec^3 x - \sec x + C$$

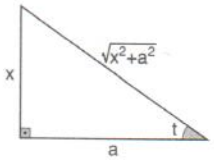
Yanıt / The Answer is D

12. $x = a\tan t$

$$dx = \frac{a}{\cos^2 t}dx$$

$$\int \frac{dx}{x\cdot\sqrt{a^2+x^2}} = \int \frac{\frac{a}{\cos^2 t}\cdot dt}{a\tan t\sqrt{a^2+a^2\tan^2 t}}$$

$$= \frac{1}{a}\int \frac{dt}{\sin t}$$



$$= \frac{1}{a}\ln\left|\frac{\sin t}{1+\cos t}\right|$$

$$= \frac{1}{a}\ln\left|\frac{\frac{x}{\sqrt{x^2+a^2}}}{1+\frac{a}{\sqrt{x^2+a^2}}}\right| + C$$

$$= \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$$

Yanıt / The Answer is B

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YÖS Soruları / YÖS Questions

1. $\int \frac{dx}{9+4x^2} = ?$

A) $\frac{2}{3}\ln x^2 + C$

B) $\sin\frac{2x}{3} + C$

C) $\frac{1}{6}\arctan\frac{2x}{3} + C$

D) $\frac{1}{3}\arcsin\frac{2x}{3} + C$

E) $\frac{1}{3}\arccos\frac{3x}{5} + C$

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

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2. $\int (3x^2 - 2x + \frac{1}{x})\,dx = ?$

A) $x^3 + x^2 - \frac{1}{x^2} + C$

B) $x^3 - x^2 - \frac{1}{x^2} + C$

C) $x^3 - x^2 - \frac{1}{x^2} + C$

D) $x^3 - x^2 + \ln x + C$

E) $x^3 - x^2 - \ln x + C$

[MARMARA ÜNİV. YÖS 2013]

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3. $\int \frac{4}{x^2 - 4}\,dx = ?$

A) $\ln|x-2|\cdot|x+2| + C$

B) $x^3 - 4x + C$

C) $\ln\left|\frac{x-2}{x+2}\right| + C$

D) $\ln\left|\frac{x+2}{x-2}\right| + C$

E) $\ln\left|\frac{x-2}{x+2}\right| + x + C$

[ANKARA ÜNİV. YÖS 2013]

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4. $\int \frac{\sqrt{x+1}}{\sqrt[3]{x+1}}\,dx = ?$

A) $\frac{6}{7}(x+1)^{\frac{7}{6}} + C$

B) $\frac{7}{6}(x+1)^{\frac{7}{6}} + C$

C) $\frac{6}{7}(x+1)^{\frac{6}{7}} + C$

D) $\frac{6}{7}(x-1)^{\frac{7}{6}} + C$

E) $\frac{7}{6}(x-1)^{\frac{7}{6}} + C$

[EGE-YÖS 2012]

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5. $\int \frac{3x^2}{(x^3+1)^5}\,dx = ?$

A) $\frac{-1}{4(x^3+1)^4} + C$

B) $\frac{-1}{x^2+4} + C$

C) $\frac{x}{x^2+1} + C$

D) $\frac{1}{2(x^3+1)^2} + C$

E) $\frac{-1}{(x^3+1)^3} + C$

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

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6. $\frac{3}{2}\int \frac{\sqrt{\ln x}}{x}\,dx = ?$

A) $\sqrt{\ln^3 x} + C$

B) $\sqrt{\ln x} + C$

C) $\frac{1}{2}x + C$

D) $-\frac{2}{3x} + C$

E) $3x\ln x + C$

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

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7. $\int f(x)\,dx = 2x^3 - 5x^2 + C$

$f(x) = ?$

A) $6x - 5$

B) $6x^2 - 10x$

C) $2x^4 - 5x^3$

D) $6x^2 - 5x + 1$

E) $3x - 1$

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

8. $\int \frac{2\ln x}{x}\,dx = ?$

A) $\ln x + C$ B) $\ln^2 x + C$ C) $x\ln x + C$

D) $x^2e^x + C$ E) $xe^x + C$

[YÖS 2010]

9. $\int (x^2 + 4x)^4 \cdot (2x + 4)\,dx = ?$

A) $(x^2 + 4x)^4 + C$ B) $4(x^2 + 4x) + C$

C) $8(x^2 + 4x) + C$ D) $\frac{(x^2 + 4x)^4}{4} + C$

E) $\frac{(x^2 + 4x)^5}{5} + C$

[YÖS 2008]

10. $\int \frac{x}{x+3}\,dx = ?$

A) $x - 3\ln|x+3| + C$ B) $x - \ln|x+3| + C$

C) $\frac{x}{\ln|x+3|} + C$ D) $\frac{\ln|x+3|}{x} + C$

E) $\ln|x+3| - x + C$

[YÖS 2008]

11. $f(1) = 9$

$f'(x) = 6x + 4$

$f(0) = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

[YÖS 2006]

12. $\int x \cdot f(x)\,dx = 2x^3 + 4x^2 + C$

$f(x) = ?$

A) $6x + 4$ B) $6x + 8$ C) $6x + 13$

D) $\frac{2}{3}x^3 + 2x$ E) $\frac{2}{3}x^3 + 2x^2$

[YÖS 2005]

YANITLAR / ANSWERS							
1. C	2. D	3. C	4. A	5. A	6. A	7. B	8. B
9. E	10. A	11. C	12. B				

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1. $\int \frac{dx}{9+4x^2} = \frac{1}{9}\int \frac{dx}{1+\left(\frac{2x}{3}\right)^2}$

$\frac{2x}{3} = u \Rightarrow \frac{2dx}{3} = du$

$dx = \frac{3du}{2}$

$\Rightarrow \frac{1}{9}\int \frac{3du}{2(1+u^2)} = \frac{1}{6}\arctan u$

$= \frac{1}{6}\arctan\frac{2x}{3} + C$

Yanıt / The Answer is C

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2 $\int (3x^2 - 2x + \frac{1}{x})\,dx = \frac{3x^3}{3} - \frac{2x^2}{2} + \ln x + C$

$= x^3 - x^2 + \ln x + C$

Yanıt / The Answer is D

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3. $\frac{4}{x^2-4} = \frac{1}{x-2} - \frac{1}{x+2}$

$\Rightarrow \int \frac{4}{x^2-4}\,dx = \int\left(\frac{1}{x-2} - \frac{1}{x+2}\right)dx$

$= \ln|x-2| - \ln|x+2| + C$

$= \ln\left|\frac{x-2}{x+2}\right| + C$

Yanıt / The Answer is C

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4. $\int \frac{\sqrt{x+1}}{\sqrt[3]{x+1}}\,dx = \int (x+1)^{\frac{1}{2}} \cdot (x+1)^{-\frac{1}{3}}\,dx$

$= \int (x+1)^{\frac{1}{6}}\,dx$

$= \frac{(x+1)^{\frac{1}{6}+1}}{\frac{1}{6}+1} = \frac{6\cdot(x+1)^{\frac{7}{6}}}{7} + C$

Yanıt / The Answer is A

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5. $x^3 + 1 = u \Rightarrow 3x^2\,dx = du$

$\int \frac{3x^2}{(x^3+1)^5}\,dx = \int \frac{du}{u^5}$

$= \int u^{-5}\cdot du$

$= \frac{u^{-4}}{-4} + C$

$= \frac{(x^3+1)^{-4}}{-4} + C$

Yanıt / The Answer is A

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6. $\left.\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}\right\} \Rightarrow \frac{3}{2}\int \frac{\sqrt{\ln x}}{x}\,dx = \frac{3}{2}\int \sqrt{u}\,du$

$= \frac{3}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \sqrt{\ln^3 x} + C$

Yanıt / The Answer is A

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7. $\frac{d}{dx}\left(\int f(x)\,dx\right)=\frac{d}{dx}(2x^3-5x^2+C)$

$f(x)=6x^2-10x$

Yanıt / The Answer is B

8. $\ln x=u \Rightarrow \frac{1}{x}dx=du$

$\int \frac{2\ln x}{x}dx=\int 2u\,du$

$=u^2+C$

$=\ln^2 x+C$

Yanıt / The Answer is B

9. $x^2+4x=u \Rightarrow (2x+4)\,dx=du$

$\int (x^2+4x)^4\cdot(2x+4)\,dx=\int u^4\cdot du=\frac{u^5}{5}+C$

$=\frac{(x^2+4x)^5}{5}+C$

Yanıt / The Answer is E

10. $\int \frac{x}{x+3}dx=\int\left(1-\frac{3}{x+3}\right)dx$

$=x-3\ln|x+3|+C$

Yanıt / The Answer is A

11. $f'(x)=6x+4$

$\Rightarrow f(x)=\int(6x+4)\,dx=3x^2+4x+c$

$f(1)=3\cdot1+4\cdot1+c=9$

$c=2$

$f(x)=3x^2+4x+2$

$f(0)=3\cdot0+4\cdot0+2=2$

Yanıt / The Answer is C

12. $\int x\,f(x)\,dx=2x^3+4x^2+c$

$\Rightarrow x\,f(x)=6x^2+8x$

$f(x)=\frac{6x^2+8x}{x}=6x+8$

Yanıt / The Answer is B

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1. $\int\left(\frac{x+1}{x}\right)dx = ?$

Yanıt / The Answer is $x + \ln|x| + c$

2. $\int(e^{3x}+5^{2x})dx = ?$

Yanıt / The Answer is $\frac{e^{3x}}{3}+\frac{5^{2x}}{2\ln 5}+c$

3. $\int\frac{x}{(x^2+2)^3}dx = ?$

Yanıt / The Answer is $\frac{-1}{4(x^2+2)^2}+c$

4. $\int(x+1)\cdot f(x)\,dx = x^2-3x+5 \Rightarrow f(2) = ?$

Yanıt / The Answer is $\frac{1}{3}$

5. $\int(\sin 8x - e^{3x})\,dx = ?$

Yanıt / The Answer is $\frac{-\cos 8x}{8}-\frac{e^{3x}}{3}+c$

6. $\int\cos^2 x\,dx = ?$

Yanıt / The Answer is $\frac{\sin 2x}{4}+\frac{x}{2}+c$

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7. $\int \frac{\ln^3 x}{x}\,dx = ?$

Yanıt / The Answer is $\frac{\ln^4 x}{4} + C$

8. $\int \frac{x^3 + x^2 - x + 4}{x+1}\,dx = ?$

Yanıt / The Answer is $\frac{x^3}{3} - x + 5\ln|x+1| + C$

9. $f(x) = \int (3x^2 - 5x + 1)\,dx \Rightarrow f'(2) = ?$

Yanıt / The Answer is 3

10. $f(x) = \int (\sin x)\,dx \Rightarrow f'(\frac{\pi}{6}) = ?$

Yanıt / The Answer is $\frac{1}{2}$

11. $\int 5\,dx = ?$

Yanıt / The Answer is $5x + C$

12. $\int 4x^2 dx = ?$

Yanıt / The Answer is $\frac{4x^3}{3} + C$

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13. $\int \frac{dx}{x^3} = ?$

Yanıt / The Answer is $-\frac{1}{2x^2} + C$

14. $\int \frac{dx}{\sqrt{x}} = ?$

Yanıt / The Answer is $2\sqrt{x} + C$

15. $\int (3 - x^2)\, dx = ?$

Yanıt / The Answer is $3x - \frac{x^3}{3} + C$

16. $\int (12x^3 - 6x^2 - 4x)\, dx = ?$

Yanıt / The Answer is $3x^4 - 2x^3 - 2x^2 + C$

17. $\int \left(\frac{1}{3} - \frac{1}{x}\right) dx = ?$

Yanıt / The Answer is $\frac{1}{3}x - \ln|x| + C$

18. $\int 4\cos x\, dx = ?$

Yanıt / The Answer is $4\sin x + C$

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19. $\int (3\sin x - 2\cos x)\,dx = ?$

Yanıt / The Answer is $-(3\cos x + 2\sin x) + C$

20. $\int \frac{1}{\sec x}\,dx = ?$

Yanıt / The Answer is $\sin x + C$

21. $\int \frac{1}{\operatorname{cosec} x}\,dx = ?$

Yanıt / The Answer is $-\cos x + C$

22. $\frac{d}{dx}\int (x^3 - 3\sin^2 x)\,dx = ?$

Yanıt / The Answer is $x^3 - 3\sin^2 x$

23. $\int \frac{2x+1}{x^2+x}\,dx = ?$

Yanıt / The Answer is $\ln|x^2 + x| + C$

24. $\int \frac{3\cos 3x}{\sin 3x}\,dx = ?$

Yanıt / The Answer is $\ln|\sin 3x| + C$

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25. $\int \cot x \, dx = ?$

Yanıt / The Answer is $\ln|\sin x| + C$

26. $\int e^{2x+3} \, dx = ?$

Yanıt / The Answer is $\frac{1}{2} e^{2x+3} + C$

27. $\int \frac{\arcsin x}{\sqrt{1-x^2}} \, dx = ?$

Yanıt / The Answer is $\frac{(\arcsin x)^2}{2} + C$

28. $\int x^2 e^x \, dx = ?$

Yanıt / The Answer is $x^2e^x - 2xe^x + 2e^x + C$

29. $\int \frac{x^2 - x - 6}{x - 3} dx = ?$

Yanıt / The Answer is $\frac{x^2}{2} + 2x + C$

30. $\int \frac{x^3 + 2x^2 + 1}{x + 1} dx = ?$

Yanıt / The Answer is $\frac{x^3}{3} + \frac{x^2}{2} - x + 2 \cdot \ln|x + 1| + C$

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1. $\int \frac{\sqrt[3]{2x+7}-\sqrt{2x+7}}{2x+7}dx = ?$

A) $\sqrt[6]{2x+7}+C$

B) $\sqrt[6]{(2x+7)^2}-\sqrt[6]{(2x+7)^3}+C$

C) $\frac{3}{2}\sqrt[3]{(2x+7)}-\sqrt{(2x+7)}+C$

D) $\frac{3}{2}\sqrt{(2x+7)}-\sqrt[3]{(2x+7)}+C$

E) $\frac{3}{2}\sqrt[3]{(2x+7)^2}+\sqrt{(2x+7)}+C$

2. $\frac{df(x)}{dx} = 3x-4$ ve(and) $f(-1)=\frac{13}{2} \Rightarrow$ **f(x) = ?**

A) $\frac{3x^2}{2}-4x+2$

B) $\frac{3x^2}{2}-4x+1$

C) $\frac{3x^2}{2}-4x+3$

D) $-x^3-2x^2-2$

E) x^2-4x+6

3. $f'(x) = 6x^2 - 4x$, $f(1) = 3 \Rightarrow$ **f(2) = ?**

A) 11

B) 14

C) 15

D) 17

E) 20

4. $\int d\cdot(\cos^2 x) = ?$

A) $\cos^2 x + C$

B) $\sin^2 x + C$

C) $\tan^2 x + C$

D) $\frac{\cos^3 x}{3}+C$

E) $\sin x \cdot \cos^2 x + C$

5. $\int \frac{3x^2-x+1}{x^3}dx = ?$

A) $3\ln|x|+\frac{1}{x}-\frac{1}{2x^2}+C$

B) $3\ln|x|-\frac{1}{x^2}-\frac{1}{3x^3}+C$

C) $\ln|x|+\frac{2}{x}-\frac{3}{x^2}+C$

D) $\frac{\ln|x|}{3}+\frac{1}{x}-\frac{2}{x^2}+C$

E) $3\ln|x|+\frac{1}{2x}-\frac{3}{3x^2}+C$

6. $\int x\cdot\sqrt[3]{(x^2-3)^2}\,dx = ?$

A) $\frac{3\cdot(x^2-3)^{\frac{2}{3}}}{4}+C$

B) $\frac{3(x^2-3)^{\frac{4}{3}}}{8}+C$

C) $\frac{3(x^2-3)^{\frac{5}{3}}}{10}+C$

D) $\frac{4(x^2-3)^{\frac{5}{4}}}{10}+C$

E) $2(x^2-3)^{\frac{1}{2}}+C$

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7. $\int \sec^2 x\, dx = ?$

A) tanx + C
B) cotx + C
C) sinx + C
D) cosx + C
E) cosx + sinx + C

8. $\frac{d}{dx}\int (\cos x - e^x + \log x)\, dx = ?$

A) cosx – e^x + logx
B) cosx + e^x + logx
C) cosx – e^x – logx
D) cosx – e^x + 2logx
E) cosx + e^x – logx

9. $\int \frac{dx}{x(\ln x)^3} = ?$

A) $-\frac{1}{4}(\ln x)^{-4} + C$
B) $-\frac{1}{3}(\ln x)^{-3} + C$
C) $-\frac{1}{2}(\ln x)^{-2} + C$
D) $\frac{1}{2}(\ln x)^{2} + C$
E) $\frac{1}{4}(\ln x)^{4} + C$

10. $\int \frac{dx}{\sqrt{1-4x^2}} = ?$

A) $-\frac{1}{4}\arcsin 2x + C$
B) $-\frac{1}{2}\arcsin 2x + C$
C) $\frac{1}{4}\arcsin 2x + C$
D) $\frac{1}{2}\arcsin 2x + C$
E) $\frac{1}{2}\arcsin 4x + C$

11. $\int \sqrt{4x^2+1}\, x\, dx = ?$

A) $\frac{1}{18}\sqrt{(4x^2+1)^3} + C$
B) $\frac{1}{12}\sqrt{(4x^2+1)^3} + C$
C) $\frac{1}{6}\sqrt{(4x^2+1)^3} + C$
D) $\frac{1}{4}\sqrt{(4x^2+1)^3} + C$
E) $\frac{1}{2}\sqrt{(4x^2+1)^3} + C$

12. $\int \frac{1}{x^2+4x+5}\, dx = ?$

A) arccot(x + 2) + C
B) arctan(x + 2) + C
C) arcsin(x + 2) + C
D) arctan(x – 2) + C
E) arccot(x – 2) + C

YANITLAR / ANSWERS							
1. C	2. B	3. A	4. A	5. A	6. C	7. A	8. A
9. C	10. D	11. B	12. B				

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1. $\frac{df(x)}{dx} = x^2 - x,\ f(3) = 4 \Rightarrow \mathbf{f(1) = ?}$

A) 2

B) $\frac{3}{2}$

C) $\frac{1}{3}$

D) $-\frac{2}{3}$

E) -2

2. $f'(x) = 6x^2 - 8x + 5,$
$f(1) = 2 \Rightarrow \mathbf{f(-1) = ?}$

A) -8

B) -10

C) -12

D) -14

E) -16

3. $\int (x^2 - 1) \cdot f(x)\, dx = 3x^3 - x^2 + 2 \Rightarrow \mathbf{f(2) = ?}$

A) $\frac{22}{3}$

B) $\frac{26}{3}$

C) $\frac{28}{3}$

D) $\frac{32}{3}$

E) $\frac{34}{3}$

4. $\int e^{\sin x} \cdot \cos x\, dx = ?$

A) $e^{\sin x} + C$

B) $e^{\cos x} + C$

C) $e^{-\sin x} + C$

D) $e^{-\cos x} + C$

E) $e + C$

5. $\int \sqrt{3x + 2}\, dx = ?$

A) $\frac{1}{3}\sqrt{(3x+2)^3} + C$

B) $\frac{1}{6}\sqrt{(3x+2)^3} + C$

C) $\frac{1}{9}\sqrt{(3x+2)^3} + C$

D) $\frac{2}{9}\sqrt{(3x+2)^3} + C$

E) $\frac{2}{15}\sqrt{(3x+2)^3} + C$

6. $\int (x-4)^5\, dx = ?$

A) $-\frac{1}{4}(x-4)^4 + C$

B) $-\frac{1}{5}(x-4)^5 + C$

C) $-\frac{1}{6}(x-4)^6 + C$

D) $\frac{1}{5}(x-4)^4 + C$

E) $\frac{1}{6}(x-4)^6 + C$

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7. $\int \sin^3 x \cdot \cos x \, dx = ?$

A) $\frac{\sin^3 x}{3} + C$

B) $\frac{\sin^4 x}{4} + C$

C) $\frac{\sin^5 x}{5} + C$

D) $\frac{\cos^3 x}{3} + C$

E) $\frac{\cos^4 x}{4} + C$

8. $\int \frac{\ln^4 x}{x} dx = ?$

A) $\frac{(\ln x)^3}{3} + C$

B) $\frac{(\ln x)^4}{4} + C$

C) $\frac{(\ln x)^5}{5} + C$

D) $\frac{5 \ln x}{4} + C$

E) $\frac{4 \ln x}{5} + C$

9. $\int \sqrt[3]{x^2+3} \; 2x \, dx = ?$

A) $\frac{3}{4}\sqrt[3]{(x^2+3)^4} + C$

B) $\frac{3}{4}\sqrt{(x^2+3)^3} + C$

C) $-\frac{3}{4}\sqrt[3]{(x^2+3)^4} + C$

D) $-\frac{3}{4}\sqrt{(x^2+3)^3} + C$

E) $-\frac{4}{3}\sqrt[3]{(x^2+3)^4} + C$

10. $\int \frac{x}{(x^2-1)^{10}} dx = ?$

A) $\frac{-2}{5(x^2-1)^9} + C$

B) $\frac{-2}{9(x^2-1)^9} + C$

C) $\frac{-1}{18(x^2-1)^9} + C$

D) $\frac{1}{18(x^2-1)^9} + C$

E) $\frac{2}{9(x^2-1)^9} + C$

11. $\int \frac{\sin x}{\cos^3 x} dx = ?$

A) $\frac{-2}{\cos^2 x} + C$

B) $\frac{-1}{\cos^2 x} + C$

C) $\frac{1}{2\cos^2 x} + C$

D) $\frac{1}{\cos^2 x} + C$

E) $\frac{2}{\cos^2 x} + C$

12. $\int \frac{4x+3}{4x+1} dx = ?$

A) $x - \ln|4x+1| + C$

B) $x + \ln|4x+1| + C$

C) $x - \frac{1}{2}\ln|4x+1| + C$

D) $x + \frac{1}{2}\ln|4x+1| + C$

E) $x - 2\ln|4x+1| + C$

YANITLAR / ANSWERS							
1. D	2. C	3. D	4. A	5. D	6. E	7. B	8. C
9. A	10. C	11. C	12. D				

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1. $f(x)=\int(\cos^2 x+\cos 2x-\frac{1}{2})dx$ ve(and) $f(0)=0 \Rightarrow f\left(\frac{\pi}{4}\right)=?$

A) 1

B) $\frac{3}{4}$

C) $\frac{7}{4}$

D) $\frac{9}{2}$

E) $\frac{9}{4}$

2. $\int f(x)dx=\arctan(2x)+\sin\frac{\pi}{2}x-\frac{\sqrt{2}\,\pi}{4}x \Rightarrow f\left(\frac{1}{2}\right)=?$

A) 1

B) $1+\frac{\pi\sqrt{2}}{2}$

C) $1-\frac{2\sqrt{2}}{4}$

D) $\frac{1}{2}$

E) $1+\frac{\pi}{2}$

3. $\int \sec^2 x\, f(x)dx=\tan^2 x(1+\tan x)+x \Rightarrow f(\frac{\pi}{4})=?$

A) 3

B) 5

C) $\frac{11}{2}$

D) $\frac{13}{2}$

E) $\frac{15}{2}$

4. $\int \tan x\ dx=?$

A) $-\ln|\sin x|+C$

B) $-\ln|\cos x|+C$

C) $\ln|\sin x|+C$

D) $\ln|\cos x|+C$

E) $\ln|\tan x|+C$

5. $\int 2^{\sin x}\cos x\ dx=?$

A) $2^{\cos x}+C$

B) $2^{\sin x}+C$

C) $2^{\tan x}+C$

D) $\frac{2^{\sin x}}{\ln 2}+C$

E) $\frac{2^{\sin x}}{e}+C$

6. $\int 2x\cdot e^{x^2}dx=?$

A) $2xe^{x^2}+C$

B) $2e^{x^2}+C$

C) $e^{x^2}+C$

D) $\frac{e^{x^2}}{2}+C$

E) $\frac{e^{2x}}{2}+C$

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7. $\int \frac{\arctan x}{1+x^2}\,dx = ?$

A) $\arcsin x + C$

B) $(\arcsin x)^2 + C$

C) $\arctan x + C$

D) $(\arctan x)^2 + C$

E) $\frac{(\arctan x)^2}{2} + C$

8. $\int \frac{2x}{1+x^4}\,dx = ?$

A) $\arcsin x + C$

B) $\arctan x + C$

C) $\arctan x^2 + C$

D) $\frac{1}{2}\arctan x + C$

E) $\frac{1}{2}\arctan x^2 + C$

9. $\int (x^2-1)\sin x\,dx = ?$

A) $(1-x^2)\cos x + 2x\sin x + 2\cos x + C$

B) $(1-x^2)\cos x - 2x\sin x + 2\cos x + C$

C) $(1-x^2)\cos x - 2x\sin x - 2\cos x + C$

D) $(1-x^2)\cos x + 2x\sin x - 2\cos x + C$

E) $(1-x^2)\cos x + 2x\sin x + \cos x + C$

10. $\int x \cdot \ln x\,dx = ?$

A) $\frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

B) $\frac{x^2}{2}\ln x + \frac{x^2}{4} + C$

C) $\frac{x^2}{2}\ln x - \frac{x^2}{2} + C$

D) $x^2\ln x - x^2 + C$

E) $x^2\ln x + x^2 + C$

11. $\int (x^3-5)e^x\,dx = ?$

A) $(x^3-5)e^x - 3x^2e^x + 6xe^x - 6e^x + C$

B) $(x^3-5)e^x - 3x^2e^x + 6xe^x + 6e^x + C$

C) $(x^3-5)e^x + 3x^2e^x - 6xe^x + 6e^x + C$

D) $(x^3-5)e^x + 3x^2e^x + 6xe^x - 6e^x + C$

E) $(x^3-5)e^x - 3x^2e^x + 6xe^x - e^x + C$

12. $\int \cos^5 x\,dx = ?$

A) $\sin x + \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

B) $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

C) $\sin x - \frac{2}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

D) $\sin x + \sin^3 x + \sin^5 x + C$

E) $\sin x + 3\sin^3 x + 5\sin^5 x + C$

YANITLAR / ANSWERS							
1. B	2. A	3. C	4. B	5. D	6. C	7. E	8. C
9. A	10. A	11. A	12. B				

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BÖLÜM 2
CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Galois, Evariste [1811 – 1832]

Fransız matematikçi. 20 yaşında ölmeden önce denklem teorisine yaptığı katkılar ile tanınır. Bir denklemin çözülmesi için grup teorisini geliştirmiştir.

French mathematician who had made major contributions to the theory of equations before he died at the age of 20. His work developed the necessarily group theory to deal with the question of whether an equation can be solved algebraically.

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DEFINITE INTEGRAL

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DEFINITE INTEGRAL

BÖLÜM 2
CHAPTER

f: [a, b] → R, y = f(x) [a, b] aralığında integrallenebilir bir fonksiyon olmak üzere,

$\int f(x)\,dx = F(x)+C \Rightarrow \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$ ifadesine f fonksiyonunun a dan b ye kadar belirli integrali denir.

ÖRNEK 1

$\int_0^2 3x^2\,dx = ?$

A) 4 B) 8 C) 16 D) 20 E) 24

ÇÖZÜM

Yanıt **B**

$\int_0^2 3x^2\,dx = \frac{3x^3}{3}\Big|_0^2 = 2^3 - 0^3 = 8$

ÖRNEK 2

$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = ?$

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

ÇÖZÜM

Yanıt **C**

$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = -\frac{1}{2}\cos 2x\Big|_0^{\frac{\pi}{4}} = -\frac{1}{2}\left(\cos\frac{\pi}{2} - \cos 0\right) = -\frac{1}{2}(0-1) = \frac{1}{2}$

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Let f: [a, b] → R be an integrable function on the interval [a, b].

The expression $\int f(x)\,dx = F(x) + C \Rightarrow \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ is called definite integral of the function f from a to b.

EXAMPLE 1

$$\int_0^2 3x^2\,dx = ?$$

A) 4 B) 8 C) 16 D) 20 E) 24

SOLUTION

The answer is B

$$\int_0^2 3x^2\,dx = \frac{3x^3}{3}\Big|_0^2 = 2^3 - 0^3 = 8$$

EXAMPLE 2

$$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = ?$$

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

SOLUTION

The answer is C

$$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = -\frac{1}{2}\cos 2x\Big|_0^{\frac{\pi}{4}} = -\frac{1}{2}\left(\cos\frac{\pi}{2} - \cos 0\right) = -\frac{1}{2}(0-1) = \frac{1}{2}$$

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BELİRLİ İNTEGRALİN ÖZELLİKLERİ

a, b, c ∈ R,

1. $\int_a^b k\cdot f(x)\,dx = k\cdot\int_a^b f(x)\,dx \qquad (k\in R)$

2. $\int_a^b (f(x)\pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

3. f(x) tek fonksiyon.

$$\int_{-a}^{a} f(x)\,dx = 0$$

f(x) çift fonksiyon.

$$\int_{-a}^{a} f(x)\,dx = 2\cdot\int_0^a f(x)\,dx$$

4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

5. $a < c < b$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

NOTLAR

✓ f(x) tek fonksiyon ise,
f(−x) = −f(x).

✓ f(x) çift fonksiyon ise
f(−x) = f(x).

ÖRNEK 3

$\int_{-1}^{2} (x^2-2x+3)\,dx = ?$

A) 9 B) 7 C) 5 D) 3 E) 1

PEKİŞTİRME

$\int_1^2 (3x^2+4x)\,dx = ?$

ÇÖZÜM

Yanıt A

$$\int_{-1}^{2} (x^2-2x+3)\,dx = \frac{x^3}{3}-\frac{2x^2}{2}+3x\Big|_{-1}^{2}$$

$$=\left(\frac{8}{3}-2^2+3\cdot 2\right)-\left(-\frac{1}{3}-1-3\right)$$

$$=\frac{14}{3}+\frac{13}{3}=9$$

YANIT : 13

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NOTES

✓ If f(x) is odd function, then
$f(-x) = -f(x)$.

✓ If f(x) is even function, then
$f(-x) = f(x)$.

PROPERTIES of DEFINITE INTEGRAL

a, b, c ∈ R,

1. $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx \qquad (k \in R)$

2. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

3. f(x) is odd function.

$$\int_{-a}^{a} f(x)\,dx = 0$$

f(x) is even function.

$$\int_{-a}^{a} f(x)\,dx = 2 \cdot \int_0^a f(x)\,dx$$

4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

5. $a < c < b$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

REINFORCE

$\int_1^2 (3x^2 + 4x)\,dx = ?$

ANSWER : 13

EXAMPLE 3

$\int_{-1}^{2} (x^2 - 2x + 3)\,dx = ?$

A) 9 B) 7 C) 5 D) 3 E) 1

SOLUTION

The answer is A

$$\int_{-1}^{2} (x^2 - 2x + 3)\,dx = \frac{x^3}{3} - \frac{2x^2}{2} + 3x \Big|_{-1}^{2}$$

$$= \left(\frac{8}{3} - 2^2 + 3 \cdot 2\right) - \left(-\frac{1}{3} - 1 - 3\right)$$

$$= \frac{14}{3} + \frac{13}{3} = 9$$

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ÖRNEK 4

$$\int_{\frac{1}{4}}^{\frac{e^2}{4}} \frac{\ln 4x}{x}\,dx = ?$$

A) $-\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) 2

ÇÖZÜM

Yanıt E

$\ln 4x = u \Rightarrow \frac{1}{x}dx = du$

$x = \frac{1}{4} \Rightarrow \ln 4x = 0 \Rightarrow u = 0$

$x = \frac{e^2}{4} \Rightarrow \ln 4x = 2 \Rightarrow u = 2$

$$\int_{\frac{1}{4}}^{\frac{e^2}{4}} \frac{\ln 4x}{x}\,dx = \int_0^2 u\,du = \frac{u^2}{2}\Big|_0^2 = \left(\frac{2^2}{2} - \frac{0^2}{2}\right) = 2$$

PEKİŞTİRME

$$\int_{e}^{e^2} \frac{\ln x}{x}\,dx = ?$$

YANIT : 3 / 2

ÖRNEK 5

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x}\,dx = ?$$

A) $\ln 3$ B) $\ln\frac{3}{4}$ C) $\ln\frac{1}{4}$ D) 1 E) 2

ÇÖZÜM

Yanıt A

$\sin^2 x = u \Rightarrow \sin 2x\,dx = du$

$x = \frac{\pi}{6} \Rightarrow \sin^2\frac{\pi}{6} = \frac{1}{4} \Rightarrow u = \frac{1}{4}$

$x = \frac{\pi}{3} \Rightarrow \sin^2\frac{\pi}{3} = \frac{3}{4} \Rightarrow u = \frac{3}{4}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x}\,dx = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{du}{u} = \ln u\Big|_{\frac{1}{4}}^{\frac{3}{4}} = \ln\frac{3}{4} - \ln\frac{1}{4} = \ln 3$$

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REINFORCE

$\int_{e}^{e^2} \frac{\ln x}{x}\,dx = ?$

ANSWER : 3 / 2

EXAMPLE 4

$\int_{\frac{1}{4}}^{\frac{e^2}{4}} \frac{\ln 4x}{x}\,dx = ?$

A) $-\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) 2

SOLUTION

The answer is E

$\ln 4x = u \Rightarrow \frac{1}{x}dx = du$

$x = \frac{1}{4} \Rightarrow \ln 4x = 0 \Rightarrow u = 0$

$x = \frac{e^2}{4} \Rightarrow \ln 4x = 2 \Rightarrow u = 2$

$$\int_{\frac{1}{4}}^{\frac{e^2}{4}} \frac{\ln 4x}{x}\,dx = \int_{0}^{2} u\,du = \frac{u^2}{2}\Big|_0^2 = \left(\frac{2^2}{2} - \frac{0^2}{2}\right) = 2$$

EXAMPLE 5

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x}\,dx = ?$

A) $\ln 3$ B) $\ln\frac{3}{4}$ C) $\ln\frac{1}{4}$ D) 1 E) 2

SOLUTION

The answer is A

$\sin^2 x = u \Rightarrow \sin 2x\,dx = du$

$x = \frac{\pi}{6} \Rightarrow \sin^2\frac{\pi}{6} = \frac{1}{4} \Rightarrow u = \frac{1}{4}$

$x = \frac{\pi}{3} \Rightarrow \sin^2\frac{\pi}{3} = \frac{3}{4} \Rightarrow u = \frac{3}{4}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x}\,dx = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{du}{u} = \ln u\Big|_{\frac{1}{4}}^{\frac{3}{4}} = \ln\frac{3}{4} - \ln\frac{1}{4} = \ln 3$$

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ÖRNEK 6

$$\int_0^2 (x^2+3x)\cdot(2x+3)\,dx = ?$$

A) 55 B) 50 C) 44 D) 36 E) 28

ÇÖZÜM

Yanıt B

$x^2+3x = u \Rightarrow (2x+3)\,dx = du$

$x = 0 \Rightarrow x^2+3x = 0 \Rightarrow u = 0$

$x = 2 \Rightarrow x^2+3x = 0 \Rightarrow u = 10$

$$\int_0^2 (x^2+3x)\cdot(2x+3)\,dx = \int_0^{10} u\,du$$

$$= \frac{u^2}{2}\Big|_0^{10} = \frac{100}{2} - \frac{0}{2} = 50$$

ÖRNEK 7

$$\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = ?$$

A) $-\frac{2}{3}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{2}{3}$

ÇÖZÜM

Yanıt E

$$\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = \int_0^{\frac{\pi}{2}} \cos^2 x\cdot\cos x\,dx = \int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cdot\cos x\,dx$$

$\sin x = u \Rightarrow \cos x\,dx = du$

$x = 0 \Rightarrow \sin 0 = 0 \Rightarrow u = 0$

$x = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1 \Rightarrow u = 1$

$$\int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cdot\cos x\,dx = \int_0^1 (1-u^2)\,du$$

$$= u - \frac{u^3}{3}\Big|_0^1 = \left(1-\frac{1}{3}\right) - \left(0-\frac{0^3}{3}\right) = \frac{2}{3}$$

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EXAMPLE 6

$$\int_0^2 (x^2+3x)\cdot(2x+3)\,dx = ?$$

A) 55 B) 50 C) 44 D) 36 E) 28

SOLUTION

The answer is B

$x^2+3x=u \Rightarrow (2x+3)\,dx = du$

$x=0 \Rightarrow x^2+3x=0 \Rightarrow u=0$

$x=2 \Rightarrow x^2+3x=0 \Rightarrow u=10$

$$\int_0^2 (x^2+3x)\cdot(2x+3)\,dx = \int_0^{10} u\,du$$

$$= \frac{u^2}{2}\Big|_0^{10} = \frac{100}{2} - \frac{0}{2} = 50$$

EXAMPLE 7

$$\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = ?$$

A) $-\frac{2}{3}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{2}{3}$

SOLUTION

The answer is E

$$\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = \int_0^{\frac{\pi}{2}} \cos^2 x\cdot\cos x\,dx = \int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cdot\cos x\,dx$$

$\sin x = u \Rightarrow \cos x\,dx = du$

$x=0 \Rightarrow \sin 0 = 0 \Rightarrow u=0$

$x=\frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1 \Rightarrow u=1$

$$\int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cdot\cos x\,dx = \int_0^1 (1-u^2)\,du$$

$$= u - \frac{u^3}{3}\Big|_0^1 = \left(1-\frac{1}{3}\right) - \left(0-\frac{0^3}{3}\right) = \frac{2}{3}$$

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ÖRNEK 8

$$\int_{-2}^{2} (5x^4 + 3x^3 + 3x^2 + 4x + 5)\ dx = ?$$

A) 10 B) 50 C) 100 D) 115 E) 120

ÇÖZÜM

Yanıt **C**

$3x^3$, $4x$ tek fonksiyonlardır.

$$\int_{-2}^{2} (5x^4 + 3x^3 + 3x^2 + 4x + 5)\ dx = \int_{-2}^{2} (5x^4 + 3x^2 + 5)\ dx$$

$$= 2\int_{0}^{2} (5x^4 + 3x^2 + 5)\ dx$$

$$= 2\cdot\left[5\cdot\frac{x^5}{5} + \frac{3\cdot x^3}{3} + 5x\right]\Bigg|_0^2$$

$$= 2\cdot\left[2^5 + 2^3 + 5\cdot 2\right] = 100$$

ÖRNEK 9

$$\int_{0}^{2} (x-1)\cdot(x+2)\ dx = ?$$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) 1

ÇÖZÜM

Yanıt **D**

$$\int_{0}^{2} (x-1)\cdot(x+2)\ dx = \int_{0}^{2} (x^2 + x - 2)\ dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - 2x\Bigg|_0^2$$

$$= \left(\frac{2^3}{3} + \frac{2^2}{2} - 2\cdot 2\right) - \left(\frac{0}{3} + \frac{0}{2} - 2\cdot 0\right)$$

$$= \frac{2}{3}$$

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EXAMPLE 8

$$\int_{-2}^{2} (5x^4 + 3x^3 + 3x^2 + 4x + 5)\ dx = ?$$

A) 10 B) 50 C) 100 D) 115 E) 120

SOLUTION

The answer is C

$3x^3$, $4x$ are odd functions.

$$\int_{-2}^{2} (5x^4 + 3x^3 + 3x^2 + 4x + 5)\ dx = \int_{-2}^{2} (5x^4 + 3x^2 + 5)\ dx$$

$$= 2\int_{0}^{2} (5x^4 + 3x^2 + 5)\ dx$$

$$= 2 \cdot \left[5 \cdot \frac{x^5}{5} + \frac{3 \cdot x^3}{3} + 5x \right]\Bigg|_0^2$$

$$= 2 \cdot \left[2^5 + 2^3 + 5 \cdot 2 \right] = 100$$

EXAMPLE 9

$$\int_{0}^{2} (x-1) \cdot (x+2)\ dx = ?$$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) 1

SOLUTION

The answer is D

$$\int_{0}^{2} (x-1) \cdot (x+2)\ dx = \int_{0}^{2} (x^2 + x - 2)\ dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - 2x\Bigg|_0^2$$

$$= \left(\frac{2^3}{3} + \frac{2^2}{2} - 2 \cdot 2 \right) - \left(\frac{0}{3} + \frac{0}{2} - 2 \cdot 0 \right)$$

$$= \frac{2}{3}$$

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İNTEGRALİN TÜREVİ

f : [a, b] → R fonksiyonu integrallenebilir olmak üzere,

1. $F(x) = \int_a^x f(t)\,dt \Rightarrow F'(x) = f(x)$

2. $F(x) = \int_a^{u(x)} f(t)\,dt \Rightarrow F'(x) = u'(x)\cdot f(u(x))$

3. $F(x) = \int_{v(x)}^{u(x)} f(t)\,dt \Rightarrow F'(x) = u'(x)\cdot f(u(x)) - v'(x)\cdot f(v(x))$

ÖRNEK 10

$F(x) = \int_0^{\sin x} (t^2 - t)\,dt \Rightarrow F'\left(\frac{\pi}{4}\right) = ?$

A) $\frac{2-\sqrt{2}}{2}$ B) $\frac{\sqrt{2}-2}{2}$ C) $\frac{\sqrt{2}-2}{4}$

D) $\frac{2-\sqrt{2}}{4}$ E) $2-\sqrt{2}$

ÇÖZÜM

Yanıt C

$\int_0^{\sin x} (t^2 - t)\,dt \Rightarrow F'(x) = (\sin x)'\cdot(\sin^2 x - \sin x)$

$= \cos x\cdot(\sin^2 x - \sin x)$

$F'(\frac{\pi}{4}) = \cos\frac{\pi}{4}\cdot(\sin^2\frac{\pi}{4} - \sin\frac{\pi}{4})$

$= \frac{\sqrt{2}}{2}\cdot\left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}-2}{4}$

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DERIVATIVE of INTEGRAL

Let $f : [a, b] \to R$ be an integrable function.

1. $F(x) = \int_a^x f(t)\,dt \Rightarrow F'(x) = f(x)$

2. $F(x) = \int_a^{u(x)} f(t)\,dt \Rightarrow F'(x) = u'(x)\cdot f(u(x))$

3. $F(x) = \int_{v(x)}^{u(x)} f(t)\,dt \Rightarrow F'(x) = u'(x)\cdot f(u(x)) - v'(x)\cdot f(v(x))$

EXAMPLE 10

$F(x) = \int_0^{\sin x} (t^2 - t)\,dt \Rightarrow F'\left(\frac{\pi}{4}\right) = ?$

A) $\frac{2-\sqrt{2}}{2}$ B) $\frac{\sqrt{2}-2}{2}$ C) $\frac{\sqrt{2}-2}{4}$

D) $\frac{2-\sqrt{2}}{4}$ E) $2-\sqrt{2}$

SOLUTION

The answer is C

$$\int_0^{\sin x} (t^2 - t)\,dt \Rightarrow F'(x) = (\sin x)'\cdot(\sin^2 x - \sin x)$$

$$= \cos x\cdot(\sin^2 x - \sin x)$$

$$F'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4}\cdot\left(\sin^2\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2}\cdot\left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}-2}{4}$$

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ÖRNEK 11

$F(x)=\int_0^{\sqrt{x}}(t+1)^{\frac{1}{2}}\,dt \Rightarrow F'(4)=?$

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

ÇÖZÜM

Yanıt **C**

$\int_0^{\sqrt{x}}(t+2)^{\frac{1}{2}}\,dt \Rightarrow F'(x)=(\sqrt{x})'\cdot(\sqrt{x}+2)^{\frac{1}{2}}$

$=\frac{1}{2\sqrt{x}}\cdot\left(\sqrt{\sqrt{x}+2}\right)$

$F'(4)=\frac{1}{2\sqrt{4}}\cdot\left(\sqrt{\sqrt{4}+2}\right)$

$=\frac{1}{4}\cdot 2=\frac{1}{2}$

ÖRNEK 12

$F(x)=\int_x^{x^2}\frac{\ln t}{2+\ln^2 t}\,dt \Rightarrow F'(e)=?$

A) $\frac{2e}{3}$ B) $\frac{2}{3}$ C) $\frac{2e-1}{4}$ D) $\frac{2e-1}{3}$ E) $\frac{2e+1}{3}$

ÇÖZÜM

Yanıt **D**

$\int_x^{x^2}\frac{\ln t}{2+\ln^2 t}\,dt \Rightarrow F'(x)=(x^2)'\cdot\left(\frac{\ln x^2}{2+\ln^2 x^2}\right)-(x)'\cdot\left(\frac{\ln x}{2+\ln^2 x}\right)$

$=2x\cdot\frac{\ln x^2}{2+\ln^2 x^2}-\frac{\ln x}{2+\ln^2 x}$

$F'(e)=2e\cdot\frac{\ln e^2}{2+\ln^2 e^2}-\frac{\ln e}{2+\ln^2 e}$

$=2e\cdot\frac{2}{6}-\frac{1}{3}$

$=\frac{2e-1}{3}$

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EXAMPLE 11

$F(x)=\int_0^{\sqrt{x}}(t+1)^{\frac{1}{2}}\,dt \Rightarrow F'(4)=?$

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

SOLUTION

The answer is C

$\int_0^{\sqrt{x}}(t+2)^{\frac{1}{2}}\,dt \Rightarrow F'(x)=(\sqrt{x})'\cdot(\sqrt{x}+2)^{\frac{1}{2}}$

$=\frac{1}{2\sqrt{x}}\cdot\left(\sqrt{\sqrt{x}+2}\right)$

$F'(4)=\frac{1}{2\sqrt{4}}\cdot\left(\sqrt{\sqrt{4}+2}\right)$

$=\frac{1}{4}\cdot 2=\frac{1}{2}$

EXAMPLE 12

$F(x)=\int_x^{x^2}\frac{\ln t}{2+\ln^2 t}\,dt \Rightarrow F'(e)=?$

A) $\frac{2e}{3}$ B) $\frac{2}{3}$ C) $\frac{2e-1}{4}$ D) $\frac{2e-1}{3}$ E) $\frac{2e+1}{3}$

SOLUTION

The answer is D

$\int_x^{x^2}\frac{\ln t}{2+\ln^2 t}\,dt \Rightarrow F'(x)=(x^2)'\cdot\left(\frac{\ln x^2}{2+\ln^2 x^2}\right)-(x)'\cdot\left(\frac{\ln x}{2+\ln^2 x}\right)$

$=2x\cdot\frac{\ln x^2}{2+\ln^2 x^2}-\frac{\ln x}{2+\ln^2 x}$

$F'(e)=2e\cdot\frac{\ln e^2}{2+\ln^2 e^2}-\frac{\ln e}{2+\ln^2 e}$

$=2e\cdot\frac{2}{6}-\frac{1}{3}$

$=\frac{2e-1}{3}$

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MUTLAK DEĞER FONKSİYONUNUN İNTEGRALİ

$\int_a^b |f(x)|\,dx$ integrali hesaplanırken $f(x) = 0$ denkleminin kökleri bulunarak kritik noktalar belirlenir. Bu noktalara göre integral parçalanarak hesaplanır.

ÖRNEK 13

$\int_{-4}^{1} |x+2|\,dx = ?$

A) $\frac{5}{2}$ B) $\frac{7}{2}$ C) $\frac{9}{2}$ D) $\frac{11}{2}$ E) $\frac{13}{2}$

ÇÖZÜM

Yanıt **E**

$x + 2 = 0 \Rightarrow x = -2$

x		−2	
x + 2	−	0	+

$$\int_{-4}^{1} |x+2|\,dx = \int_{-4}^{-2} (-x-2)\,dx + \int_{-2}^{1} (x+2)\,dx$$

$$= -\left(\frac{x^2}{2}+2x\right)\Big|_{-4}^{-2} + \left(\frac{x^2}{2}+2x\right)\Big|_{-2}^{1}$$

$$= \frac{13}{2}$$

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INTEGRATION of the ABSOLUTE VALUE FUNCTION

While the integral $\int_a^b |f(x)|\,dx$ is found, firstly the roots of the equation $f(x) = 0$ and critical points are found. Integral is found by part up to the these points.

EXAMPLE 13

$$\int_{-4}^{1} |x+2|\,dx = ?$$

A) $\frac{5}{2}$ B) $\frac{7}{2}$ C) $\frac{9}{2}$ D) $\frac{11}{2}$ E) $\frac{13}{2}$

SOLUTION

The answer is E

$x + 2 = 0 \Rightarrow x = -2$

x		-2	
$x+2$	$-$	0	$+$

$$\int_{-4}^{1} |x+2|\,dx = \int_{-4}^{-2} (-x-2)\,dx + \int_{-2}^{1} (x+2)\,dx$$

$$= -\left(\frac{x^2}{2}+2x\right)\Big|_{-4}^{-2} + \left(\frac{x^2}{2}+2x\right)\Big|_{-2}^{1}$$

$$= \frac{13}{2}$$

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ÖRNEK 14

$$\int_0^2 |x^2-3x+2|\,dx = ?$$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{5}{6}$ D) 1 E) $\frac{6}{5}$

ÇÖZÜM

Yanıt **D**

$x^2-3x+2=0 \Rightarrow (x-2)\cdot(x-1)=0$

$x=2,\ x=1$

x		1		2	
x^2-3x+2	+	0	−	0	+

$$\int_0^2 |x^2-3x+2|\,dx = \int_0^1 (x^2-3x+2)\,dx + \int_1^2 -(x^2-3x+2)\,dx$$

$$= \frac{x^3}{3}-\frac{3x^2}{2}+2x\Big|_0^1 - \left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)\Big|_1^2 = 1$$

ÖRNEK 15

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx = ?$$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

ÇÖZÜM

Yanıt **E**

$x \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow \sin x < 0$

$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow \sin x > 0$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx = \int_{-\frac{\pi}{2}}^{0} -\sin x\,dx + \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= \cos x\Big|_{-\frac{\pi}{2}}^{0} - \cos x\Big|_0^{\frac{\pi}{2}} = 2$$

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EXAMPLE 14

$$\int_0^2 |x^2-3x+2|\,dx = ?$$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{5}{6}$ D) 1 E) $\frac{6}{5}$

SOLUTION

The answer is D

$x^2-3x+2=0 \Rightarrow (x-2)\cdot(x-1)=0$

$x=2\,,\ x=1$

x		1		2	
x^2-3x+2	+	0	−	0	+

$$\int_0^2 |x^2-3x+2|\,dx = \int_0^1 (x^2-3x+2)\,dx + \int_1^2 -(x^2-3x+2)\,dx$$

$$= \frac{x^3}{3}-\frac{3x^2}{2}+2x\Big|_0^1 - \left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)\Big|_1^2 = 1$$

EXAMPLE 15

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx = ?$$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

SOLUTION

The answer is E

$x \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow \sin x < 0$

$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow \sin x > 0$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx = \int_{-\frac{\pi}{2}}^{0} -\sin x\,dx + \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= \cos x\Big|_{-\frac{\pi}{2}}^{0} - \cos x\Big|_0^{\frac{\pi}{2}} = 2$$

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1. $\int_{-1}^{2} x^3\,dx = ?$

A) $\frac{7}{2}$

B) $\frac{15}{4}$

C) 4

D) $\frac{17}{4}$

E) $\frac{9}{2}$

2. $\int_{0}^{4} \sqrt{2x+1}\,dx = ?$

A) $\frac{25}{3}$

B) $\frac{26}{3}$

C) 9

D) $\frac{28}{3}$

E) $\frac{29}{4}$

3. $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1-x^2}} = ?$

A) $\frac{\pi}{6}$

B) $\frac{\pi}{3}$

C) $\frac{2\pi}{3}$

D) π

E) $\frac{5\pi}{6}$

4. $\int_{-1}^{1} (x^2 + 2x - 1)\,dx = ?$

A) 2

B) 1

C) $\frac{4}{3}$

D) $-\frac{1}{3}$

E) $-\frac{4}{3}$

5. $\int_{0}^{2} \left(\frac{2x^3+8x+1}{x^2+4}\right) dx = ?$

A) $\frac{14+\pi}{8}$

B) $\frac{20+\pi}{8}$

C) $\frac{24+\pi}{8}$

D) $\frac{28+\pi}{8}$

E) $\frac{32+\pi}{8}$

6. $\int_{9}^{16} \frac{dx}{\sqrt{x}+x} = ?$

A) $\ln\frac{9}{25}$

B) $\ln\frac{16}{25}$

C) $\ln\frac{16}{9}$

D) $\ln\frac{9}{16}$

E) $\ln\frac{25}{16}$

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7. $\int_1^3 x \cdot y \cdot dy = 9 \Rightarrow x = ?$

A) 2

B) $\frac{9}{4}$

C) $\frac{5}{2}$

D) $\frac{11}{4}$

E) 3

8. $\int_0^{e-1} \frac{x-1}{x+1} dx = ?$

A) e – 1

B) e – 2

C) e – 3

D) e – 4

E) e – 5

9. $\int_0^{\frac{\pi}{2}} x \cdot \sin x \, dx = ?$

A) 5

B) 4

C) 3

D) 2

E) 1

10. $f(x) = \int_0^{\cos x} t \cdot dt \Rightarrow f'\left(\frac{\pi}{6}\right) = ?$

A) $-\frac{\sqrt{3}}{4}$

B) $\frac{\sqrt{3}}{4}$

C) $-\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{2}}{4}$

E) $\sqrt{3}$

11. $\int_{-3}^{3} |x^2 - 4| \, dx = ?$

A) $\frac{46}{3}$

B) $\frac{44}{3}$

C) $\frac{38}{3}$

D) $\frac{35}{3}$

E) $\frac{29}{3}$

12. $\int_0^2 |x^2 - 1| \, dx = ?$

A) – 4 B) – 2 C) 2

D) 4 E) 6

YANITLAR / ANSWERS							
1. B	2. B	3. A	4. E	5. E	6. E	7. B	8. C
9. E	10. A	11. A	12. C				

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1. $\int_{-1}^{2} x^3\,dx = \left.\frac{x^{3+1}}{3+1}\right|_{-1}^{2} = \left.\frac{x^4}{4}\right|_{-1}^{2}$

$= \frac{2^4-(-1)^4}{4} = \frac{15}{4}$

Yanıt / The Answer is B

2. $\int_{0}^{4} \sqrt{2x+1}\,dx = \int \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$\left.\begin{array}{l} 2x+1=u \\ 2dx=du \\ dx=\frac{du}{2} \end{array}\right\} \Rightarrow = \frac{1}{2}\cdot\frac{u^{\frac{2}{3}}}{\frac{3}{2}}$

$= \frac{1}{3}\cdot u^{\frac{3}{2}}$

$= \frac{1}{3}\sqrt{(2x+1)^3}\,\Big|_0^4$

$= \frac{1}{3}\left(\sqrt{(2\cdot4+1)^3} - \sqrt{(2\cdot0+1)^3}\right)$

$= \frac{1}{3}\left(\sqrt{9^3} - \sqrt{1^3}\right) = \frac{1}{3}(27-1) = \frac{26}{3}$

Yanıt / The Answer is B

3. $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1-x^2}} = \arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{1}{2}$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

Yanıt / The Answer is A

4. $\int_{-1}^{1} (x^2+2x-1)\,dx = \left(\frac{x^3}{3} + \frac{2\cdot x^2}{2} - x\right)\Big|_{-1}^{1}$

$= \left(\frac{1}{3}+1-1\right) - \left(-\frac{1}{3}+1+1\right)$

$= \frac{1}{3}+\frac{1}{3}-2$

$= \frac{2}{3}-2 = -\frac{4}{3}$

Yanıt / The Answer is E

5. $\int_{0}^{2}\left(\frac{2x^3+8x+1}{x^2+4}\right)dx = \int_{0}^{2}\left(2x+\frac{1}{x^2+4}\right)dx$

$= 2\int_0^2 x\,dx + \frac{1}{4}\int_0^2 \frac{dx}{1+\left(\frac{x}{2}\right)^2}$

$2t = x \Rightarrow 2dt = dx$

$= x^2\Big|_0^2 + \frac{1}{2}\int_0^2 \frac{dt}{1+t^2} = x^2\Big|_0^2 + \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big|_0^2$

$= 4 + \frac{1}{2}(\arctan 1 - \arctan 0)$

$= 4 + \frac{1}{2}\cdot\frac{\pi}{4} = \frac{32+\pi}{8}$

Yanıt / The Answer is E

6. $\frac{1}{\sqrt{x}+x} = \frac{1}{\sqrt{x}(\sqrt{x}+1)} = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x}+1}$

$\int_9^{16} \frac{dx}{\sqrt{x}+x} = \int_9^{16} \frac{1}{\sqrt{x}}\,dx - \int_9^{16} \frac{1}{\sqrt{x}+1}\,dx$

$= 2\sqrt{x}\Big|_9^{16} - 2(\sqrt{x}+1) + 2\ln|\sqrt{x}+1|\Big|_9^{16}$

$= 8-6-10+2\ln 5+8-2\ln 4$

$= \ln 25 - \ln 16 = \ln\frac{25}{16}$

Yanıt / The Answer is E

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7. $\int_1^3 x\cdot y\cdot dy = x\int_1^3 y\,dy = x\cdot\frac{y^2}{2}\Big|_1^3 = \frac{x}{2}[9-1] = 9$

$\Rightarrow 4x = 9$

$x = \frac{9}{4}$

Yanıt / The Answer is B

8. $\int_0^{e-1}\frac{x-1}{x+1}dx = \int_0^{e-1}\left(1-\frac{2}{x+1}\right)dx$

$= x - 2\ln|x+1|\Big|_0^{e-1}$

$= e-1-2\ln e-0+2\ln 1$

$= e-3$

Yanıt / The Answer is C

9.

Türev / Derivative	İntegral / Integral
+x	sinx
−1	−cosx
+0	−sinx

$\int_0^{\frac{\pi}{2}} x\cdot\sin x\,dx = -x\cos x+\sin x\Big|_0^{\frac{\pi}{2}}$

$= -\frac{\pi}{2}\cos\frac{\pi}{2}+\sin\frac{\pi}{2}+0\cdot\cos 0-\sin 0 = 1$

Yanıt / The Answer is E

10. $f'(x) = (\cos x)'\,F(\cos x) - 0'\;F(0)$

$= -\sin x\cos x$

$f'(\frac{\pi}{6}) = -\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{4}$

Yanıt / The Answer is A

11.

x		−3		−2		2		3	
	+	+	+	0	−	−	0	+	+

$\int_{-3}^{3}|x^2-4|dx = \int_{-3}^{-2}(x^2-4)dx+\int_{-2}^{2}(4-x^2)dx+\int_2^3(x^2-4)dx$

$= \left(\frac{x^3}{3}-4x\right)\Big|_{-3}^{-2}+\left(4x-\frac{x^3}{3}\right)\Big|_{-2}^{2}+\left(\frac{x^3}{3}-4x\right)\Big|_2^3$

$= \left[\left(-\frac{8}{3}+8\right)-(-9+12)\right]+\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$

$+\left[(9-12)-\left(\frac{8}{3}-8\right)\right]$

$= \frac{16}{3}-3+\frac{16}{3}+\frac{16}{3}-3+\frac{16}{3} = \frac{64}{3}-6 = \frac{46}{3}$

Yanıt / The Answer is A

12. $\int_0^2|x^2-1|\,dx = \int_0^1(-x^2+1)dx+\int_1^2(x^2-1)dx$

$= \left(-\frac{x^3}{3}+x\right)\Big|_0^1+\left(\frac{x^3}{3}-x\right)\Big|_1^2$

$= \left(-\frac{1}{3}+1\right)+\left[\left(\frac{8}{3}-2\right)-\left(\frac{1}{3}-1\right)\right]$

$= \frac{2}{3}+\left[\frac{2}{3}+\frac{2}{3}\right]$

$= \frac{2}{3}+\frac{4}{3} = 2$

Yanıt / The Answer is C

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زنگنه ZTC مجموعه موسسات

YÖS Soruları / YÖS Questions

1. $\int_a^b (4x-3)dx = 68 \,,\, b-a=4 \Rightarrow b+a=?$

A) 10 B) 15 C) 18 D) 20 E) 24

[MARMARA ÜNİV. YÖS 2013]

2. $\int_1^3 \frac{1}{x\ln 2}\,dx = ?$

A) $\frac{\ln 3}{\ln 2}$ B) $2\ln 2$ C) $\frac{1}{\ln 2}$ D) $\ln 4$ E) e^{2x}

[İSTANBUL ÜNİV. YÖS 2013]

3. $\int_0^1 \frac{x^2}{x^3+2}\,dx = ?$

A) 1 B) e C) $\frac{1}{3}\ln\left(\frac{3}{2}\right)$ D) $\ln 3$ E) $\frac{1}{9}\ln 2$

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

4. $\int_1^e \frac{x^2+1}{x}\,dx = ?$

A) 0 B) 1 C) e^2 D) $\frac{e^2+1}{2}$ E) 3

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

5. $\int_0^{\frac{\pi}{2}} \cos x \cdot \sin x\,dx = ?$

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{\pi}{2}$ E) π

[KARADENİZ TEKNİK ÜNİV. YÖS 2013]

6. $\int_{-\pi}^{\pi} \cos(2x)\,d(2x) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

[YILDIZ TEKNİK ÜNİV. YÖS 2013]

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7. $\int_0^{3\pi/2} \sin^2 x \sin 2x\, dx = ?$

A) 1 B) $\frac{3}{2}$ C) $\frac{1}{2}$ D) 0 E) -1

[YILDIZ TEKNİK ÜNİV. YÖS 2012]

8. $\int_1^e \frac{\sqrt{\ln x}}{2x} dx = ?$

A) e B) 1 C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $e-1$

[ANADOLU ÜNİV. YÖS 2011]

9. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{d}{dx} \cos^2 x\, dx = ?$

A) 1 B) $\frac{1}{2}$ C) 0 D) $\frac{1}{3}$ E) $\frac{1}{4}$

[ANKARA ÜNİV. YÖS 2011]

10. $\int_{-\pi}^{\pi} \sin\frac{2x}{3} dx = ?$

A) $\frac{8}{9}$ B) $\frac{2}{9}$ C) -2 D) 0 E) 5

[YILDIZ TEKNİK ÜNİV. YÖS 2011]

11. $\int_{-2}^{4} \frac{d}{dx}\left(\frac{x^2}{2}\right) dx = ?$

A) 2 B) 4 C) 6 D) 8 E) 9

[YÖS 2010]

12. $\int_2^3 \frac{3\,dx}{x^2+x-2} = ?$

A) $\ln\frac{8}{7}$ B) $\ln\frac{8}{5}$ C) $\ln\frac{8}{3}$ D) $\ln\frac{3}{8}$ E) $\ln\frac{5}{8}$

[YÖS 2010]

YANITLAR / ANSWERS							
1. A	2. A	3. C	4. D	5. B	6. C	7. C	8. D
9. C	10. D	11. C	12. B				

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1. $\int_a^b (4x-3)dx = 68$

$$\frac{4x^2}{2} - 3x + C = 68$$

$$2x^2 - 3x + C = 68$$

$$2b^2 - 3b + C - 2a^2 + 3a - C = 68$$

$$2(b^2 - a^2) - 3(b-a) = 68$$

$$(b-a)(2b+2a-3) = 68$$

$$4 \cdot (2b+2a-3) = 68$$

$$2 \cdot (b+a) - 3 = 17$$

$$2 \cdot (b+a) = 20$$

$$b + a = 10$$

Yanıt / The Answer is A

2. $\int_1^3 \frac{1}{x\ln 2}\,dx = \frac{1}{\ln 2}\int_1^3 \frac{1}{x}\,dx = \frac{1}{\ln 2}\ln x\Big|_1^3$

$$= \frac{1}{\ln 2}[\ln 3 - \ln 1]$$

$$= \frac{\ln 3}{\ln 2}$$

Yanıt / The Answer is A

3. $\left.\begin{matrix} x^3 + 2 = u \\ 3x^2 dx = du \end{matrix}\right\} \Rightarrow \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u|$

$$= \frac{1}{3}\ln|x^3 + 2|\Big|_0^1$$

$$= \frac{1}{3}(\ln 3 - \ln 2)$$

$$= \frac{1}{3}\ln\frac{3}{2}$$

Yanıt / The Answer is C

4. $\int_1^e \frac{x^2+1}{x}\,dx = \int_1^e x + \frac{1}{x}\,dx = \left(\frac{x^2}{2} + \ln x\right)\Big|_1^e$

$$= \left(\frac{e^2}{2} + \ln e\right) - \left(\frac{1}{2} + \ln 1\right)$$

$$= \left(\frac{e^2}{2} + 1\right) - \left(\frac{1}{2} + 0\right)$$

$$= \frac{e^2}{2} + \frac{1}{2} = \frac{e^2+1}{2}$$

Yanıt / The Answer is D

5. $\int_0^{\frac{\pi}{2}} \cos x \cdot \sin x\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{2}\,dx = -\frac{\cos 2x}{4}\Big|_0^{\pi/2}$

$$= -\frac{1}{4}(\cos\pi - \cos 0)$$

$$= -\frac{1}{4}(-2)$$

$$= \frac{1}{2}$$

Yanıt / The Answer is B

6. $\int_{-\pi}^{\pi} \cos(2x)\,d(2x) = \int_{-\pi}^{\pi} \cos(2x)\,2\,dx$

$$= \sin 2x\Big|_{-\pi}^{\pi}$$

$$= \sin 2\pi - \sin(-2\pi)$$

$$= 2\sin 2\pi$$

$$= 0$$

Yanıt / The Answer is C

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7. $\sin^2 x = u \Rightarrow \sin 2x\,dx = du$

$x = \frac{3\pi}{2} \Rightarrow u = 1$

$x = 0 \Rightarrow u = 0$

$$\int_0^{3\pi/2} \sin^2 x \sin 2x\,dx = \int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$

Yanıt / The Answer is C

8. $\left.\begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array}\right\} \Rightarrow \int_1^e \frac{\sqrt{\ln x}}{2x}dx = \int \frac{\sqrt{u}}{2}du$

$$= \frac{1}{3}(\ln x)^{\frac{3}{2}}\Big|_1^e$$

$$= \frac{1}{3} - 0 = \frac{1}{3}$$

Yanıt / The Answer is D

9. $$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{d}{dx}\cos^2 x\,dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} -\sin 2x\,dx$$

$$= \frac{\cos 2x}{2}\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{1}{4} - \frac{1}{4} = 0$$

Yanıt / The Answer is C

10. $$\int_{-\pi}^{\pi} \sin\frac{2x}{3}dx = \frac{-3}{2}\cos\frac{2x}{3}\Big|_{-\pi}^{\pi}$$

$$= -\frac{3}{2}\left[\cos\frac{2\pi}{3} - \cos\frac{2\pi}{3}\right]$$

$$= 0$$

Yanıt / The Answer is D

11. $$\int_{-2}^{4} \frac{d}{dx}\left(\frac{x^2}{2}\right)dx = \frac{x^2}{2}\Big|_{-2}^{4}$$

$$= \frac{16}{2} - \frac{4}{2}$$

$$= 6$$

Yanıt / The Answer is C

12. $$\frac{3}{x^2+x-2} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\frac{3}{x^2+x-2} = \frac{Ax - A + Bx + 2B}{x^2+x-2}$$

$\left.\begin{array}{l} A+B=0 \\ 2B-A=3 \end{array}\right\} \Rightarrow \begin{array}{l} A=-1 \\ B=1 \end{array}$

$$\int_2^3 \frac{3\,dx}{x^2+x-2} = \int_2^3 \left(\frac{1}{x-1} - \frac{1}{x+2}\right)dx$$

$$= \ln|x-1|\Big|_2^3 - \ln|x+2|\Big|_2^3$$

$$= \ln 2 - \ln 5 + \ln 4 = \ln\frac{8}{5}$$

Yanıt / The Answer is B

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1. $\int_1^2 5^{-2x+3}\,dx = ?$

Yanıt / The Answer is $\frac{12}{5\ln 5}$

2. $\int_{-1}^{2} (2x+1)(x^2+x+1)\,dx = ?$

Yanıt / The Answer is **24**

3. $\int_1^e \frac{\ln x}{x}\,dx = ?$

Yanıt / The Answer is $\frac{1}{2}$

4. $\int_e^{e^2} \frac{dx}{x(\ln x)^2} = ?$

Yanıt / The Answer is $\frac{1}{2}$

5. $\int_1^2 \frac{2x^3-3x^2+1}{x^2}\,dx = ?$

Yanıt / The Answer is $\frac{1}{2}$

6. $\int_{1-e}^{2} \frac{dx}{x+e} = ?$

Yanıt / The Answer is **ln|2 + e|**

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7. $\int_4^5 \frac{2}{(x-3)^3}dx = ?$

Yanıt / The Answer is $\frac{3}{4}$

8. $\int_{\frac{1}{2}}^3 \frac{1}{x^2}dx = ?$

Yanıt / The Answer is $\frac{5}{3}$

9. $\int_3^4 \frac{e^{\ln x}}{x}dx = ?$

Yanıt / The Answer is **1**

10. $\int_0^1 (x^2+3)x\ dx = ?$

Yanıt / The Answer is $\frac{7}{4}$

11. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x}dx = ?$

Yanıt / The Answer is $\sqrt{2}-1$

12. $\int_0^1 x^2 e^x\ dx = ?$

Yanıt / The Answer is **e – 2**

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13. $\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\sqrt{1-\cos^2\theta}}\, d\theta = ?$

Yanıt / The Answer is $\frac{\pi}{4}$

14. $\int_{-2}^{1} |x|dx = ?$

Yanıt / The Answer is $\frac{5}{2}$

15. $\int_2^4 |x-3|dx = ?$

Yanıt / The Answer is **1**

16. $f(x) = \int_0^x e^t\, dt \Rightarrow f'(\ln 3) = ?$

Yanıt / The Answer is **3**

17. $f(x) = \int_1^x \frac{1}{u}\, du \Rightarrow f'(\frac{1}{3}) = ?$

Yanıt / The Answer is **3**

18. $f(x) = \int_{2x}^{3x+1} t\, dt \Rightarrow f'(1) = ?$

Yanıt / The Answer is **8**

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1. $\int_{-1}^{5} dx = ?$

A) 3
B) 4
C) 5
D) 6
E) 7

2. $\int_{0}^{2} e^{2x}\,dx = ?$

A) $\frac{1}{2}e^2$
B) $\frac{1}{2}e^4$
C) $\frac{1}{2}(e^2-1)$
D) $\frac{1}{2}(e^4-1)$
E) $\frac{e^4}{4}$

3. $\int_{1}^{\sqrt{3}} \frac{1}{x^2+1}\,dx = ?$

A) $\frac{\pi}{3}$
B) $\frac{\pi}{4}$
C) $\frac{\pi}{6}$
D) $\frac{\pi}{8}$
E) $\frac{\pi}{12}$

4. $\int_{-2}^{2} \frac{3(x^4+x^3)}{x^2}\,dx = ?$

A) 12
B) 14
C) 16
D) 18
E) 20

5. $\int_{1}^{e} \frac{x+1}{x^2}\,dx = ?$

A) $\frac{2e-1}{e}$
B) $\frac{e-1}{e}$
C) $\frac{e+1}{e}$
D) $\frac{2e+1}{e}$
E) $\frac{2e+1}{2}$

6. $\int_{0}^{2} e^{x^2}\,2x\,dx = ?$

A) $e - 1$
B) $e^2 - 1$
C) $e^3 - 1$
D) $e^4 - 1$
E) $e^5 - 1$

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7. $\int_{e^4}^{e^7} d(\ln x) = ?$

A) 1
B) 2
C) 3
D) 4
E) 7

8. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \tan x \cdot \sec^2 x \, dx = ?$

A) $\frac{1}{6}$
B) $\frac{1}{4}$
C) $\frac{1}{3}$
D) $\frac{1}{2}$
E) 1

9. $\int_{e^2}^{e^4} \ln x \, dx = ?$

A) $e^2(3e^2 - 1)$
B) $e^2(2e^2 - 1)$
C) $e^2(e^2 - 1)$
D) $e^2(2e^2 + 1)$
E) $e^2(3e^2 + 1)$

10. $f(x) = \begin{cases} x^2 - 1 & , x < -1 \\ x + 1 & , -1 \le x < 0 \end{cases} \Rightarrow \int_{-2}^{0} f(x)\,dx = ?$

A) $\frac{7}{6}$
B) $\frac{11}{6}$
C) $\frac{13}{6}$
D) $\frac{8}{3}$
E) $\frac{17}{6}$

11. $\int_{-2}^{2} |x^2 - 1| \, dx = ?$

A) 0
B) 2
C) 4
D) 6
E) 8

12. $f(x) = \int_{4}^{3x} \sqrt{5t+1}\, dt \Rightarrow f'(1) = ?$

A) 4
B) 8
C) 12
D) 14
E) 16

YANITLAR / ANSWERS

1. D	2. D	3. E	4. C	5. A	6. D	7. C	8. C
9. A	10. B	11. C	12. C				

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1. $\int_1^2 (3x+4)\,dx = ?$

A) $\frac{9}{2}$

B) $\frac{11}{2}$

C) $\frac{13}{2}$

D) $\frac{15}{2}$

E) $\frac{17}{2}$

2. $\int_{\pi}^{2\pi} \sin x\,dx = ?$

A) –2

B) –1

C) 0

D) 1

E) 2

3. $\int_1^3 \frac{1}{x^2}\,dx = ?$

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) 1

D) $\frac{4}{3}$

E) $\frac{5}{3}$

4. $\int_2^6 \frac{1}{x+2}\,dx = ?$

A) 2

B) 4

C) 6

D) ln2

E) ln4

5. $\int_0^{\pi/2} 2\sin 2x\,dx = ?$

A) –4

B) –2

C) 0

D) 2

E) 4

6. $\int_0^1 (2x^2+1)^2 4x\,dx = ?$

A) $\frac{16}{3}$

B) $\frac{20}{3}$

C) $\frac{22}{3}$

D) $\frac{26}{3}$

E) $\frac{32}{3}$

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7. $\int_0^1 2\cdot 3^{2x}\,dx = ?$

A) $\frac{2}{\ln 3}$

B) $\frac{4}{\ln 3}$

C) $\frac{6}{\ln 3}$

D) $\frac{8}{\ln 3}$

E) $\frac{10}{\ln 3}$

8. $\int_1^e \frac{\ln^2 x - \ln x}{x}\,dx = ?$

A) $-\frac{1}{6}$

B) $-\frac{1}{3}$

C) $-\frac{e}{6}$

D) $-\frac{e}{3}$

E) $-\frac{e^2}{6}$

9. $\int_0^4 \frac{3x}{x^2+1}\,dx = ?$

A) $\frac{1}{2}\ln 17$

B) $\ln 17$

C) $\frac{3}{2}\ln 17$

D) $\frac{1}{2}\ln 16$

E) $\frac{3}{2}\ln 16$

10. $\int_1^{\sqrt{3}} d(\arctan x) = ?$

A) 15° B) 30° C) 45°

D) 60° E) 75°

11. $\int_0^1 \sqrt{3x+1}\,dx = ?$

A) $\frac{7}{9}$ B) $\frac{8}{9}$ C) $\frac{10}{9}$

D) $\frac{13}{9}$ E) $\frac{14}{9}$

12. $\int_{-2}^1 (3x^2 + a)\,dx = 21 \Rightarrow a = ?$

A) 3 B) 4 C) 5

D) 6 E) 8

YANITLAR / ANSWERS

1. E	2. A	3. B	4. D	5. D	6. D	7. D	8. A
9. C	10. A	11. E	12. B				

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1. $\int_0^1 x^2 (x^3+2)^2 \, dx = ?$

A) 1

B) $\frac{4}{3}$

C) $\frac{19}{9}$

D) 2

E) $\frac{7}{3}$

2. $\int_0^a (4x-5)\,dx = 12 \Rightarrow a = ?$

A) 0

B) 1

C) 2

D) 3

E) 4

3. $\int_2^4 xy\,dy = 12 \Rightarrow x = ?$

A) 2

B) 3

C) 4

D) 5

E) 6

4. $\int_1^2 \frac{2}{x^2+2x} \, dx = ?$

A) $\ln \frac{1}{4}$

B) $\ln \frac{1}{2}$

C) $\ln \frac{3}{2}$

D) $\ln \frac{5}{2}$

E) $\ln 3$

5. $\int_{-2}^0 x\sqrt{2x^2+1} \, dx = ?$

A) -10

B) -5

C) $-\frac{13}{3}$

D) $-\frac{7}{2}$

E) $\frac{14}{3}$

6. $\int_1^3 (x+1)e^{x^2+2x} \, dx = ?$

A) $\frac{e^3}{2}(e^{12}-1)$

B) $\frac{e^2}{3}(e^9-1)$

C) $\frac{e^{12}}{3}-1$

D) $e^{15}-15$

E) $\frac{1}{2}e^3(e^9+1)$

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7. $\int_0^2 x^2 e^{x^3}\,dx = ?$

A) $\frac{1}{3}\left(e^8-1\right)$

B) $\frac{e}{3}\left(e^6-1\right)$

C) $\frac{1}{3}e^9-1$

D) e^6-1

E) $\frac{1}{6}\left(e^6-1\right)$

8. $\int_0^1 \arcsin x\,dx = ?$

A) $\frac{\pi+1}{2}$

B) $\frac{\pi-2}{2}$

C) $\frac{\pi+2}{2}$

D) $\frac{\pi-1}{3}$

E) $\frac{\pi-4}{4}$

9. $\int_0^4 |x-2|\,dx = ?$

A) –2

B)–4

C) 1

D) 2

E) 4

10. $\int_{-3}^{3} (x+|x|)\,dx = ?$

A) 1

B) 3

C) 6

D) 9

E) 12

11. $\int_0^{\frac{1}{\sqrt{6}}} x\cdot\cos(\pi x^2)\,dx = ?$

A) $\frac{1}{2\pi}$

B) $\frac{1}{4}$

C) $\frac{1}{4\pi}$

D) $\frac{1}{6}$

E) $\frac{1}{8}$

12. $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{1-\cos 2x}\,dx = ?$

A) $-\frac{\sqrt{3}}{4}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\sqrt{3}$ D) $\sqrt{2}$ E) $\frac{\sqrt{3}}{4}$

YANITLAR / ANSWERS							
1. C	2. E	3. A	4. C	5. C	6. A	7. A	8. B
9. E	10. D	11. C	12. D				

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1. $\int_1^{e^2} \frac{\ln x}{x}\,dx = ?$

A) -4

B) -2

C) 2

D) 4

E) 6

2. $\int_0^1 \frac{4x^2-2x+1}{x+1}\,dx = ?$

A) $7 + 7\ln 2$

B) $7\ln 2 + 4$

C) $4 - 7\ln 2$

D) $7\ln 2 - 4$

E) $7\ln 2 - 2$

3. $\int_0^{\ln 3} \frac{e^x}{e^x+1}\,dx = ?$

A) $\ln 2$

B) $\ln 4$

C) $\ln \frac{1}{2}$

D) $\ln \frac{9}{4}$

E) $\ln 8$

4. $\int_1^{\sqrt{3}} \frac{2x+1}{x^2+1}\,dx = ?$

A) $\ln 4 + \frac{\pi}{6}$

B) $\ln 2 + \frac{\pi}{2}$

C) $\ln 4 + \frac{\pi}{12}$

D) $\ln 2 + \frac{\pi}{12}$

E) $\ln 2 + \frac{\pi}{3}$

5. $\int_1^2 x \ln x\,dx = ?$

A) $\ln 2 - 1$

B) $\frac{5}{4}$

C) $\ln 4 - 3$

D) $\ln 4 - \frac{3}{4}$

E) $\ln 8 - \frac{1}{2}$

6. $\int_0^{\frac{\pi}{4}} \sin x \cdot \sin 2x\,dx = ?$

A) $\sqrt{2}$

B) $\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{2}}{3}$

D) $\frac{\sqrt{2}}{4}$

E) $\frac{\sqrt{2}}{6}$

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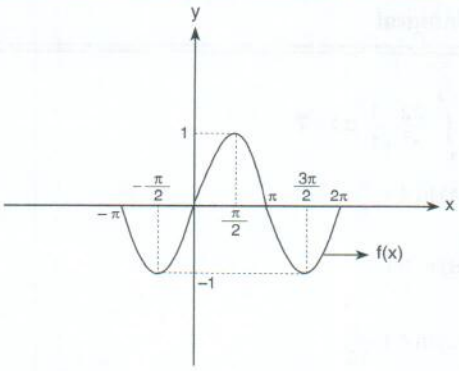
7.



$f(x)=\sin x \Rightarrow \int_{\pi/2}^{\pi} \sin x\,dx = ?$

A) 6 B) 4 C) 3

D) 2 E) 1

8. $\int_0^1 \frac{3x^3}{x^2+3}\,dx = ?$

A) $\frac{3}{2}\ln\frac{e}{4}$

B) $\frac{3}{2}\cdot\ln\frac{27e}{64}$

C) $\frac{3}{2}\ln\frac{e}{4}$

D) $\frac{3}{2}\ln\frac{3}{4}$

E) $\frac{3}{2}\ln\frac{1}{4}$

9. $f^{-1}(x)=\frac{3x-1}{x+4} \Rightarrow \int_{-1}^{1} d(f(x)) = ?$

A) $\frac{13}{4}$

B) $\frac{15}{4}$

C) $\frac{17}{4}$

D) $\frac{19}{4}$

E) $\frac{21}{4}$

10. $\int_1^2 x^2\cdot\ln x\,dx = ?$

A) $\frac{4}{3}\ln 2+\frac{7}{9}$

B) $\frac{8}{3}\ln 2-\frac{7}{9}$

C) $\frac{8}{3}\ln 2+\frac{7}{9}$

D) $\frac{4}{3}\ln 2$

E) $\frac{8}{3}\ln 2$

11. $f(x)=\int_3^{2x}(u^3+3^{\cos u})\,du \Rightarrow f'(0)=?$

A) 2

B) 4

C) 6

D) 8

E) 12

12. $f(x)=\int_2^{x^2} t\cdot\cos t\,dt \Rightarrow f'(x)=?$

A) $\cos x^2$ B) $2x^3\cdot\cos x^2$ C) $x^3\cdot\cos x^2$

D) $3x^3\cdot\cos x^2$ E) $4x^3\cdot\cos x^2$

YANITLAR / ANSWERS							
1. C	2. D	3. A	4. D	5. D	6. E	7. E	8. B
9. A	10. B	11. C	12. B				

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BÖLÜM 3
CHAPTER 3

TARİHSEL NOT / HISTORICAL NOTE

De Morgan, Augustus [1806 – 1871]

İngiliz matematikçi ve mantıkçı. Cebirin sembolik dille anlatılmasına katkıda bulunmuş, sembolik mantığın başlamasını sağlamıştır. Kendi ismiyle anılan De Morgan kuralını formülleştirmiştir. 1838'deki makalede, tümevarım gösterimleri ile ilgili bilgi vermiştir.

British mathematician and logician who was responsible for developing a more symbolic approach to algebra, and who played a considerable role in the beginnings of symbolic logic. His name is remembered in De Morgan's laws, which he formulated. In an article of 1838, he clarified the notion of mathematical induction.

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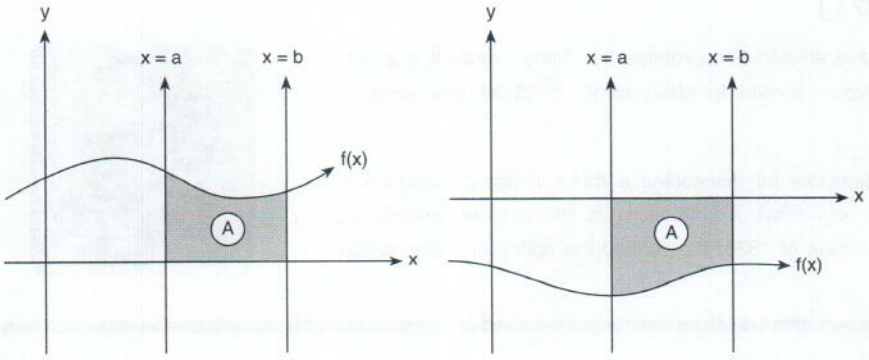
APPLICATIONS of INTEGRAL

BÖLÜM 3
CHAPTER

Alan Hesabı

1. $y = f(x)$ fonksiyonu, x – ekseni, $x = a$, $x = b$ doğruları tarafından sınırlandırılan bölgenin alanı, aşağıdaki gibi hesaplanır.



$$A = \int_a^b f(x)\,dx \qquad A = -\int_a^b f(x)\,dx$$

ÖRNEK 1

Taralı alan = ? br²

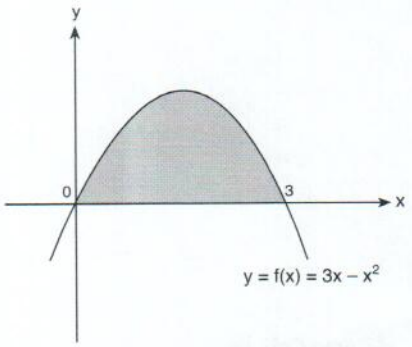
A) $\frac{27}{2}$

B) 9

C) $\frac{27}{4}$

D) $\frac{27}{5}$

E) $\frac{9}{2}$



ÇÖZÜM

Yanıt **E**

$$TA = \int_0^3 (3x - x^2)\,dx = \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)\Big|_0^3 = \left(\frac{27}{2} - \frac{27}{3}\right) - 0 = \frac{27}{6} = \frac{9}{2}$$

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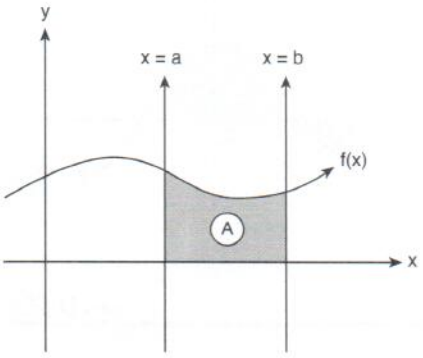
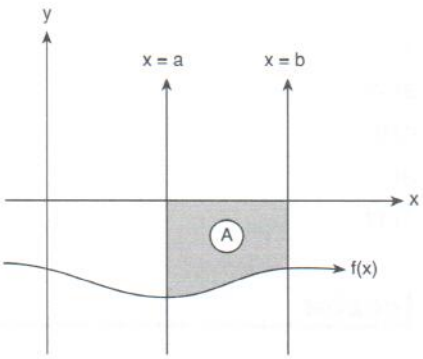
Area Calculus

1. The area of the region bounded by the function y = f(x), the lines x = a, x = b and the x – axis is founded as follows.



$$A = \int_a^b f(x)\,dx$$



$$A = -\int_a^b f(x)\,dx$$

EXAMPLE 1

The shaded area = ? u^2

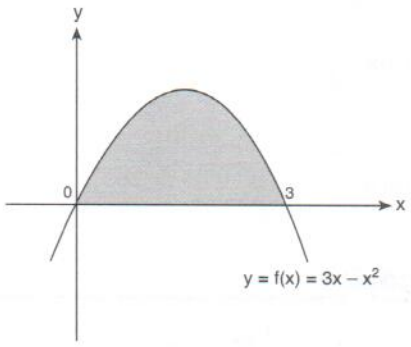
A) $\frac{27}{2}$

B) 9

C) $\frac{27}{4}$

D) $\frac{27}{5}$

E) $\frac{9}{2}$



SOLUTION

The answer is E

$$SA = \int_0^3 (3x - x^2)\,dx = \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)\Big|_0^3 = \left(\frac{27}{2} - \frac{27}{3}\right) - 0 = \frac{27}{6} = \frac{9}{2}$$

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ÖRNEK 2

$A_1 = 5\ br^2$, $A_2 = 12\ br^2$

$\Rightarrow \int_{-5}^{2} f(x)\,dx = ?$

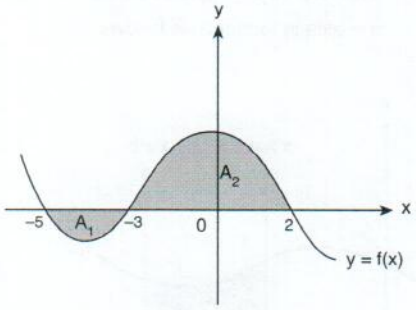
A) –17

B) –7

C) 0

D) 7

E) 17



ÇÖZÜM

Yanıt **D**

$$A_1 = -\int_{-5}^{-3} f(x)\,dx = 5 \Rightarrow \int_{-5}^{-3} f(x)\,dx = -5 \quad , \quad A_2 = \int_{-3}^{2} f(x)\,dx = 12$$

$$\Rightarrow \int_{-5}^{2} f(x)\,dx = \int_{-5}^{-3} f(x)\,dx + \int_{-3}^{2} f(x)\,dx = -5 + 12 = 7$$

ÖRNEK 3

S = ? br^2

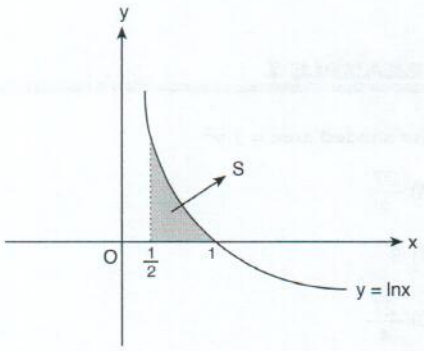
A) $\frac{1}{2}\ln 2 - \frac{1}{2}$

B) $\frac{1}{2}\ln 2 + \frac{1}{2}$

C) $\ln 2 - \frac{1}{2}$

D) $\ln 2 + \frac{1}{2}$

E) $\ln 2$



ÇÖZÜM

Yanıt **A**

$$\left.\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x}dx \\ dv = dx \Rightarrow v = x \end{array}\right\} \Rightarrow \int x\ln x = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - x + C$$

$$S = \int_{\frac{1}{2}}^{1} \ln x\,dx = x\ln x - x\Big|_{\frac{1}{2}}^{1} = 1\cdot\ln 1 - 1 - \frac{1}{2}\ln\frac{1}{2} + \frac{1}{2} = -\frac{1}{2} - \frac{1}{2}\ln 2$$

$$= -\frac{1}{2} - \frac{1}{2}[\ln 1 - \ln 2] = \frac{1}{2}\ln 2 - \frac{1}{2}$$

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EXAMPLE 2

$A_1 = 5\ u^2$, $A_2 = 12\ u^2$

$\Rightarrow \int_{-5}^{2} f(x)\,dx = ?$

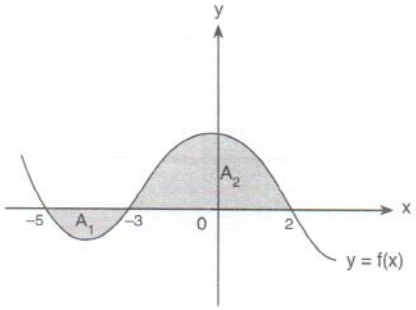
A) –17

B) –7

C) 0

D) 7

E) 17



SOLUTION

The answer is D

$$A_1 = -\int_{-5}^{-3} f(x)\,dx = 5 \Rightarrow \int_{-5}^{-3} f(x)\,dx = -5 \quad , \quad A_2 = \int_{-3}^{2} f(x)\,dx = 12$$

$$\Rightarrow \int_{-5}^{2} f(x)\,dx = \int_{-5}^{-3} f(x)\,dx + \int_{-3}^{2} f(x)\,dx = -5 + 12 = 7$$

EXAMPLE 3

$S = ?\ u^2$

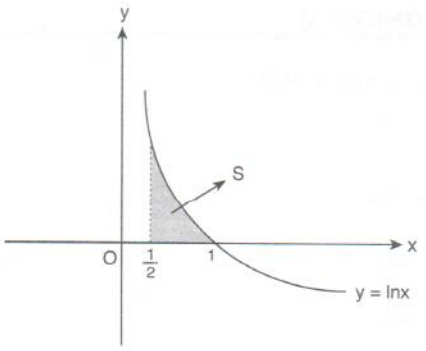
A) $\frac{1}{2}\ln 2 - \frac{1}{2}$

B) $\frac{1}{2}\ln 2 + \frac{1}{2}$

C) $\ln 2 - \frac{1}{2}$

D) $\ln 2 + \frac{1}{2}$

E) $\ln 2$



SOLUTION

The answer is A

$$\left.\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x}dx \\ dv = dx \Rightarrow v = x \end{array}\right\} \Rightarrow \int x\ln x = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - x + C$$

$$S = \int_{\frac{1}{2}}^{1} \ln x\,dx = x\ln x - x\Big|_{\frac{1}{2}}^{1} = 1\cdot\ln 1 - 1 - \frac{1}{2}\ln\frac{1}{2} + \frac{1}{2} = -\frac{1}{2} - \frac{1}{2}\ln 2$$

$$= -\frac{1}{2} - \frac{1}{2}[\ln 1 - \ln 2] = \frac{1}{2}\ln 2 - \frac{1}{2}$$

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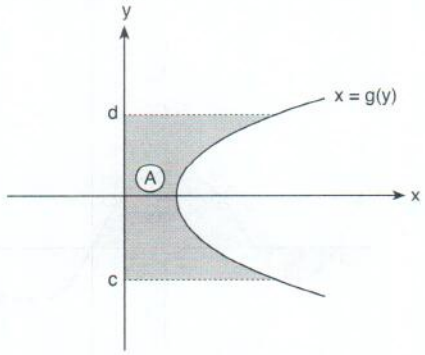
2. y ekseninin sağı, x = g(y) eğrisi ve y = c ile y = d doğruları arasında kalan bölgenin alanı:



$$A = \int_c^d g(y)\,dy$$

ÖRNEK 4

Taralı alan = ? br^2

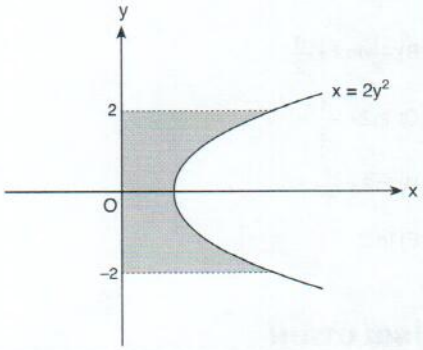
A) $\frac{16}{3}$

B) $\frac{32}{3}$

C) 16

D) $\frac{64}{3}$

E) $\frac{80}{3}$



ÇÖZÜM

Yanıt **B**

$$\text{Taralı alan} = \int_{-2}^{2} 2y^2\,dy = \frac{2}{3}y^3\Big|_{-2}^{2}$$

$$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

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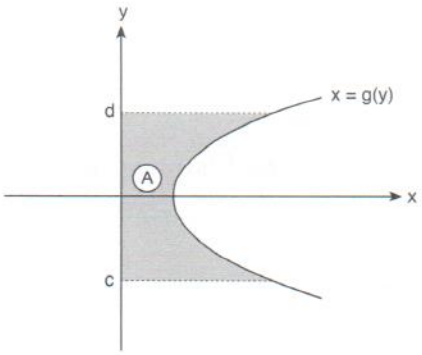
2. A region bounded on the left by the y-axis, on the right by a curve x = g(y), and lying between y = c and y = d.



$$A = \int_c^d g(y)\,dy$$

EXAMPLE 4

The shaded area = ? u^2

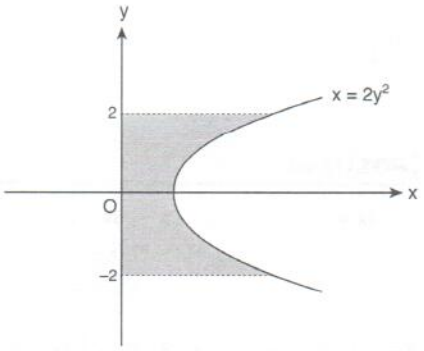
A) $\frac{16}{3}$

B) $\frac{32}{3}$

C) 16

D) $\frac{64}{3}$

E) $\frac{80}{3}$



SOLUTION

The answer is B

$$\text{The shaded area} = \int_{-2}^{2} 2y^2\,dy = \frac{2}{3}y^3\Big|_{-2}^{2}$$

$$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

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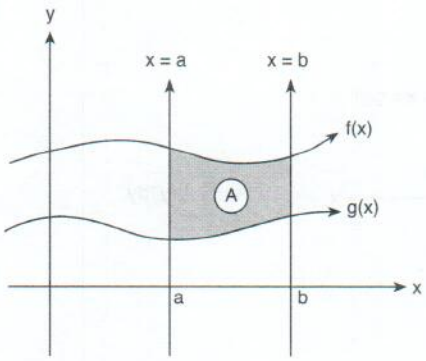
3. $y = f(x)$, $y = g(x)$ fonksiyonları ile $x = a$, $x = b$ doğruları tarafından sınırlanan bölgenin alanı aşağıdaki gibi bulunur.



$$A = \int_a^b |f(x) - g(x)|\,dx$$

ÖRNEK 5

Taralı alan = ? br²

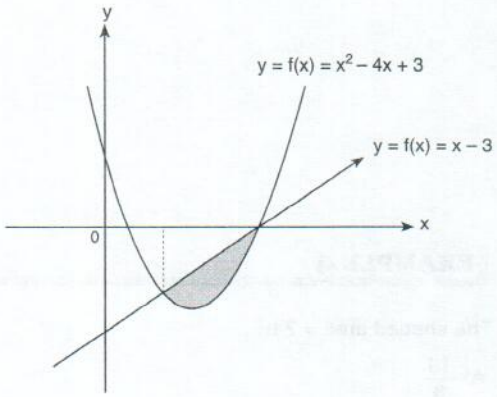
A) $\frac{1}{6}$

B) $\frac{2}{3}$

C) 1

D) $\frac{4}{3}$

E) $\frac{5}{3}$



SOLUTION

The answer is **A**

$x^2 - 4x + 3 = x - 3 \Rightarrow x^2 - 5x + 6 = 0$

$(x - 2)\cdot(x - 3) = 0$

$x = 2,\ x = 3$

$$TA = \int_2^3 \left|x^2 - 4x + 3 - (x - 3)\right| dx = \int_2^3 |x^2 - 5x + 6|\,dx$$

$$= \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right)\Bigg|_2^3$$

$$= \left|\left(9 - \frac{45}{2} + 18\right) - \left(\frac{8}{3} - 10 + 12\right)\right|$$

$$= \left|\frac{9}{2} - \frac{14}{3}\right| = \frac{1}{6}$$

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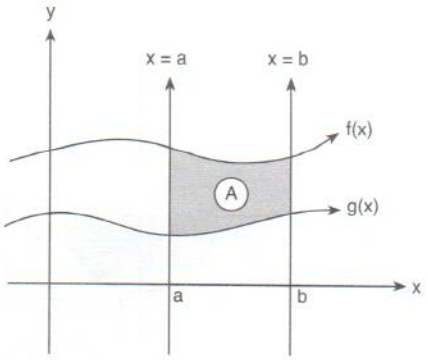
3. The area of the region bounded by the functions y = f(x), y = g(x), the lines x = a and x = b are founded as follow.



$$A = \int_a^b |f(x) - g(x)|\, dx$$

EXAMPLE 5

The shaded area = ? u^2

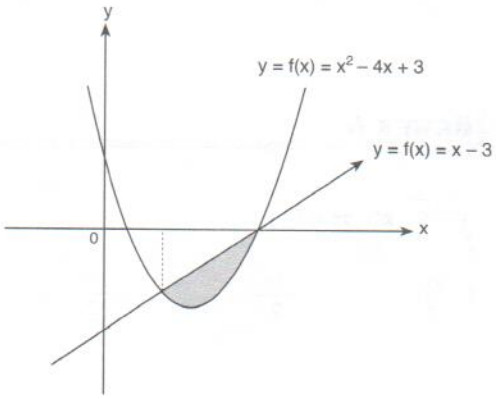
A) $\frac{1}{6}$

B) $\frac{2}{3}$

C) 1

D) $\frac{4}{3}$

E) $\frac{5}{3}$



SOLUTION

The answer is A

$x^2 - 4x + 3 = x - 3 \Rightarrow x^2 - 5x + 6 = 0$

$(x - 2)\cdot(x - 3) = 0$

$x = 2,\ x = 3$

$$SA = \int_2^3 \left|x^2 - 4x + 3 - (x - 3)\right| dx = \int_2^3 |x^2 - 5x + 6|\, dx$$

$$= \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right)\Bigg|_2^3$$

$$= \left|\left(9 - \frac{45}{2} + 18\right) - \left(\frac{8}{3} - 10 + 12\right)\right|$$

$$= \left|\frac{9}{2} - \frac{14}{3}\right| = \frac{1}{6}$$

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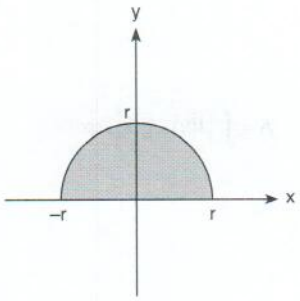
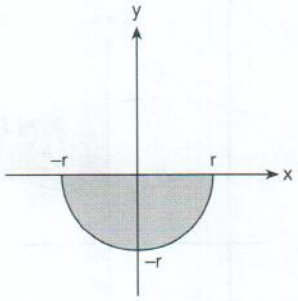
Alan Yardımıyla İntegral Bulma

$x^2 + y^2 = r^2 \Rightarrow y = \pm\sqrt{r^2 - x^2}$



$y = \sqrt{r^2 - x^2}$



$y = -\sqrt{r^2 - x^2}$

$$\int_{-r}^{r} f(x) = \frac{\pi r^2}{2}$$

NOTLAR

İfade	Yöntem
✓ $\sqrt{a^2 - x^2}$	$x = a\sin t$
✓ $\sqrt{x^2 - a^2}$	$x = a\sec t$
✓ $\sqrt{x^2 + a^2}$	$x = a\tan t$

ÖRNEK 6

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx = ?$$

A) $\frac{9\pi}{2}$ B) $\frac{7\pi}{2}$ C) $\frac{5\pi}{2}$ D) $\frac{3\pi}{2}$ E) $\frac{\pi}{2}$

ÇÖZÜM

Yanıt **A**

$\sqrt{9 - x^2} = y$

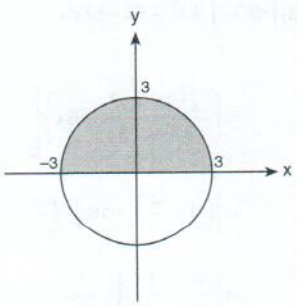
$9 - x^2 = y^2$ [Merkezi (0,0) yarıçapı 3 br olan çemberin denklemi]

$x^2 + y^2 = 9$

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{\pi \cdot r^2}{2}$$

$$= \frac{\pi \cdot 3^2}{2}$$

$$= \frac{9\pi}{2}$$



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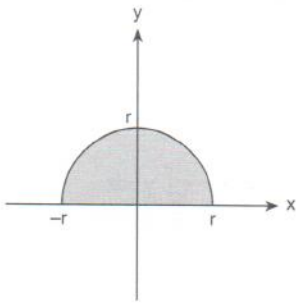
NOTES

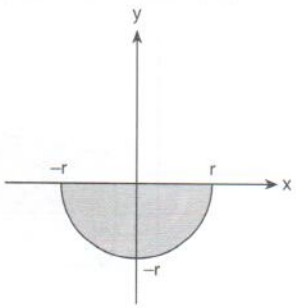
Expression	Method
✓ $\sqrt{a^2-x^2}$	$x = a\sin t$
✓ $\sqrt{x^2-a^2}$	$x = a\sec t$
✓ $\sqrt{x^2+a^2}$	$x = a\tan t$

Finding Integral with Area

$x^2 + y^2 = r^2 \Rightarrow y = \pm\sqrt{r^2 - x^2}$



$y = \sqrt{r^2 - x^2}$



$y = -\sqrt{r^2 - x^2}$

$$\int_{-r}^{r} f(x) = \frac{\pi r^2}{2}$$

EXAMPLE 6

$$\int_{-3}^{3} \sqrt{9-x^2}\, dx = ?$$

A) $\frac{9\pi}{2}$ B) $\frac{7\pi}{2}$ C) $\frac{5\pi}{2}$ D) $\frac{3\pi}{2}$ E) $\frac{\pi}{2}$

SOLUTION

The answer is **A**

$\sqrt{9-x^2} = y$

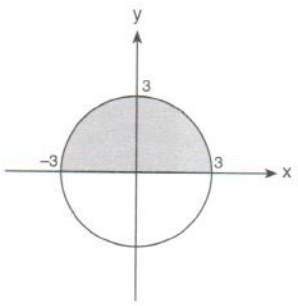
$9 - x^2 = y^2$ [Equation of circle that centre is (0,0) and radius is 3]

$x^2 + y^2 = 9$

$$\int_{-3}^{3} \sqrt{9-x^2}\, dx = \frac{\pi \cdot r^2}{2}$$

$$= \frac{\pi \cdot 3^2}{2}$$

$$= \frac{9\pi}{2}$$



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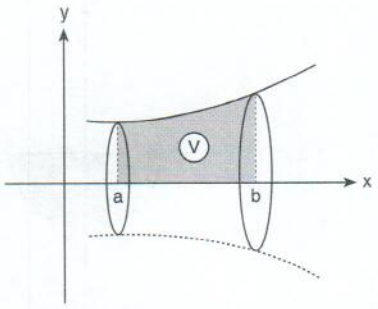
Hacim Hesabı

y = f(x) fonksiyonu, x ekseni ve x = a, x = b doğruları arasında kalan bölgenin x – ekseni etrafında döndürülmesiyle oluşan cismin hacmi aşağıdaki gibi bulunur.



$$\Rightarrow V = \pi \int_a^b y^2\,dx$$

ÖRNEK 7

$V = ?\pi$ br^3

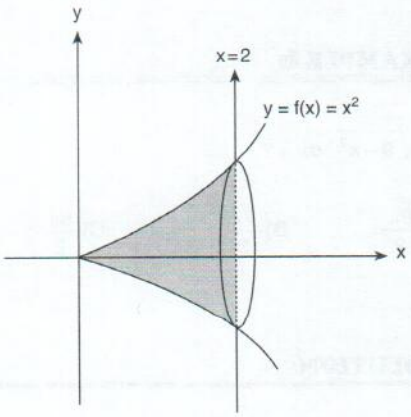
A) 16

B) $\frac{32}{3}$

C) 8

D) $\frac{32}{5}$

E) 4



ÇÖZÜM

Yanıt **D**

$$V = \pi \int_0^2 [f(x)]^2\,dx$$

$$= \pi \int_0^2 (x^2)^2\,dx$$

$$= \pi \int_0^2 x^4\,dx$$

$$= \pi \cdot \frac{x^5}{5}\Big|_0^2 = \frac{32}{5}\pi$$

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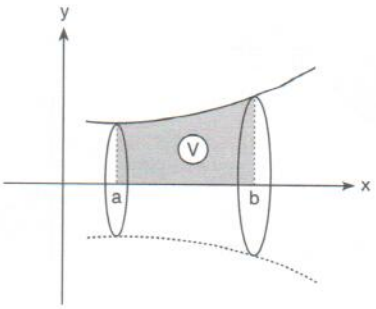
Volume Calculus

The volume of the solid rotating around the x – axis is founded as follow that A region bounded by the function y = f(x), the lines x = a, x = b and x – axis.



$$\Rightarrow V = \pi \int_a^b y^2 \, dx$$

EXAMPLE 7

V = ? π u³

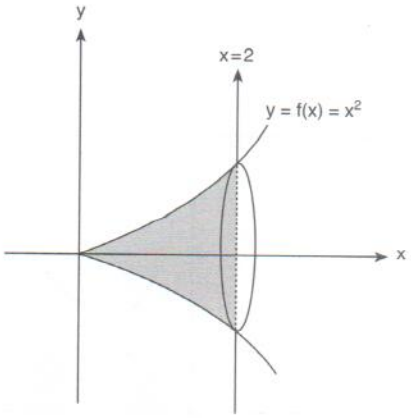
A) 16

B) $\frac{32}{3}$

C) 8

D) $\frac{32}{5}$

E) 4



SOLUTION

The answer is D

$$V = \pi \int_0^2 [f(x)]^2 \, dx$$

$$= \pi \int_0^2 (x^2)^2 \, dx$$

$$= \pi \int_0^2 x^4 \, dx$$

$$= \pi \cdot \frac{x^5}{5} \Big|_0^2 = \frac{32}{5}\pi$$

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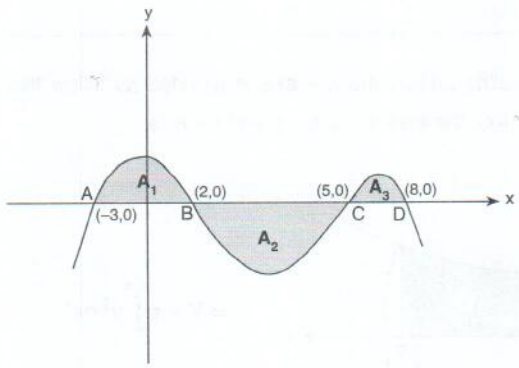
1.



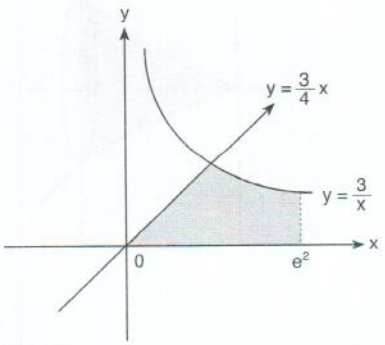
$A_1 = 12\ br^2(u^2)$, $A_2 = 18\ br^2(u^2)$, $A_3 = 9\ br^2(u^2)$

$\Rightarrow \int_{-3}^{8} f(x)\,dx = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

2.



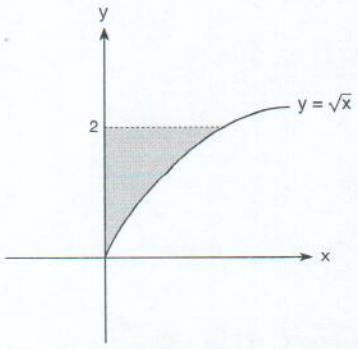
Taralı alan / The shaded area = ? $br^2(u^2)$

A) 4 – ln8 B) 5 – ln8

C) $\frac{15}{2}$ – ln8 D) 6 – ln8

E) 4 – ln8

3.

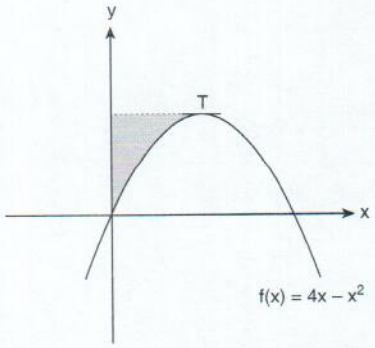


Taralı alan / The shaded area = ? $br^2(u^2)$

A) $\frac{4}{3}$ B) 2 C) $\frac{8}{3}$

D) 3 E) $\frac{10}{3}$

4.



Taralı alan / The shaded area = ? $br^2(u^2)$

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) $\frac{8}{3}$

D) $\frac{10}{3}$ E) $\frac{16}{3}$

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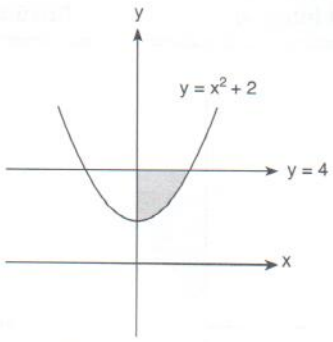
5.

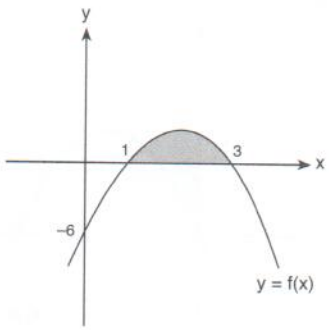


Taralı alan / The shaded area = ? $br^2(u^2)$

A) $\frac{1}{3}\sqrt{2}$ B) $\frac{2}{3}\sqrt{2}$ C) $\sqrt{2}$

D) $\frac{4}{3}\sqrt{2}$ E) $\frac{5}{2}\sqrt{3}$

6.

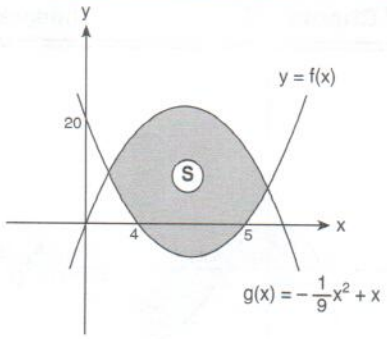


Taralı alan / The shaded area = ? $br^2(u^2)$

A) 1 B) $\frac{3}{2}$ C) $\frac{4}{3}$

D) 2 E) $\frac{8}{3}$

7.

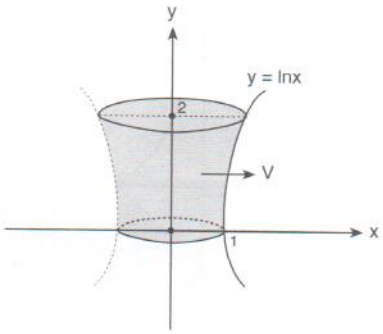


S = ? $br^2(u^2)$

A) $\frac{3580}{27}$ B) $\frac{3581}{27}$ C) $\frac{3583}{27}$

D) $\frac{3587}{27}$ E) $\frac{3592}{27}$

8.



V = ? br^3 (u^3)

A) $\frac{\pi}{2}(e^2 - 1)$ B) $\frac{\pi}{2}(e^3 - 1)$ C) $\frac{\pi}{2}(e^4 - 1)$

D) $\pi(e^2 - 1)$ E) $\pi(e^4 - 1)$

YANITLAR / ANSWERS							
1. C	2. C	3. C	4. C	5. D	6. E	7. C	8. C

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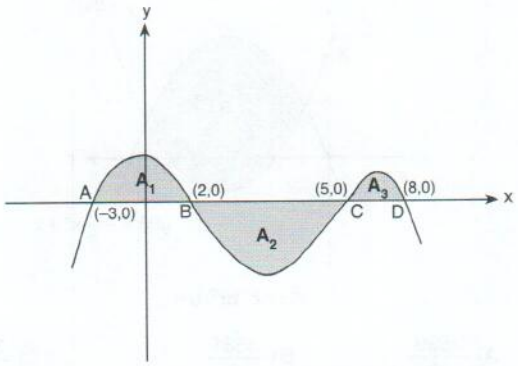
1.

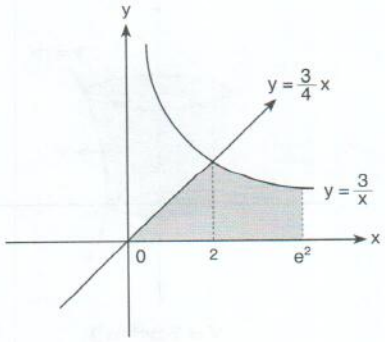


$$\int_{-3}^{8} f(x)\,dx = \int_{-3}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx + \int_{5}^{8} f(x)\,dx$$

$$= 12 - 18 + 9 = 3 \text{ br}^2(u^2)$$

Yanıt / The Answer is C

2.



$$\frac{3}{4}x = \frac{3}{x} \Rightarrow x^2 = 4$$

$$x = 2$$

$$TA/SA = \int_0^2 \frac{3x}{4}\,dx + \int_2^{e^2} \frac{3}{x}\,dx$$

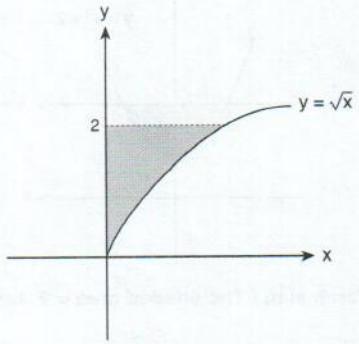
$$= \frac{3x^2}{8}\Big|_0^2 + 3\ln|x|\Big|_2^{e^2}$$

$$= \frac{3}{2} + 3\cdot 2 - 3\ln 2$$

$$= \frac{15}{2} - \ln 8 \text{ br}^2(u^2)$$

Yanıt / The Answer is C

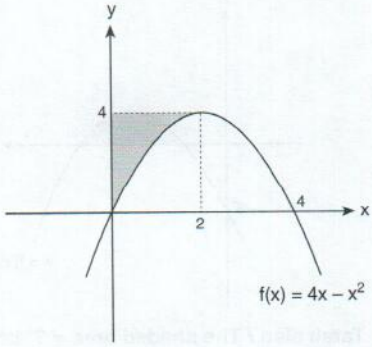
3.



$$TA/SA = \int_0^2 y^2\,dy = \frac{y^3}{3}\Big|_0^2 = \frac{8}{3} \text{ br}^2(u^2)$$

Yanıt / The Answer is C

4.



$$4x - x^2 = 0 \Rightarrow x_1 = 0\,,\ x_2 = 4$$

$$r = 2 \Rightarrow k = 4\cdot 2 - 2^2 = 4$$

$$TA/SA = 4\cdot 2 - \int_0^2 (4x - x^2)\,dx$$

$$= 8 - \left(2x^2 - \frac{x^3}{3}\right)\Big|_0^2$$

$$= 8 - 8 + \frac{8}{3} = \frac{8}{3} \text{ br}^2(u^2)$$

Yanıt / The Answer is C

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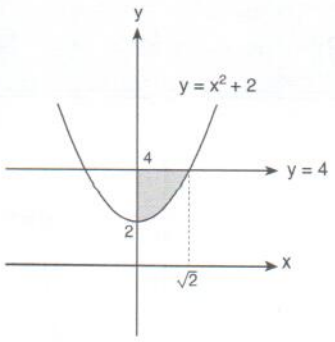
5.



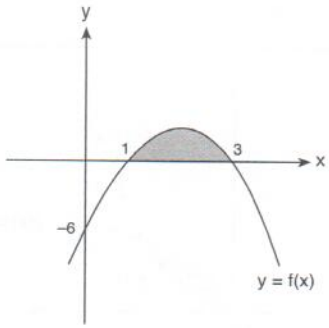
$$TA/SA = \int_0^{\sqrt{2}} (4-(x^2+2))dx$$

$$= 2x - \frac{x^3}{3}\Big|_0^{\sqrt{2}}$$

$$= 2\sqrt{2} - \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}\ br^2(u^2)$$

Yanıt / The Answer is D

6.



$$f(x) = -2(x-1)(x-3)$$

$$= -2x^2 + 8x - 6$$

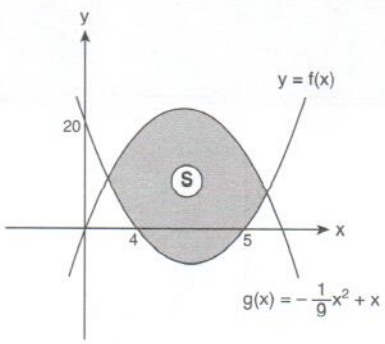
$$TA/SA = \int_1^3 (-2x^2+8x-6)\,dx = -\frac{2}{3}x^3 + 4x^2 - 6x\Big|_1^3$$

$$= -18 + 36 - 18 + \frac{2}{3} - 4 + 6$$

$$= \frac{8}{3}\ br^2(u^2)$$

Yanıt / The Answer is E

7.



$$S = \int_3^6 \left(-\frac{1}{9}x^2 + x - x^2 + 9x - 20\right)dx$$

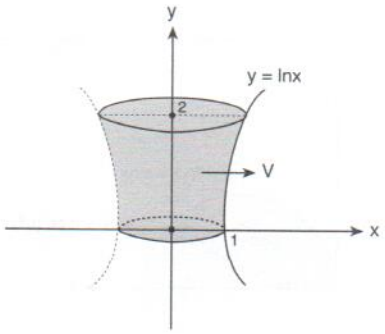
$$= \left(-\frac{1}{27}x^3 + 5x^2 - \frac{x^3}{3} + 20x\right)\Big|_3^6$$

$$= -\frac{8}{27} + 180 - 72 + 120 + 1 - 45 + 9 - 60$$

$$= \frac{3583}{27}\ br^2(u^2)$$

Yanıt / The Answer is C

8.



$$V = \int_0^2 \pi e^{2y}dy = \pi\frac{e^{2y}}{2}\Big|_0^2$$

$$= \frac{\pi}{2}(e^4 - 1)$$

Yanıt / The Answer is C

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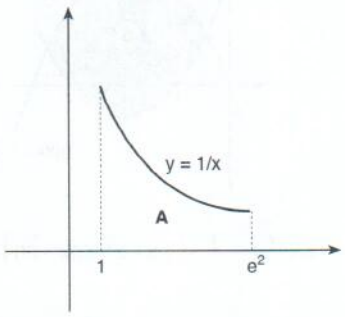
1.



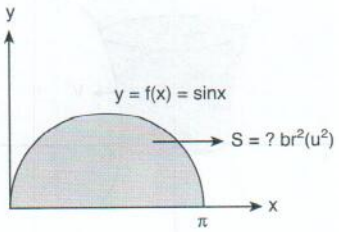
$y = \frac{1}{x}$, $x = 1$, $x = e^2$ $\Rightarrow$ **A = ?**

A) 2 B) e C) 4

D) e^2 E) 6

[İSTANBUL ÜNİV. YÖS 2013]

2.

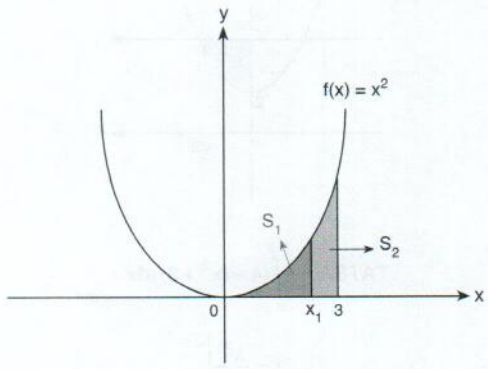


A) 1 B) 5 C) $\sqrt{2}$

D) $\sqrt{3}$ E) 2

[YILDIZ TEKNİZ ÜNİ. YÖS 2012]

4.



$S_2 = 2S_1$

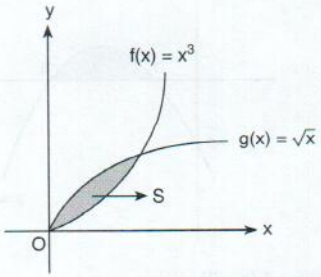
x_1 **= ?**

A) 1 B) 2 C) $\sqrt{2}$

D) $\sqrt[3]{9}$ E) $\sqrt[3]{5}$

[YILDIZ TEKNİZ ÜNİ. YÖS 2011]

4.



S = ? br²(u²)

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{5}{12}$ D) $\frac{7}{12}$ E) $\frac{11}{12}$

[YÖS 2010]

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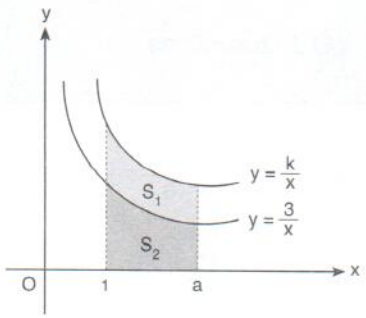
5.

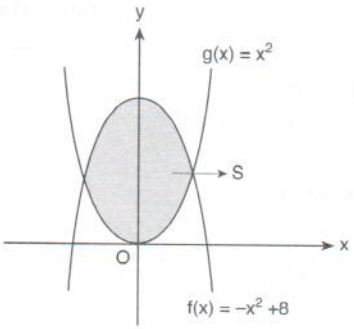


$S_1 = 8\ br^2$, $S_2 = 12\ br^2$ $\Rightarrow$ **k = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

[YÖS 2009]

6.

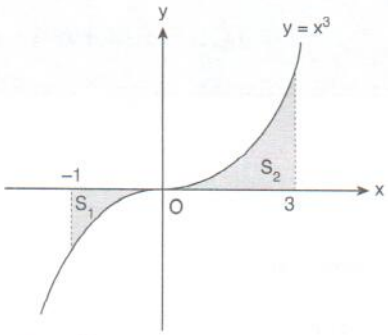


S = ? br^2

A) $\frac{32}{3}$ B) $\frac{64}{3}$ C) $\frac{81}{5}$ D) 16 E) 24

[YÖS 2008]

7.

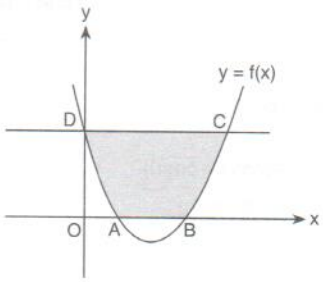


$S_1 + S_2$ = ? br^2

A) 16 B) 18 C) 20 D) $\frac{41}{2}$ E) $\frac{43}{2}$

[YÖS 2008]

8.



DC // AB, $f(x) = x^2 - 5x + 6$ $\Rightarrow$ **A(ABCD) = ? $br^2(u^2)$**

A) 16 B) 24 C) $\frac{52}{3}$ D) $\frac{62}{3}$ E) $\frac{82}{3}$

[YÖS 2007]

YANITLAR / ANSWERS

1. A	2. E	3. D	4. C	5. B	6. B	7. D	8. D

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1.

$$A = \int_1^{e^2} \frac{1}{x}\,dx$$

$$A = \ln x\Big|_1^{e^2} = \ln e^2 - \ln 1$$

$$= 2 - 0 = 2$$

Yanıt / The Answer is A

2.

$$S = \int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi}$$

$$= -(\cos\pi - \cos 0)$$

$$= -(-1-1) = 2$$

Yanıt / The Answer is E

3.

$$S_2 = \int_{x_1}^{3} x^2\,dx = \frac{x^3}{3}\Big|_{x_1}^{3} = 9 - \frac{x_1^3}{3}$$

$$S_1 = \int_0^{x_1} x^2\,dx = \frac{x^3}{3}\Big|_0^{x_1} = \frac{x_1^3}{3}$$

$$S_2 = 2\cdot S_1 \Rightarrow 9 - \frac{x_1^3}{3} = 2\cdot\frac{x_1^3}{3}$$

$$x_1^3 = 9 \Rightarrow x_1 = \sqrt[3]{9}$$

Yanıt / The Answer is D

4. $f(x) = g(x) \Rightarrow x^3 = \sqrt{x}$

$x = 0,\ x = 1$

$$S = \int_0^1 \left(\sqrt{x} - x^3\right)dx$$

$$= \frac{2x^{\frac{3}{2}}}{3} - \frac{x^4}{4}\Big|_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{4}\right) - 0 = \frac{5}{12}$$

Yanıt / The Answer is C

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5. $S_2 = \int_1^a \frac{3}{x}\,dx = 3\cdot\ln x\Big|_1^a = 12$

$\Rightarrow 3(\ln a - \ln 1) = 12$

$\ln a = 4 \Rightarrow a = e^4$

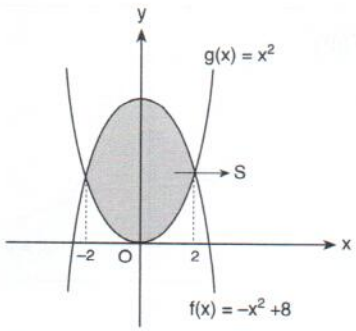
$S_1 + S_2 = \int_1^{e^4} \frac{k}{x}\,dx = k\cdot\ln x\Big|_1^{e^4} = 20$

$= k\cdot(\ln e^4 - \ln 1) = 20$

$= k\cdot 4 = 20 \Rightarrow k = 5$

Yanıt / The Answer is B

6.



$x^2 = -x^2 + 8$

$2x^2 = 8 \Rightarrow x^2 = 4 \Rightarrow x = \mp 2$

$S = \int_{-2}^{2} \left(-x^2 + 8 - x^2\right) dx$

$= \int_{-2}^{2} \left(-2x^2 + 8\right) dx = \frac{-2x^3}{3} + 8x\Big|_{-2}^{2}$

$= \left(\frac{-16}{3} + 16\right) - \left(\frac{16}{3} - 16\right) = 32 - \frac{32}{3} = \frac{64}{3}\ br^2(u^2)$

Yanıt / The Answer is B

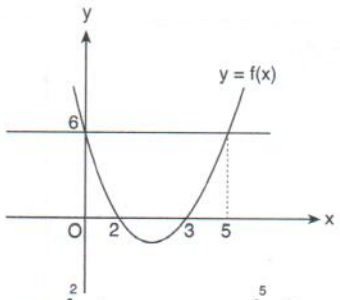
7. $S_1 = -\int_{-1}^{0} x^3\,dx = -\frac{x^4}{4}\Big|_{-1}^{0} = \frac{1}{4}$

$S_2 = \int_0^3 x^3\,dx = \frac{x^4}{4}\Big|_0^3 = \frac{81}{4}$

$S_1 + S_2 = \frac{82}{4} = \frac{41}{2}\ br^2(u^2)$

Yanıt / The Answer is D

8.



$A(ABCD) = 6\cdot 5 - \int_0^2 (x^2 - 5x + 6)\,dx - \int_3^5 (x^2 - 5x + 6)\,dx$

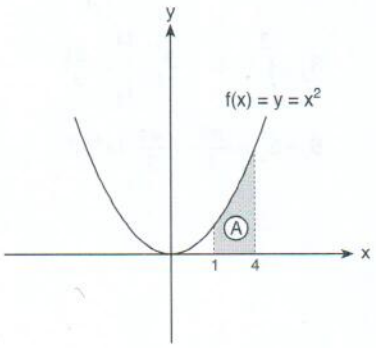
$= 30 - \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\Big|_0^2\right) - \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\Big|_3^5\right)$

$= 30 - \frac{14}{3} - \frac{14}{3} = \frac{62}{3}$

Yanıt / The Answer is D

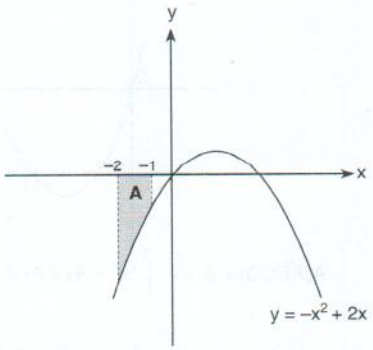
www.ztcprep.com

1. **A = ? br²(u²)**



Yanıt / The Answer is **21**

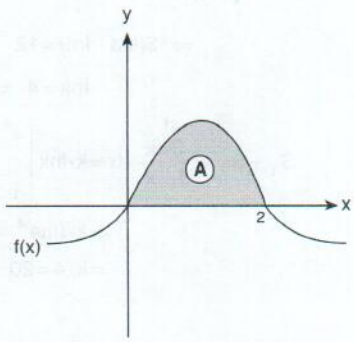
2. **A = ? br²(u²)**



Yanıt / The Answer is $\frac{16}{3}$

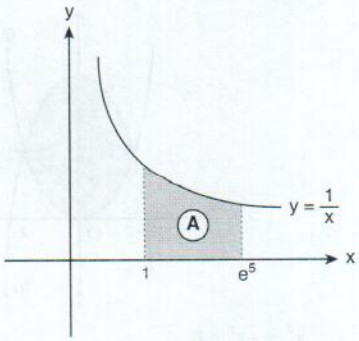
3. $f(x) = 2x^2 - x^3$

$\Rightarrow$ **A = ? br²(u²)**



Yanıt / The Answer is $\frac{4}{3}$

4. **A = ? br²(u²)**



Yanıt / The Answer is **5**

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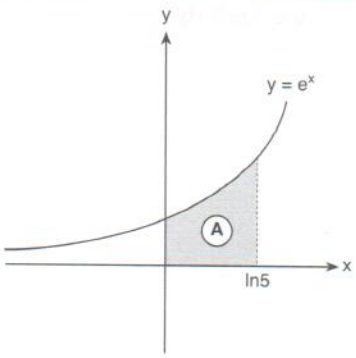
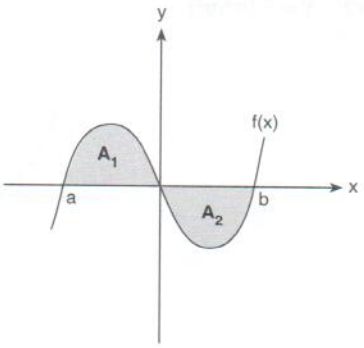
5. **A = ? br²(u²)**



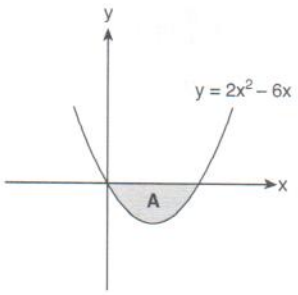
Yanıt / The Answer is **4**

6. $A_1 = 8$ br² (u²),
$A_2 = 3$ br² (u²),
$\Rightarrow \int_a^b f(x)\,dx = ?$



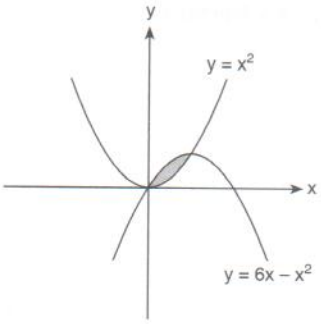
Yanıt / The Answer is **5**

7. **A = ? br²(u²)**



Yanıt / The Answer is **9**

8. **Taralı alan = ? br² (u²)**
The shaded area



Yanıt / The Answer is **9**

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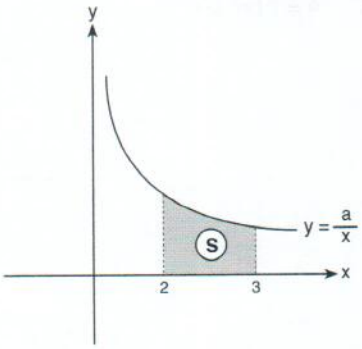
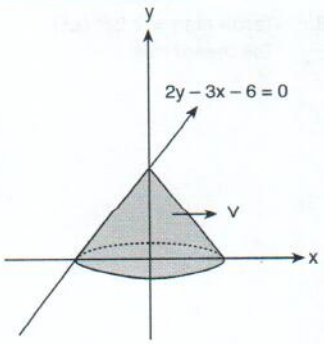
9. $S = \ln\frac{9}{4}\, br^2\,(u^2)$

$\Rightarrow a = ?$



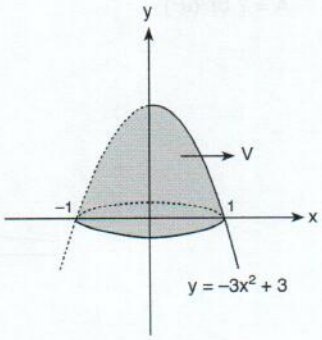
Yanıt / The Answer is **2**

10. $V = ?\ br^3(u^3)$



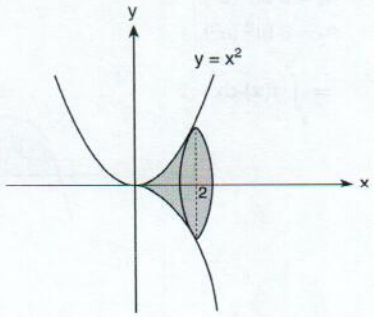
Yanıt / The Answer is 4π

11. $V = ?\ br^3(u^3)$



Yanıt / The Answer is $\frac{3\pi}{2}$

12. $V = ?\ br^3(u^3)$



Yanıt / The Answer is $\frac{32\pi}{5}$

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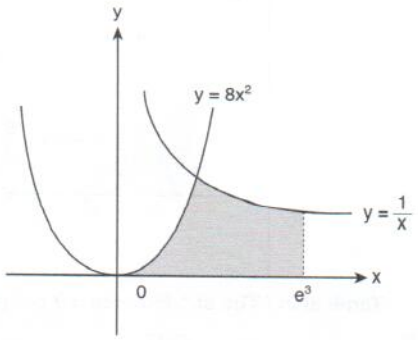
1.

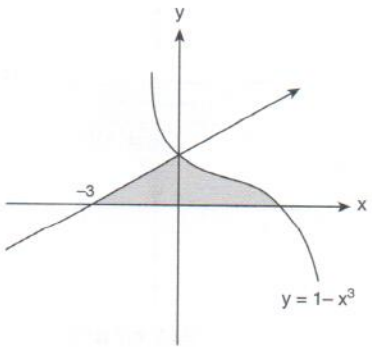


Taralı alan / The shaded area = ? br² (u²)

A) $\ln 2 + \frac{5}{3}$ B) $\ln 2 + \frac{7}{3}$ C) $\ln 2 + \frac{8}{3}$

D) $\ln 2 + \frac{10}{3}$ E) $\ln 2 + \frac{11}{3}$

2.

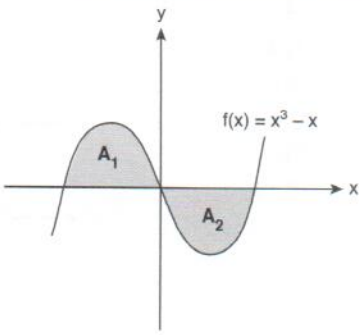


Taralı alan / The shaded area = ? br² (u²)

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{7}{2}$

D) $\frac{7}{4}$ E) $\frac{9}{4}$

3. $A_1 + A_2 = ?\ br^2\ (u^2)$



A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1

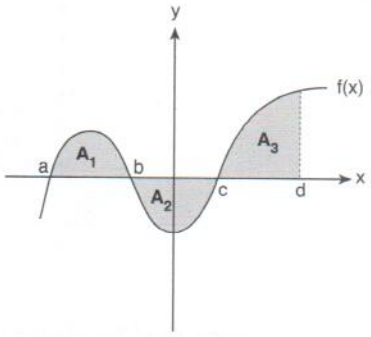
D) 2 E) 4

4. $A_1 = 8\ br^2\ (u^2)$,

$A_2 = 12\ br^2\ (u^2)$,

$\int_a^d f(x)\,dx = 10\ br^2\ (u^2)$

$\Rightarrow$ **$A_3 = ?\ br^2\ (u^2)$**



A) 8 B) 10 C) 12

D) 14 E) 16

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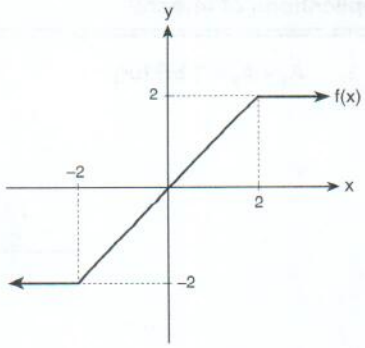
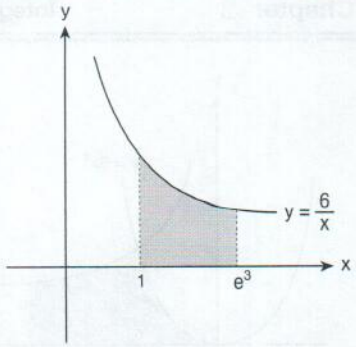
5. $\int_{-2}^{4} f(x)\,dx = ?$



A) 4
B) 6
C) 8
D) 10
E) 12

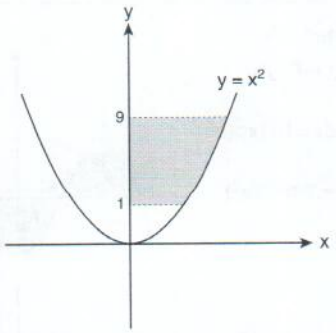
7.



Taralı alan / The shaded area = ? br² (u²)

A) 9 B) 12 C) 16 D) 18 E) 24

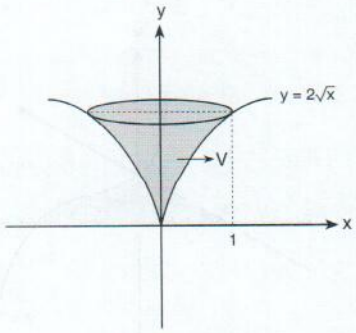
6.



Taralı alan / The shaded area = ? br² (u²)

A) $\frac{26}{3}$ B) $\frac{32}{3}$ C) $\frac{38}{3}$ D) $\frac{46}{3}$ E) $\frac{52}{3}$

8.



V= ? br³ (u³)

A) $\frac{\pi}{5}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{2}$ D) $\frac{2\pi}{3}$ E) $\frac{2\pi}{5}$

YANITLAR / ANSWERS							
1. D	2. E	3. B	4. D	5. C	6. E	7. D	8. E

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KAYNAKÇA

- Douglas Downing, Ph. D., **CALCULUS** , **Barron's**., New York. **2006**
- Frank Ayres JR., **THEORY and PROBLEMS of CALCULUS** , **Schaum's Outline Series**., New York. **1964**
- L BOSTOCK B. Sc. , S CHANDLER B.Sc. , E SIMITH B. Sc. , **STP MATHEMATICS 9A**, **Stanley Thornes Ltd**., Hong Kong., **1977**
- M.L. BOAS , **MATHEMATICAL METHODS in the PHYSICAL SCIENCES, Wiley International Edition**, New York – London., **1966**
- **METROPOL YAYINLARI** ÖSS HAZIRLIK DERGİLERİ **(20 Sayı)**, **2004 - 2005**
- **METROPOL YAYINLARI** ÖSS MATEMATİK SORU BANKASI **Birikim Matbaası**., Ankara., **2006**
- TAYFUR ŞANSAN, **DICTIONARY of MATH** (Açıklamalı, Örnekli Matematik Terimleri Sözlüğü), **Kiyap Yayın Pazarlama**., İstanbul. **1979**
- A.C. BURDETTE, **ANALYTIC GEOMETRY**, **Academic Press**, New York – London., **1971**
- **Milli Eğitim Bakanlığı 9 ve 10. Sınıf Matematik Ders Kitapları**

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BIBLIOGRAPHY

- Douglas Downing, Ph. D., **CALCULUS** , **Barron's**., New York. **2006**
- Frank Ayres JR., **THEORY and PROBLEMS of CALCULUS** , **Schaum's Outline Series**., New York. **1964**
- L BOSTOCK B. Sc. , S CHANDLER B.Sc. , E SIMITH B. Sc. , **STP MATHEMATICS 9A**, **Stanley Thornes Ltd**., Hong Kong., **1977**
- M.L. BOAS , **MATHEMATICAL METHODS in the PHYSICAL SCIENCES, Wiley International Edition**, New York – London., **1966**
- **METROPOL YAYINLARI** ÖSS HAZIRLIK DERGİLERİ **(20 Sayı)**, **2004 - 2005**
- **METROPOL YAYINLARI** ÖSS MATEMATİK SORU BANKASI **Birikim Matbaası**., Ankara., **2006**
- TAYFUR ŞANSAN, **DICTIONARY of MATH** (Açıklamalı, Örnekli Matematik Terimleri Sözlüğü), **Kiyap Yayın Pazarlama**., İstanbul. **1979**
- A.C. BURDETTE, **ANALYTIC GEOMETRY**, **Academic Press**, New York – London., **1971**
- **Milli Eğitim Bakanlığı 9 ve 10. Sınıf Matematik Ders Kitapları**

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